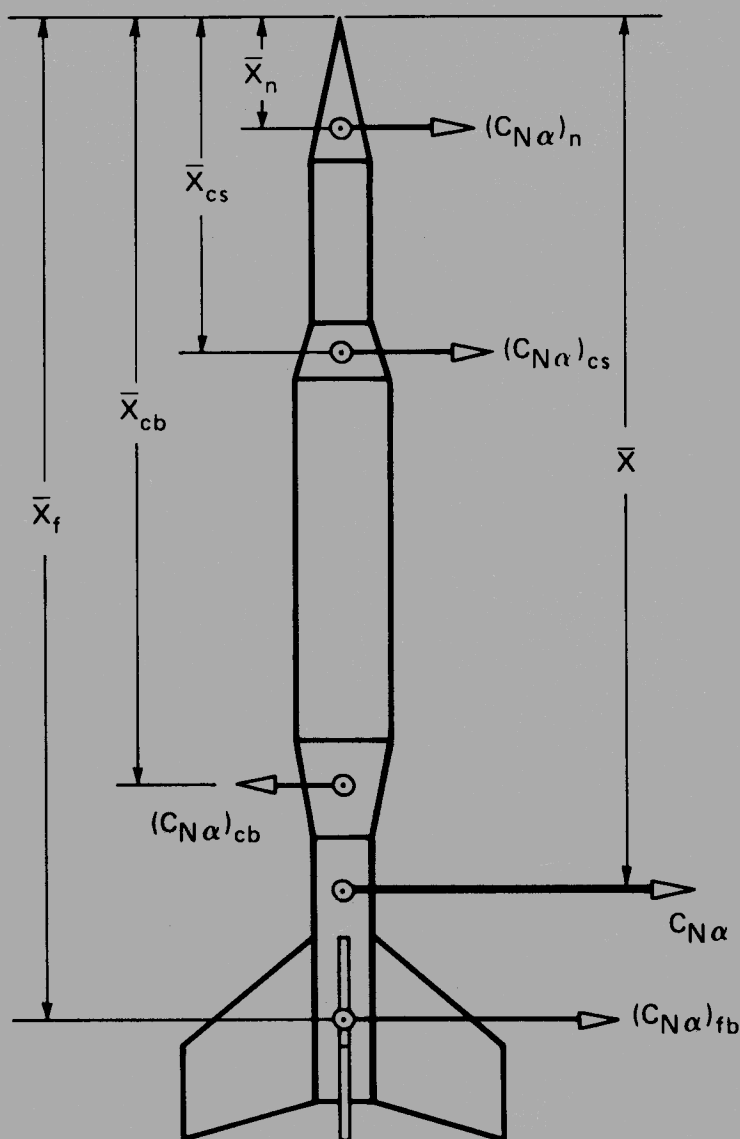


33

TECHNICAL INFORMATION REPORT



CALCULATING THE CENTER OF PRESSURE

OF A MODEL ROCKET



**CALCULATING
THE CENTER OF
PRESSURE
OF A MODEL
ROCKET**

WRITTEN BY

**JAMES
BARROWMAN**

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PROLOGUE

This report presents an easy to use method for accurately calculating the exact center of pressure of a subsonic rocket flying at small angles-of-attack.

The basic equations for determining the center of pressure were theoretically derived by the author for a research and development project which was presented at the National Association of Rocketry Annual Meet (NARAM-8) in August of 1966. The equations in this report were subsequently published by the Educational Services Offices of NASA in the widely distributed Information For You pamphlet entitled, "Calculating the Center of Pressure of a Rocket".

The report herein is a significant improvement over the NASA pamphlet in that all the complex equations have been reduced to chart form and all the formerly prevalent math operations such as squaring and taking square roots have been completely eliminated. The number of arithmetic operations and the corresponding chances of making mathematical mistakes as a result have been greatly reduced.

While some insight into center of pressure, center of gravity and stability criteria have been included in this report a more thorough presentation treating these subjects separately has been prepared and is available as CENTURI's Technical Information Report TIR-30.

INTRODUCTION

For your model rockets to have safe, predictable flights, they must be stable. A model rocket will be stable only if its center of pressure is behind its center of gravity. Before you fly any model rocket, you must be sure that it is stable. This obviously boils down to finding the locations of the rocket's center of gravity (C.G.) and center of pressure (C.P.).

If you are designing a rocket and want to find its C.G. location without building it, you must calculate the C.G. from a knowledge of the weights of its component parts (body tube, nose cone, engine, etc.). This, too, is relatively easy. A technique for predicting a rocket's C.G. location before it actually is built is given in Appendix A. The center of gravity (C.G.) is important to stability not because the rocket balances there; but because when the rocket wobbles in free flight, it will rotate only about the center of gravity (C.G.).

CENTER OF GRAVITY

The center of gravity of a rocket is the point at which all the weight of the rocket seems to be concentrated. That is, there is as much weight distributed ahead of the rocket's center of gravity as there is behind it. Another name for the C.G. is the rocket's balance point. Finding the center of gravity of a rocket that is already built is very simple. It involves balancing the rocket with the engine and parachute inserted on a string and marking the location of the string where the rocket stays level.

FIGURE 1

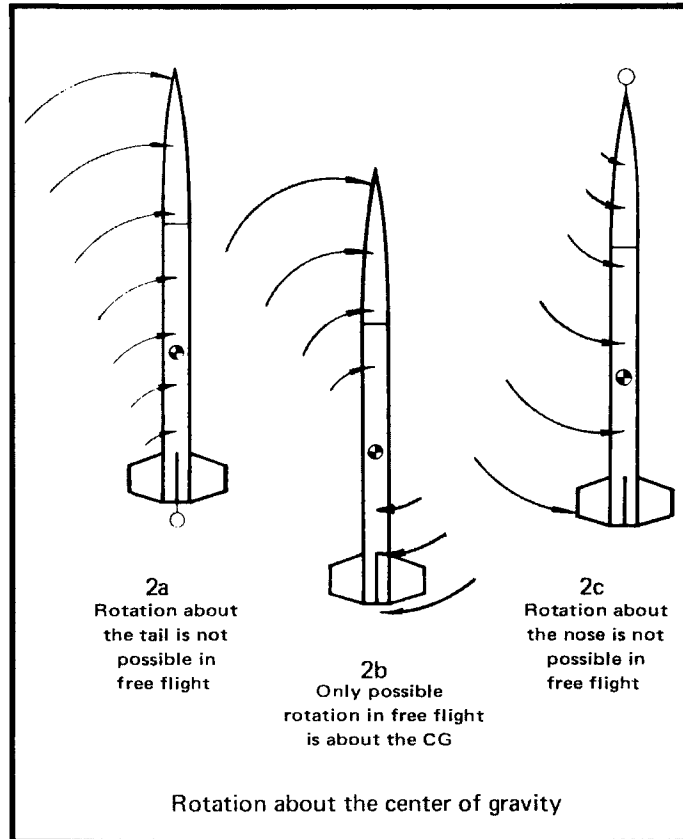
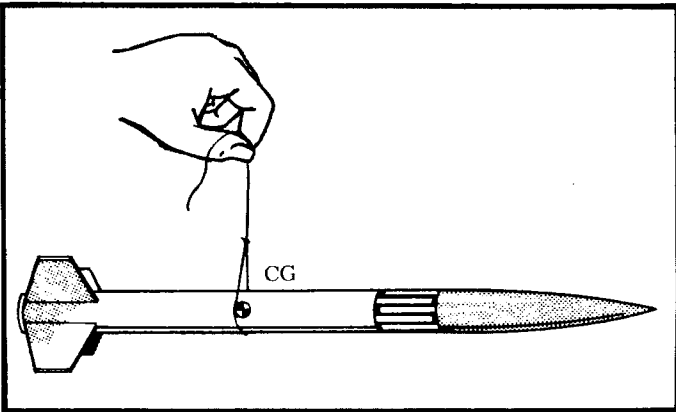



FIGURE 2

The engine used, of course, should be the most powerful with the longest delay that is ever expected to be flown in the rocket. This results in the maximum weight at the back and shifts the C.G. back as far as possible.

Note that the symbol  is used to represent the center of gravity on drawings.

CENTER OF PRESSURE

The center of pressure is similar to the center of gravity except that the forces involved are the air pressure forces acting on the rocket while it is flying. A formal definition is then -- The center of pressure of a rocket is the point at which all the air pressure forces on the rocket seem to be concentrated. That is, there is as much air pressure force distributed on the rocket ahead of the center of pressure as there is behind it. In the figure below, the size of the air pressure forces that are distributed over the length of the rocket and on the fins are represented by the length of the arrows along the side of the rocket.

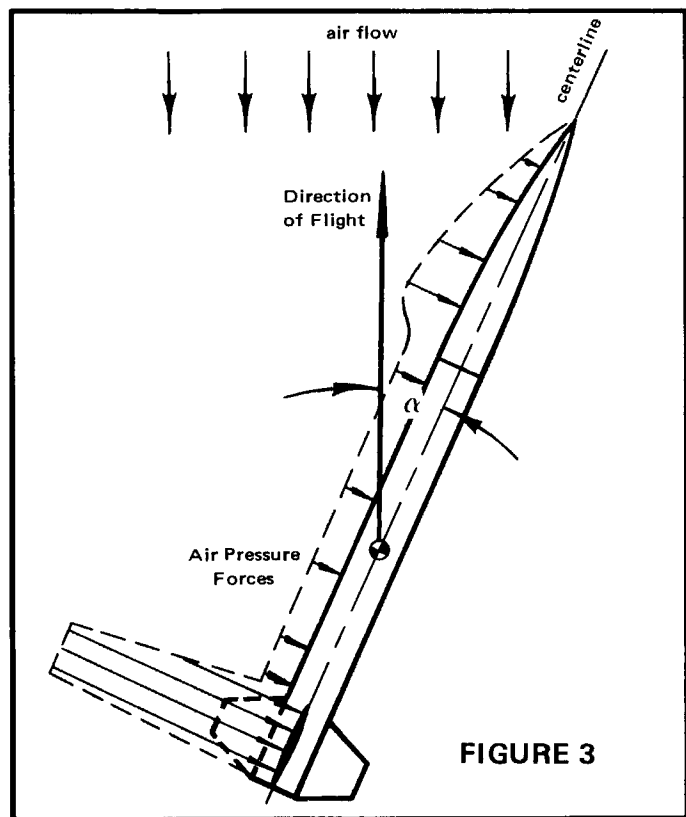


FIGURE 3

As you can see, the rocket is at an angle (highly exaggerated in the above figure) to the direction it is flying. As a result, it is at an angle to the direction of the air flow over it. This is called the angle-of-attack and is represented by the Greek letter alpha, α .

As long as the rocket isn't disturbed it will fly straight into the air flow. Now if the rocket is disturbed say by a wind gust, thrust misalignment, or cocked fin, then it will fly at an angle-of-attack. Then a stable model rocket will make continual corrections during its flight as it attempts to return to zero, just as a person manually makes constant corrections to maintain a straight path when driving a car or riding a bicycle.

Notice that the air pressure forces pictured above are all perpendicular to the rocket centerline. These are called the normal (mathematical term meaning perpendicular) forces acting on the rocket. There are also axial forces on the rocket which act parallel to the centerline (or axis) of the rocket. Although the axial air pressure forces (commonly called aero-

dynamic drag) are important in calculating the altitude performance of a rocket, they are not at all important in determining its center of pressure and resulting stability. If you are also interested in the affects of the axial air pressure forces on the flight of your rockets, we suggest that you study CENTURI's TIR-100 on model rocket performance.

The distribution of normal forces shown above represents how the forces actually act on a typical model rocket that is flying at an angle-of-attack (α).

However, since there is a point (the center of pressure) along the length of the rocket where there is as much normal force ahead of it as there is behind it, all the forces that are distributed along the length of the rocket can be added up into a single force that acts only at the center of pressure. The symbol \odot will be used in the drawings to represent the location of the center of pressure.

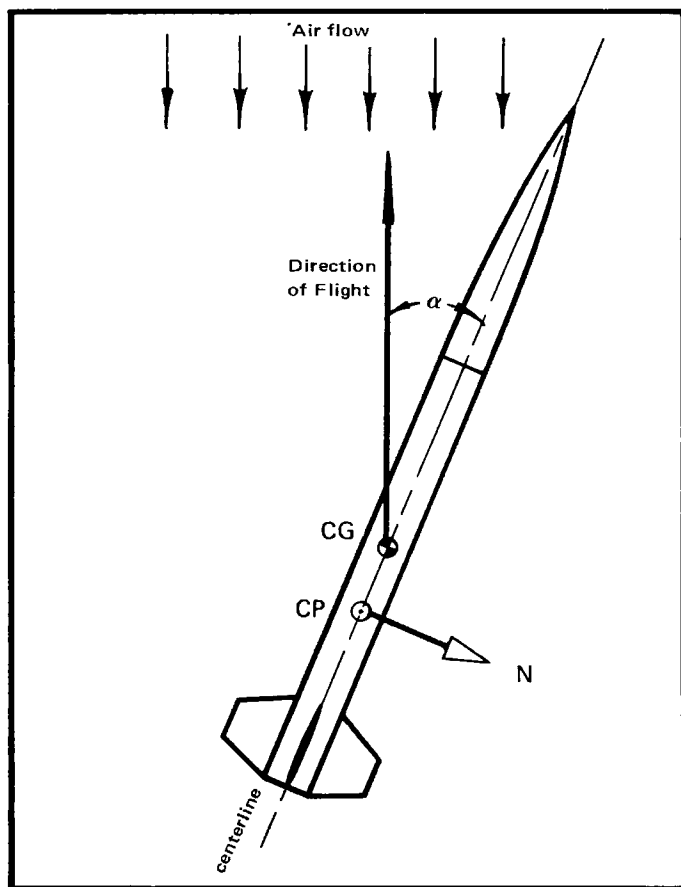


FIGURE 4

This sum of all the distributed normal forces is called the normal force and is represented by the letter N. The normal force, N, is the force which brings the rocket back to zero angle-of-attack and it will be discussed more thoroughly in a later section.

The angle-of-attack at which a rocket flies has a strong effect on the size and shape of the normal force distribution on the rocket. In turn, the shape of the normal force distribution determines the C.P. location. It has been found that the C.P. moves forward as the angle-of-attack increases. This fact is very important since it can affect a rocket's stability.

STABILITY CRITERIA


Knowing that a rocket is stable is not enough. You must also know how much stability the rocket has. The farther the rocket's C.P. is behind its C.G. the more stable the rocket will be. This is because the aerodynamic normal force which is pushing at the C.P. location has a longer lever arm distance to the rocket's pivot point (the C.G.) and so can return it to zero α proportionally faster. This distance between center of pressure (C.P.) and the center of gravity (C.G.) is called the static margin.

Thus, the larger its static margin the more stable it will be providing, of course, that the rocket's C.P. is behind its C.G.

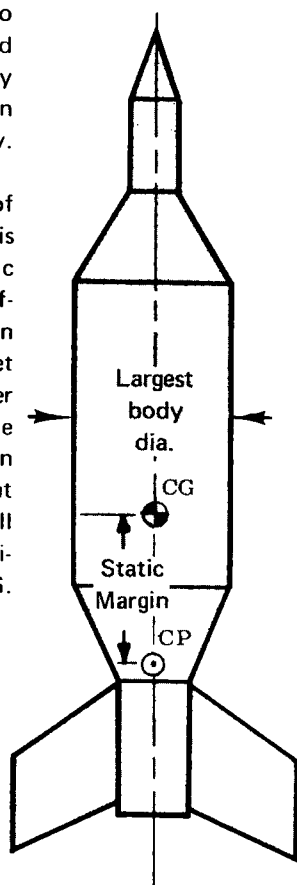
The importance of the effect of the angle-of-attack (α) on C.P. location now becomes apparent. As a rocket's angle-of-attack increases; the C.P. moves forward; the static margin decreases; and therefore the rocket becomes less stable. It is possible that the C.P. might even move forward of the C.G. causing the bird to become unstable and do flip-flops.

Obviously, you want your rockets to fly at as small an angle-of-attack as possible. Not only does it insure stability but it improves altitude performance as well, since aerodynamic drag increases proportionally as angle-of-attack increases. Drag is a minimum at zero angle-of-attack. At first glance, then, it appears that the maximum static margin is desirable. However, there is one additional factor to be considered. If a rocket has a high static margin it can actually be too stable for windy-day flying. The flight path will not be anywhere near vertical but instead the rocket will consistently arc over and head into the wind. This phenomena is called weathercocking and the reasons why it occurs are covered in detail in CENTURI's TIR-30 on stability.

It turns out that a good rule of thumb developed over the years is simply to be sure that the static margin is at least equal to and preferably is just somewhat greater than the largest diameter of the rocket as shown. Model rocketeers refer to this reference diameter as the caliber of their rocket and so when you hear someone talking about "one caliber stability", you will know that the C.P. is one maximum body diameter behind the C.G.

FIGURE 5 

Largest body diameter = Static Margin



One caliber static margin stability insures good safe flight characteristics and at the same time it minimizes the effects of crosswinds and gusts.

The remainder of this report is devoted to presenting an accurate method for finding the center of pressure of any rocket when it is flying near zero angle-of-attack (α). The static margin of your original model rocket designs can then be verified with confidence prior to flight. The additional effort expended in performing a center of pressure analysis is flight insurance and protection for the total investment in time, effort and money spent in the construction of that super bird.

3. ELEMENTS OF THE THEORETICAL CENTER OF PRESSURE CALCULATIONS

ASSUMPTIONS

The assumptions that we used in finding any equations are very important since the assumptions indicate exactly what the mathematical equations can and cannot physically simulate. The basic assumptions used in deriving the equations in this report are as follows:

- 1) The angle-of-attack of the rocket is near zero (less than 10^0).
- 2) The speed of the rocket is much less than the speed of sound (not more than 600 feet per second).
- 3) The air flow over the rocket is smooth and does not change rapidly.
- 4) The rocket is thin compared to its length.
- 5) The nose of the rocket comes smoothly to a point.
- 6) The rocket is an axially symmetric rigid body.
- 7) The fins are thin flat plates.

Although some of the above assumptions seem quite restrictive, the vast majority of model rockets conform to these requirements. However, before analyzing your rocket, you must be sure that it is not one of the few that violate these assumptions. For example, you cannot analyze a boost glider using the equations in this report. A boost glider violates assumptions 3), 4), 5), 6), and at times 1).

NORMAL FORCE TERMINOLOGY

At angles-of-attack near zero (assumption 1) the normal force acting on a rocket depends on the shape of the rocket, the density of the air, the velocity, the size of the rocket, and the angle-of-attack. In equation form:

$$N = C_{N\alpha} \frac{1}{2} \rho V^2 \alpha A_r$$

Where:

- N is the total normal air pressure force acting on the rocket.
- $C_{N\alpha}$ is the dimensionless coefficient that accounts for the shape of the rocket.
- ρ the Greek letter rho is the density of the air.
- V is the rocket's velocity or speed. V^2 means velocity is squared or multiplied by itself.
- A_r is a reference area that indicates the size of the rocket. The reference area generally used is the cross-sectional area at the base of the nose.
- α the Greek letter alpha is the angle-of-attack.

It can be seen from the above formula that the total normal aerodynamic force (N) is larger, as expected, for larger rockets because the reference area (A_r) will be correspondingly larger. Also we note from the formula that when the angle-of-attack (α) is zero that there is no normal force (N). Similarly, the normal force (N), when α is 2 degrees, is twice as large than if α was 1 degree. Thus at a given velocity, as the angle-of-attack becomes larger the tendency of a stable rocket to realign itself to zero angle-of-attack is increased.

Another important influence on the magnitude of the normal force is the velocity of the rocket. The normal force (N) is seen to be proportional to the square of velocity (V^2). This means that doubling the velocity gives a rocket four times the force tending to return it to zero from a given disturbed angle-of-attack ($2^2 = 4$), while tripling the velocity gives nine times the correcting force ($3^2 = 9$).

This is the reason why model rockets are designed to really scoot off the launch pad and never lift off in the grand stately slow grace of a Saturn booster. Once the model rocket leaves the launch rod it is in free flight and the aerodynamic normal forces acting at the C.P. must at this time be of reasonable magnitude to provide adequate pivoting about the C.G. Obviously, the higher the rocket's velocity as it leaves the launch rod the better. This, of course, presumes the rocket is stable and has the C.P. at least one maximum body diameter (caliber) behind the C.G.

Both the angle-of-attack and the velocity are outside influences on the rocket. $C_{N\alpha}$, on the other hand, indicates the influence that the rocket configuration itself has on the normal force.

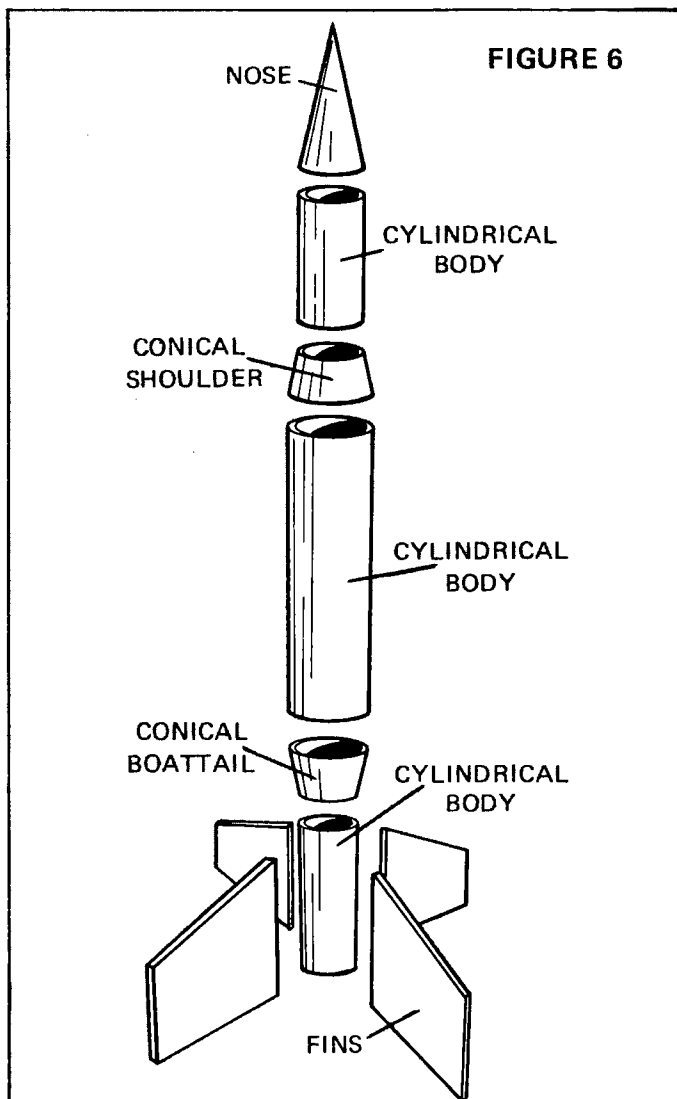
Our concern is in verifying that the rocket has this static margin by actually calculating the center of pressure using known $C_{N\alpha}$'s for known shapes. For velocities much less than the speed of sound (assumption 2), $C_{N\alpha}$ depends only on the shape of the rocket. Since the calculation of the rocket's C.P. is a direct effect of the shape of the rocket being analyzed, then $C_{N\alpha}$ is essential and not N .

Appendix D shows why $C_{N\alpha}$ validly represents the normal force in the center of pressure calculations even though it directly is only one of the factors in determining normal forces

For simplicity and convenience $C_{N\alpha}$ will be referred to as the normal force acting on the rocket in the rest of the report.

ANALYSIS BY REGIONS

In order to determine the center of pressure of a rocket, the rocket is divided into regions and each region is analyzed separately. Then the separate results are combined to obtain the value for the entire rocket. The particular set of equations in this paper is for a rocket that can be divided as shown in Figure 6. If there is more than one conical shoulder, conical boattail, and/or set of fins on the rocket, these should also be analyzed separately and then be included in the combination calculations.



CENTER OF PRESSURE LOCATIONS

In order to be meaningful, the center of pressure locations of all the regions of the rocket must be measured from the same reference point on the rocket. In this report the common reference point is the tip of the nose as shown below. For consistency, the C.G. should also be measured from the nose tip. Then simply subtracting the C.G. from the C.P. gives the dimension which is compared to the maximum body diameter for determining stability.

REGION LOCATIONS

In order to measure the C.P.'s of the different regions from the nose tip, the equations include the distance between the different portions and the nose tip. The definition of the symbols for the locations of the conical shoulder, conical boattail, and fins are shown in Figure 8. Notice that there are no bars above these X's. A bar is used above an X only when that X is the total distance of each region's C.P. to the reference nose tip. The small extra distance to the actual C.P. of a component part is denoted by ΔX (pronounced delta X) with the proper subscript.

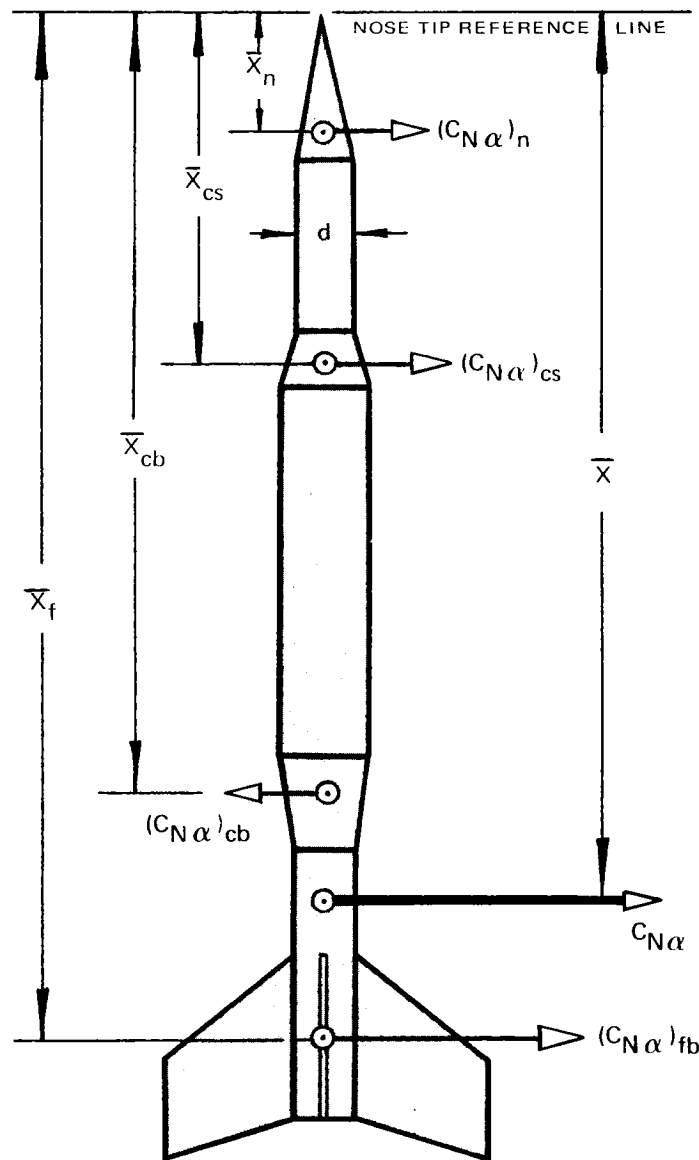
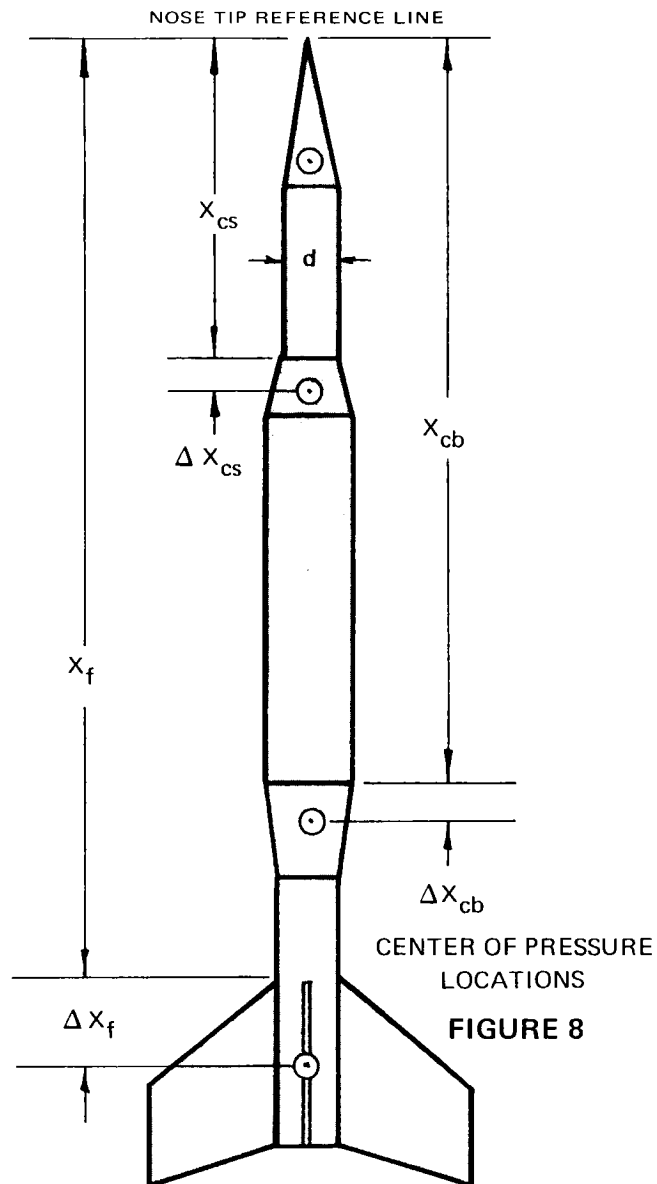


FIGURE 7
AERODYNAMIC NORMAL FORCES ACTING ON THE DIFFERENT REGIONS OF A TYPICAL ROCKET

SUBSCRIPT NOTATION

The subscripts added to $C_N \alpha$ and \bar{X} (pronounced x bar) indicate to which part of the rocket the symbol refers. For example, the force on the nose is indicated by $(C_N \alpha)_n$. If a symbol has no subscript, then it refers to the entire rocket. The subscripts used in this report and their meanings are as follows:

- cb = Conical Boattail
- cs = Conical Shoulder
- f = Fins
- fb = Fins in the presence of the body
- n = Nose



CENTER OF PRESSURE LOCATIONS

FIGURE 8

The equations for all the region $C_{N\alpha}$'s and C.P.'s were derived using calculus. No attempt has been made to present the actual derivations of equations as it requires a familiarity with the differentiation and integration processes of calculus. Instead, the remainder of the report simply presents the results and how to apply them to your model rocketry use.

Rocketeers who have had advanced mathematics courses should find enough information to satisfy their curiosity in the following list of references which were actually used in deriving the theoretical center of pressure equations.

- 1) Shapiro, A. H.; The Dynamics and Thermodynamics of Compressible Fluid Flow, Vol. 1; Ronald; New York; 1953.
- 2) Mayo, E. E.; Cone Cylinder and Ogive Cylinder Geometric and Mass Characteristics; Memo to Code 721.2 Files at NASA GSFC; 20 Sept. 1965.
- 3) Pitts, W. C.; Nielsen, J. N.; Kaatari, G. E.; Lift and Center of Pressure of Wing-Body-Tail Combinations at Subsonic, Transonic, and Supersonic Speeds; NACA TR-1307; G. P. O., Washington, D.C.; 1953.
- 4) Miles, J. W.; Unsteady Supersonic Flow; A. R. D. C.; Baltimore; 1955: Section 12.4.
- 5) McNerney, J. D.; Aerobee 350 Wind Tunnel Test Analysis; Space General Corporation; El Monte, Calif; January 1963.
- 6) Hoerner, Dr. S. F.; Fluid-Dynamic Drag; Midland Park, New Jersey; 1965.

4. EQUATIONS FOR FINDING THE CENTER OF PRESSURE OF THE ROCKET

The equations for each separate region are presented in the following order: nose, conical shoulder, conical boattail, and fins. The final section shows how to combine the results from all the regions into a value of $C_{N\alpha}$ and \bar{X} for the entire rocket.

NOSE

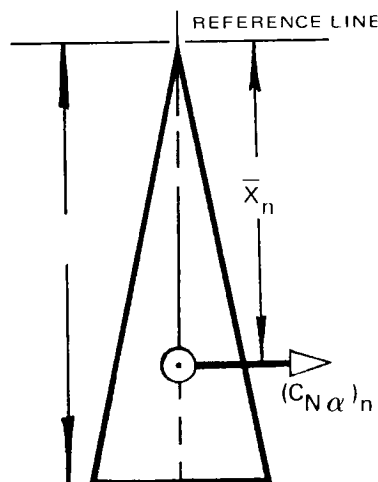
In general, the normal force $(C_{N\alpha})_n$ on the nose is identical for all shapes and always has the value:

$$(C_{N\alpha})_n = 2$$

On the other hand, the center of pressure (C.P.) location on the nose varies with each different nose shape.

The distance from the tip of the nose to the center of pressure location of a cone-shaped nose is,

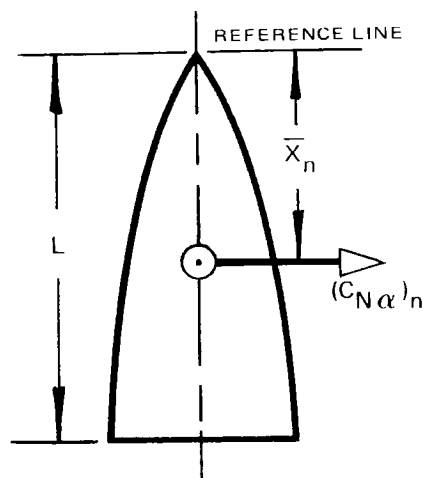
$$\bar{X}_n = \frac{2}{3}L$$



OGIVE NOSE

The distance from the tip of the nose to the center of pressure location of ogive-shaped nose is,

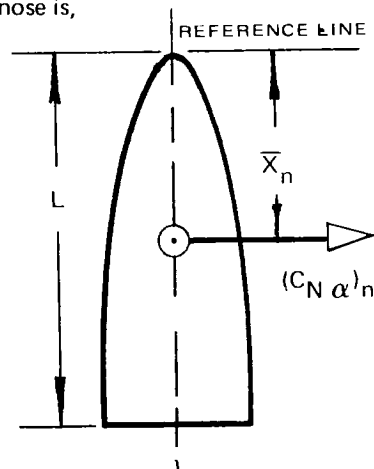
$$\bar{X}_n = .466 L$$



PARABOLIC NOSE

The distance from the tip of the nose to the center of pressure location of a parabolic nose is,

$$\bar{X}_n = \frac{1}{2}L$$

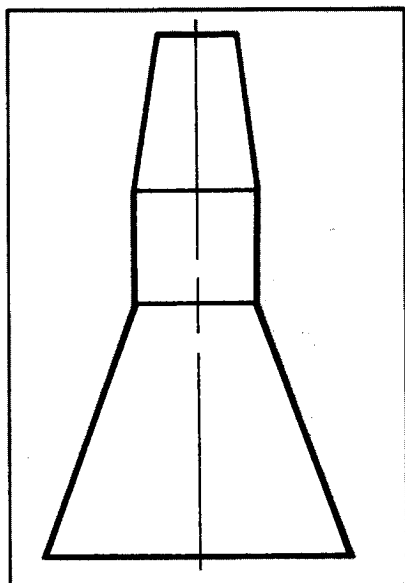


SPECIAL SHAPE NOSES

In addition to the basic shapes, there are some special frequently used nose shapes that warrant discussion.

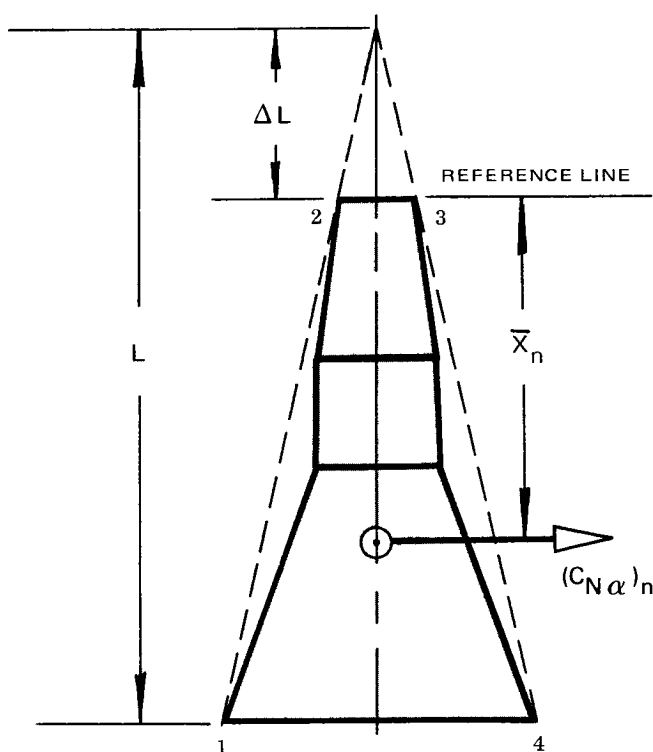
A rounded-off ogive has the same C.P. location as a parabolic nose.

The mercury capsule shape,



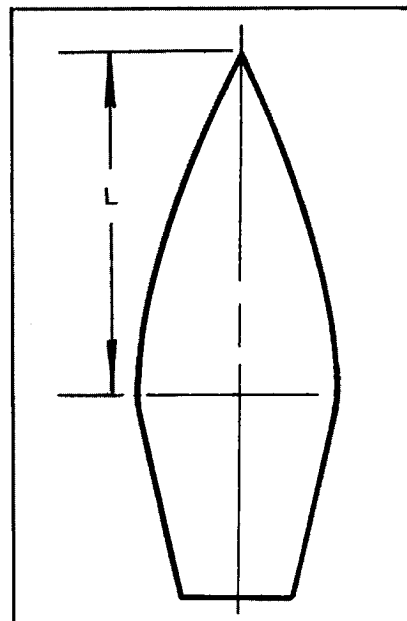
violates assumption 5 and therefore cannot be directly analyzed.

However, it has been found that such a shape can be simplified by drawing an outline of the shape and then connecting and extending its four outer-most corners (1, 2, 3, 4) to make an equivalent cone (dotted line). This technique was actually used in the initial preliminary design procedure for the Mercury, Gemini, and Apollo projects.



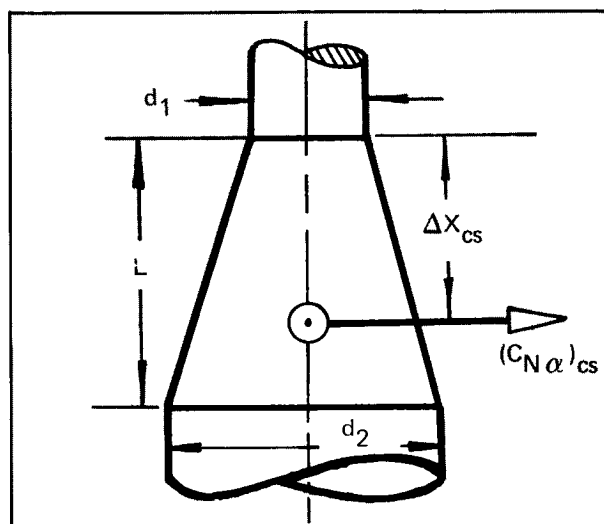
The equivalent cone is then analyzed using the equations for a cone. Remember, though, that the \bar{X}_n must be calculated using the length, L , and then the length ΔL (see drawing) must be subtracted from it to give the value of \bar{X}_n from the true front of the nose. This technique works for any nose shape that is similar to the Mercury capsule shape. The Gemini capsule and several other CENTURI nose-cone shapes fall into this category. CENTURI's RECRUITER kit, which has a similar nose-cone shape, will be analyzed as one of the examples in Section 8.

Another special shape is the HONEST JOHN nose cone.



Up to the largest diameter (shown by vertical dotted line), this shape is an ogive. The portion of the nose cone behind the thickest diameter can subsequently be analyzed as a conical boattail. Essentially, only the ogive portion should be considered to be the nose. This ogive is analyzed using the equation given for an ogive, using the dimension L shown as the nose length. Also the dimension "d" shown in Figure 7 should be the diameter at the base of the ogive.

CONICAL SHOULDER



The force on a conical shoulder is,

$$(C_{N\alpha})_{cs} = 2 \left[\left(\frac{d_2}{d}\right)^2 - \left(\frac{d_1}{d}\right)^2 \right]$$

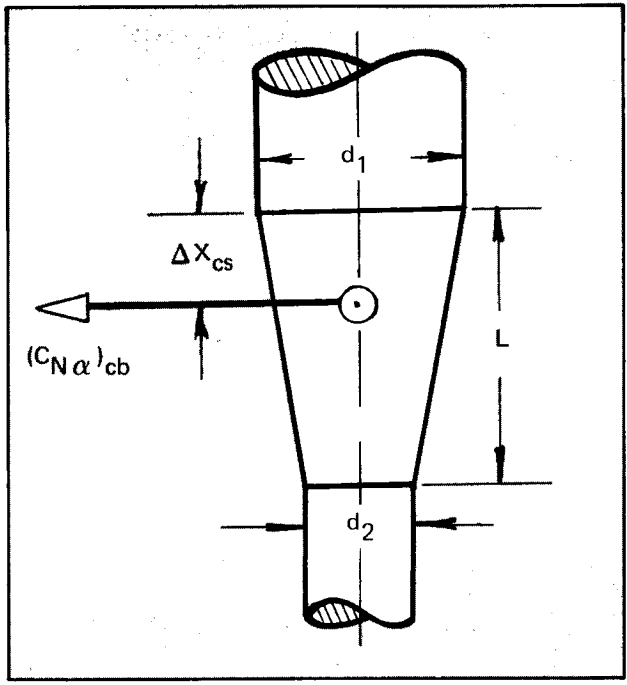
where "d" is the diameter at the base of the nose.

The center of pressure location of a conical shoulder is,

$$\bar{X}_{cs} = X_{cs} + \Delta X_{cs} = X_{cs} + \frac{L}{3} \left[1 + \frac{1 - \frac{d_1}{d_2}}{1 - \left(\frac{d_1}{d_2}\right)^2} \right]$$

where X_{cs} is the distance from the tip of the nose to the front of the conical shoulder (see Figure 8).

CONICAL BOATTAIL



The force on a conical boattail is,

$$(C_{N\alpha})_{cb} = 2 \left[\left(\frac{d_2}{d}\right)^2 - \left(\frac{d_1}{d}\right)^2 \right]$$

where "d" is the diameter at the base of the nose. Note that this is the same equation for the conical shoulder, however the force on a conical boattail comes out negative.

The center of pressure location of a conical boattail is,

$$\bar{X}_{cb} = X_{cb} + \Delta X_{cb} = X_{cb} + \frac{L}{3} \left[1 + \frac{1 - \frac{d_1}{d_2}}{1 - \left(\frac{d_1}{d_2}\right)^2} \right]$$

where X_{cb} is the distance from the tip of the nose to the front of the conical boattail (see Figure 8).

For small angles-of-attack less than 10 degrees, the force on any cylindrical body portion is so small it can be neglected as can be seen from the following figure of reference 6.

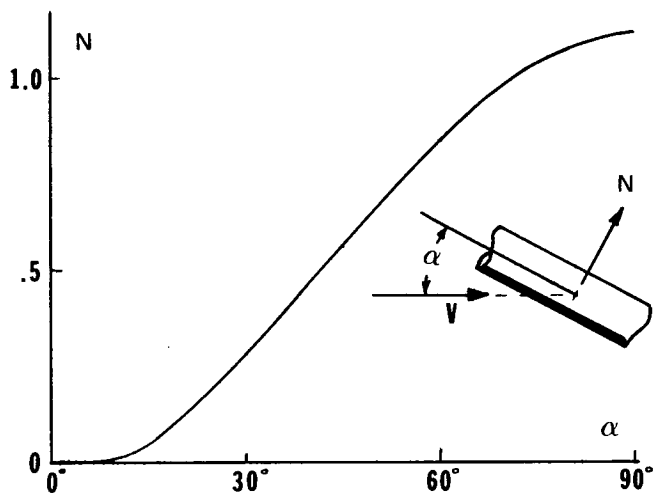
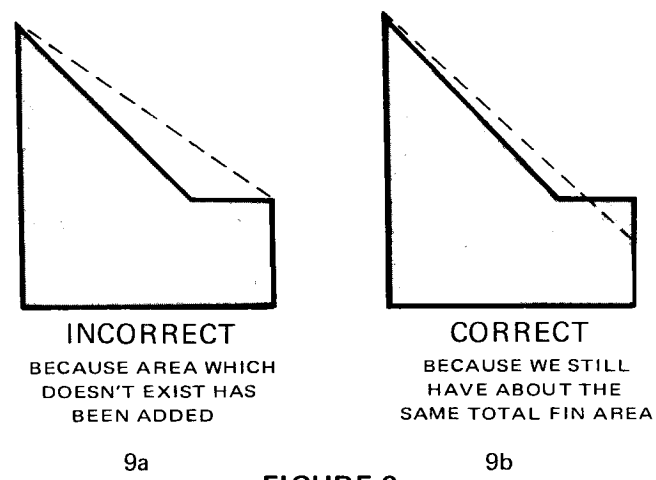


Figure 3-18 of reference 6 gives the normal force acting on circular cylinders, wires, and cables inclined to the air flow direction. This data was collected from wind tunnel tests performed primarily in the years 1918 and 1919. It is quite interesting to realize that our Space Age hobby of Model Rocketry is benefiting by engineering work done specifically to improve World War I Biplane performance.

FINS

Any fin that is not too complicated in shape may be simplified into an idealized shape that has only four straight line edges with the root and the tip parallel. It is very important when simplifying complicated fin shapes to be sure that the idealized shape has about the same or slightly less area as the actual fin. For example, Figure 9a shows an improperly idealized fin shape, while Figure 9b shows a correct simplification.



INCORRECT
BECAUSE AREA WHICH DOESN'T EXIST HAS BEEN ADDED

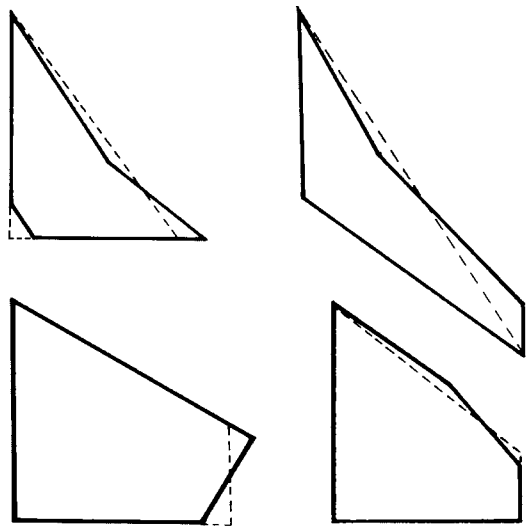
CORRECT
BECAUSE WE STILL HAVE ABOUT THE SAME TOTAL FIN AREA

9a

9b

FIGURE 9

The solid lines are the actual fin shape and the dotted lines show the changes made to simplify the shape. Some additional examples of properly idealized fin shapes are shown below:



OTHER IDEALIZED FIN SHAPES

FIGURE 10

The idealized fin shape and the dimensions associated with it are shown in Figure 11.

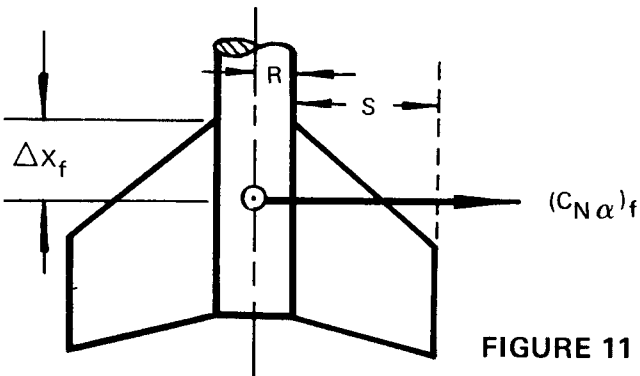
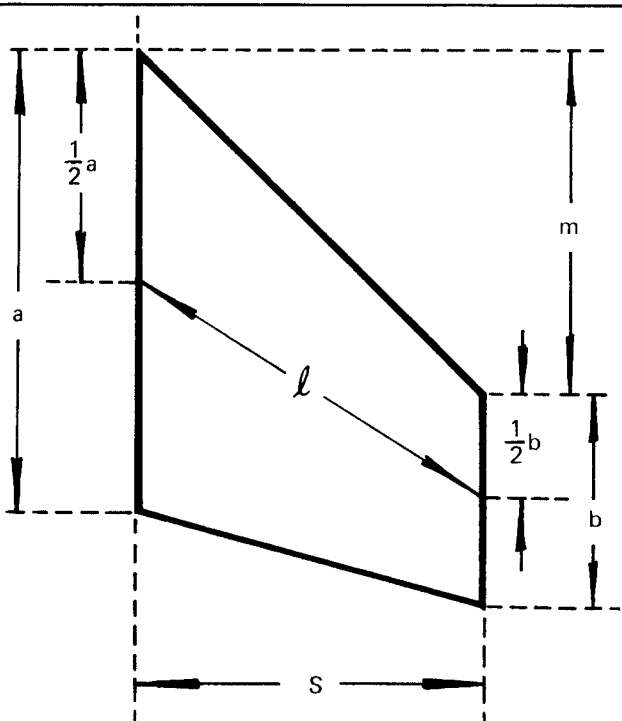


FIGURE 11

Of course, Figure 11 shows a generalized fin shape. Special cases are handled by proper use of the given dimensions. For example, a triangular fin would have $b = 0$. Similarly, a rectangular fin has $a = b$; $l = S$; and $m = 0$.

In terms of the dimensions, the force on the fins of a rocket having n fins is,

$$(C_N \alpha)_f = \frac{4n \left(\frac{s}{d}\right)^2}{1 + \sqrt{1 + \left(\frac{2l}{a+b}\right)^2}}$$

Where the number of identically shaped fins, n , can only be 3, 4, or 6. If your rocket has any other number of fins, these equations cannot be used. For multistage models, n refers to the number of fins on each stage. Of course, the fins on each stage must be analyzed separately and included separately in the combination equations.

FIN INTERFERENCE FACTOR

In addition, this air flow over the fins is influenced somewhat by the air flow over the body section to which the fins are attached. To account for this, the fin force for either 3 or 4 fins is multiplied by an interference factor,

$$K_{fb} = 1 + \frac{-R}{S + R} \quad (\text{For } n = 3 \text{ or } 4)$$

where "r" is the radius of the body between the fins and "s" is the fin semi-span shown in Figure 11. (Remember the subscript fb meant fins in presence of body). For 6 fins, however, the interference between the fins themselves cancels half the effect of the fins being attached to the body. In this case,

$$K_{fb} = 1 + \frac{.5R}{S + R} \quad (\text{For } n = 6)$$

where "r" is the radius of the body between the fins and "s" is shown in Figure 11. The total force on the tail in the presence of the body is then:

$$(C_N \alpha)_{fb} = K_{fb} (C_N \alpha)_f$$

The fin center of pressure is located in the same place on any two fins of the same size and shape. Since all the fins on a particular stage of a rocket are the same size and shape, the center of pressure location of the tail does not depend on the number of fins.

$$\begin{aligned} \bar{X}_f &= X_f + \Delta X_f \\ &= X_f + \frac{m(a+2b)}{3(a+b)} + \frac{1}{6} \left(a + b - \frac{ab}{a+b} \right) \end{aligned}$$

where X_f is the distance from the nose tip to the front edge of the fin root (see Figure 8).

COMBINATION CALCULATIONS

The total force on the entire rocket is the sum of all the forces on the separate regions, therefore:

TOTAL NORMAL FORCE

$$C_{N\alpha} = (C_{N\alpha})_n + (C_{N\alpha})_{cs} + (C_{N\alpha})_{cb} + (C_{N\alpha})_{fb}$$

The center of pressure of the entire rocket is found by taking a moment balance about the nose tip and solving for the total center of pressure location. (An explanation of moment balance and reasons why it works is given in Appendix B).

CENTER OF PRESSURE OF THE ENTIRE ROCKET

$$\bar{X} = \frac{(C_{N\alpha})_n \bar{X}_n + (C_{N\alpha})_{cs} \bar{X}_{cs} + (C_{N\alpha})_{cb} \bar{X}_{cb} + (C_{N\alpha})_{fb} \bar{X}_f}{C_{N\alpha}}$$

Again, remember that any additional conical shoulders, conical boattails or sets of fins must be included as extra terms in the combination equations. The additional terms fit into the combination equations in exactly the same manner as the terms for a single conical shoulder, conical boattail or fins. See the analysis of the ARCON-II two-stage bird in Chapter 8 for an example.

Also, if the rocket you are analyzing doesn't have one of the regions included in the combination equations, then simply drop the associated term from the equations. For example, if the rocket doesn't have either a conical shoulder or a conical boattail (the JAVELIN is such a rocket, again see Section 8 on examples) the combination equations can be written:

$$C_{N\alpha} = (C_{N\alpha})_n + (C_{N\alpha})_{fb}$$

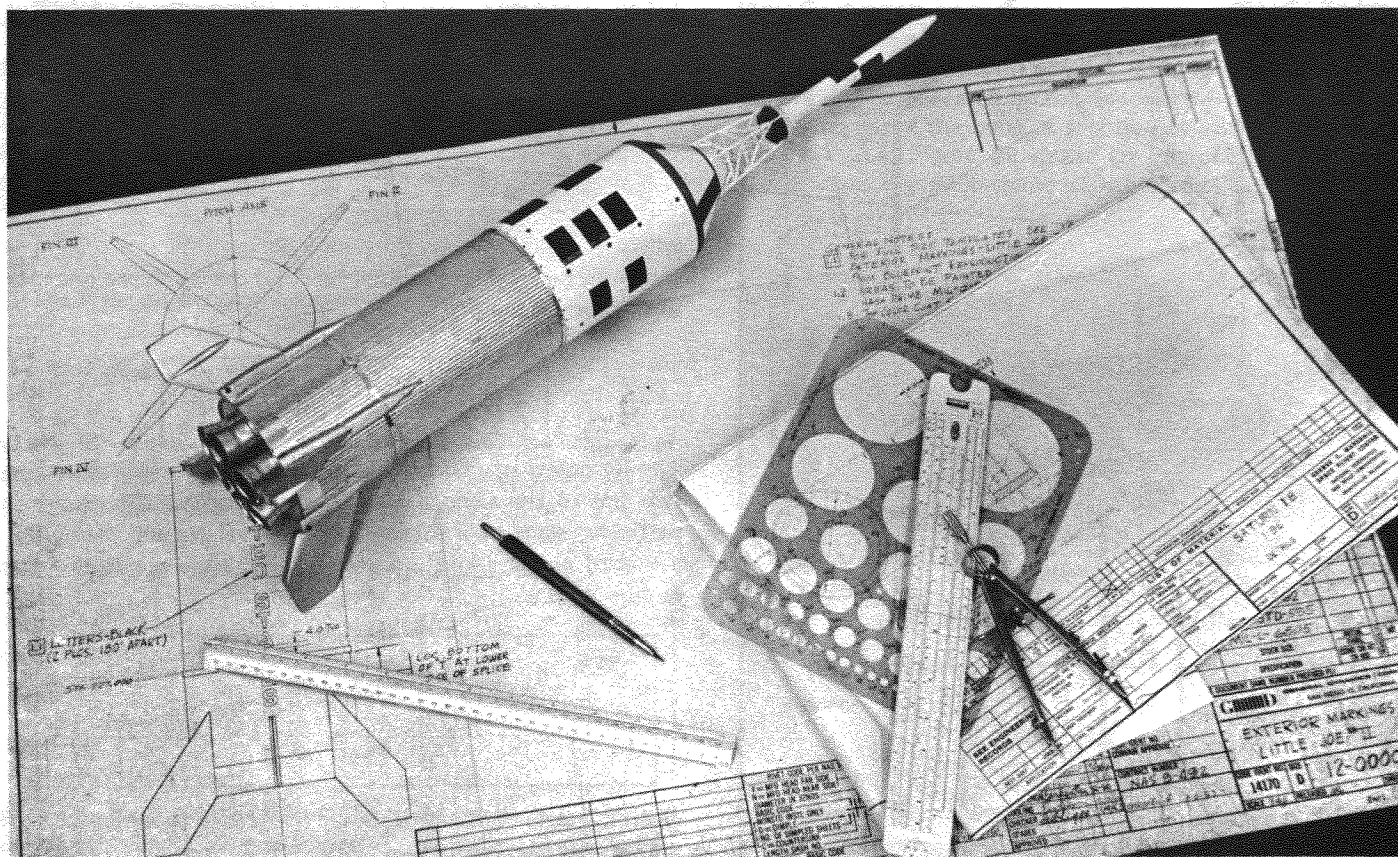
$$\bar{X} = \frac{(C_{N\alpha})_n \bar{X}_n + (C_{N\alpha})_{fb} \bar{X}_f}{C_{N\alpha}}$$

However, the above combination equations are good only for rockets having one set of fins and no shoulder or boattail.

5. SIMPLIFIED CHARTS OF THE CENTER OF PRESSURE EQUATIONS

In order to reduce the number of individual calculations, the more complex normal force and C.P. equations have been reduced to chart form. This allows even persons not well versed in reading equations to compute the center of pressure of a model rocket. In addition, it can save time and effort for those who can use the equations.

The six charts that follow give the forces and C.P. locations for conical shoulders, conical boattails, and fins alone. The nose and combination calculations are still found by using the equations. A step-by-step procedure for using the charts is given in the next section.



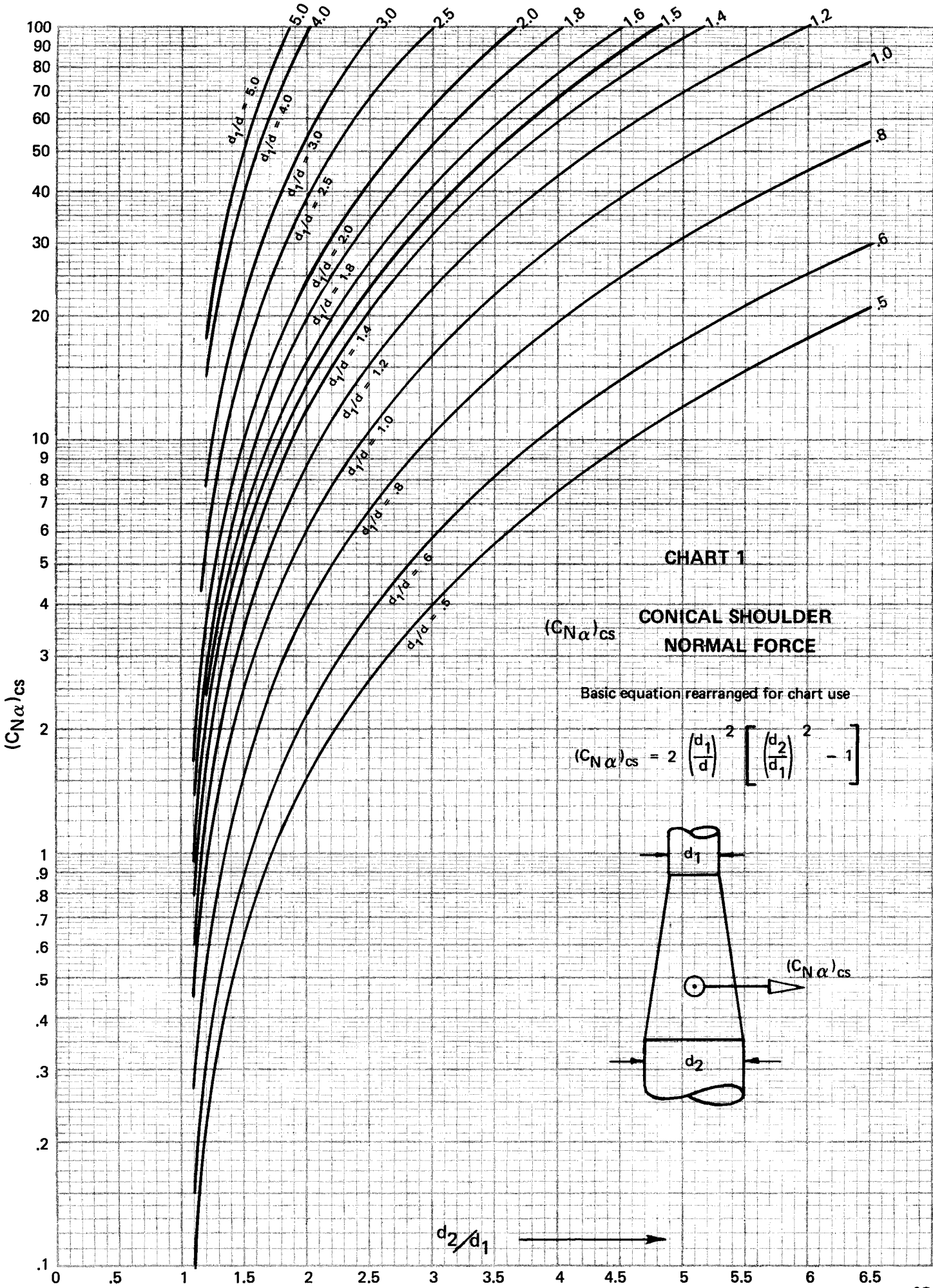


CHART 2

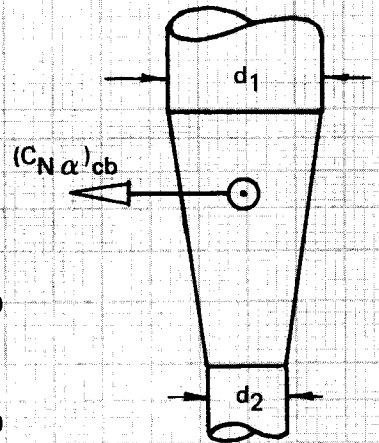
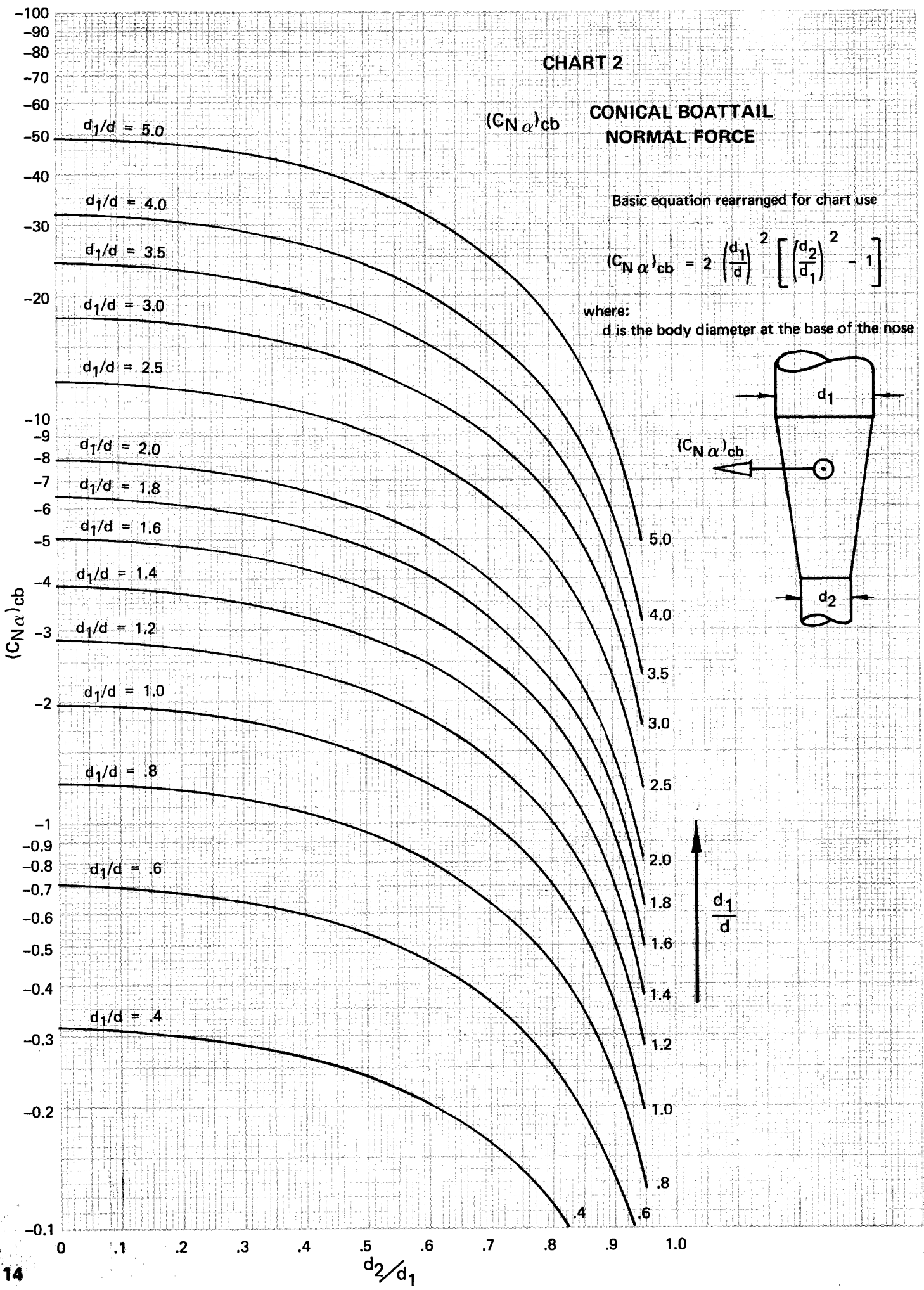
$(C_{N\alpha})_{cb}$ CONICAL BOATTAIL NORMAL FORCE

Basic equation rearranged for chart use

$$(C_{N\alpha})_{cb} = 2 \left(\frac{d_1}{d} \right)^2 \left[\left(\frac{d_2}{d_1} \right)^2 - 1 \right]$$

where:

d is the body diameter at the base of the nose



$\frac{d_1}{d}$

CHART 3

CONICAL SHOULDER AND CONICAL BOATTAIL
CENTER OF PRESSURE

$$\bar{X}_{cs} = X_{cs} + \left(\frac{\Delta X_{cs}}{L}\right) L, \quad \bar{X}_{cb} = X_{cb} + \left(\frac{\Delta X_{cb}}{L}\right) L$$

Use the above equations with the values found on this chart to find the center of pressure.

$$\frac{\Delta X_{cs}}{L}$$

.7

.6

.5

.4

.3

.2

.1

0



$$\frac{\Delta X_{cb}}{L}$$

.7

.6

.5

.4

.3

.2

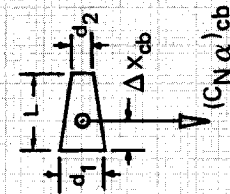
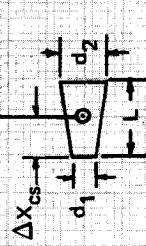
.1

0

BASIC CHART EQUATION

$$\frac{\Delta X_{cs}}{L} = \frac{\Delta X_{cb}}{L} = \frac{1}{3} \left[\frac{1 - \frac{d_1}{d_2}}{1 - \left(\frac{d_1}{d_2}\right)^2} \right]$$

CONICAL BOATTAIL CONICAL SHOULDER



4.8

4.6

4.4

4.2

4.0

3.8

3.6

3.4

3.2

3.0

2.8

2.6

2.4

2.2

2.0

1.8

1.6

1.4

1.2

1.0

.8

.6

.4

.2

0

d_2/d_1

$(C_{N\alpha})_f$ - FIN NORMAL FORCE

CHART VALUES ARE FOR 4 FINS

For 3 fins - multiply by .75

For 6 fins - multiply by 1.5

Basic equation for chart

$$(C_{N\alpha})_f = \frac{4n\left(\frac{S}{d}\right)^2}{1 + \sqrt{1 + \left(\frac{2l}{a+b}\right)^2}}$$

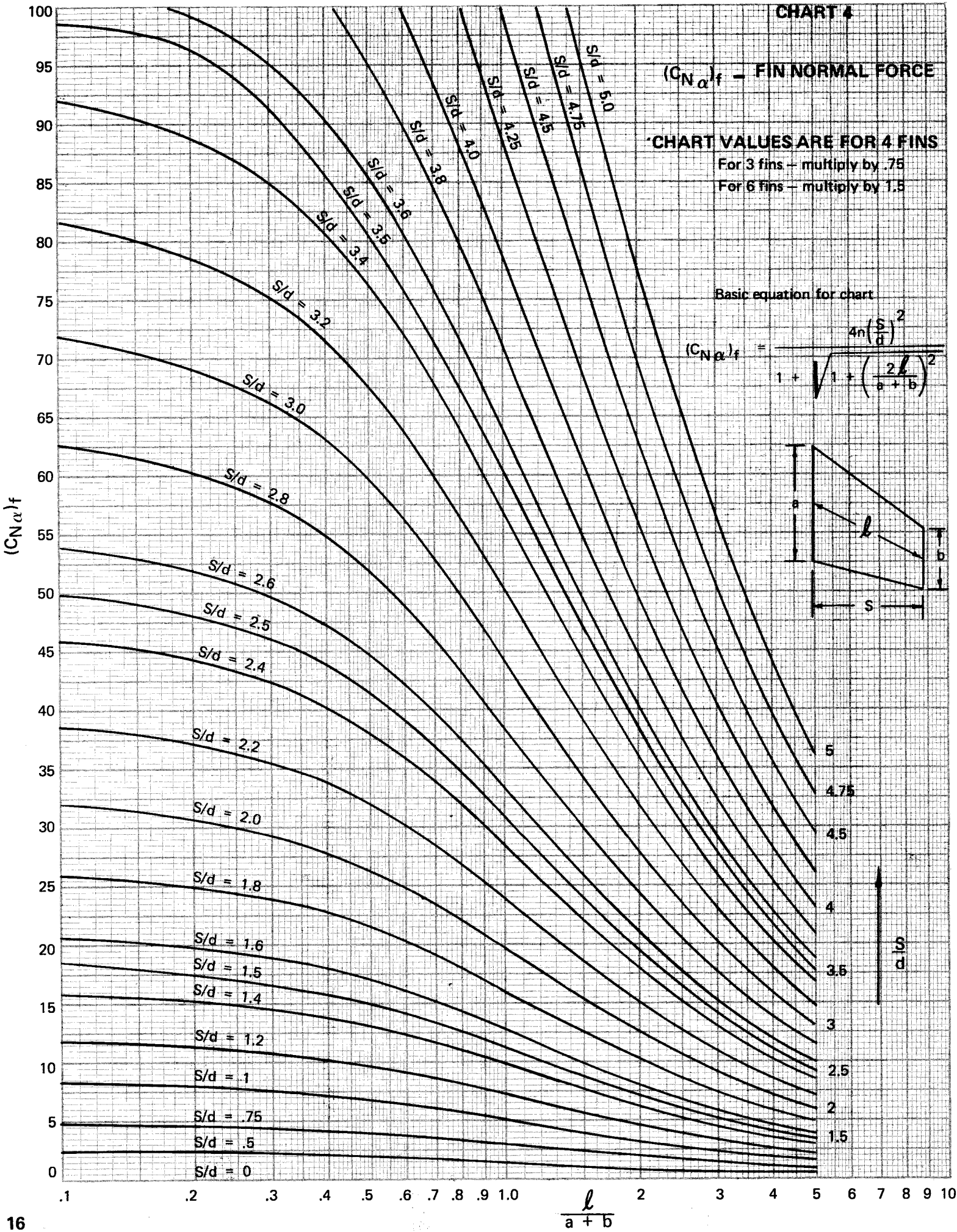
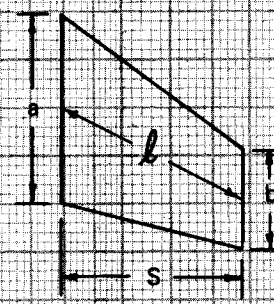


CHART 5

K_{fb} — FIN INTERFERENCE FACTOR



Basic K_{fb} equation rearranged for plotting as a function of the ratio R/S

$$K_{fb} = 1 + \frac{\frac{R}{S}}{1 + \frac{R}{S}} \quad \text{for 3 and 4 fins}$$

$$K_{fb} = 1 + \frac{.5 \frac{R}{S}}{1 + \frac{R}{S}} \quad \text{for 6 fins}$$

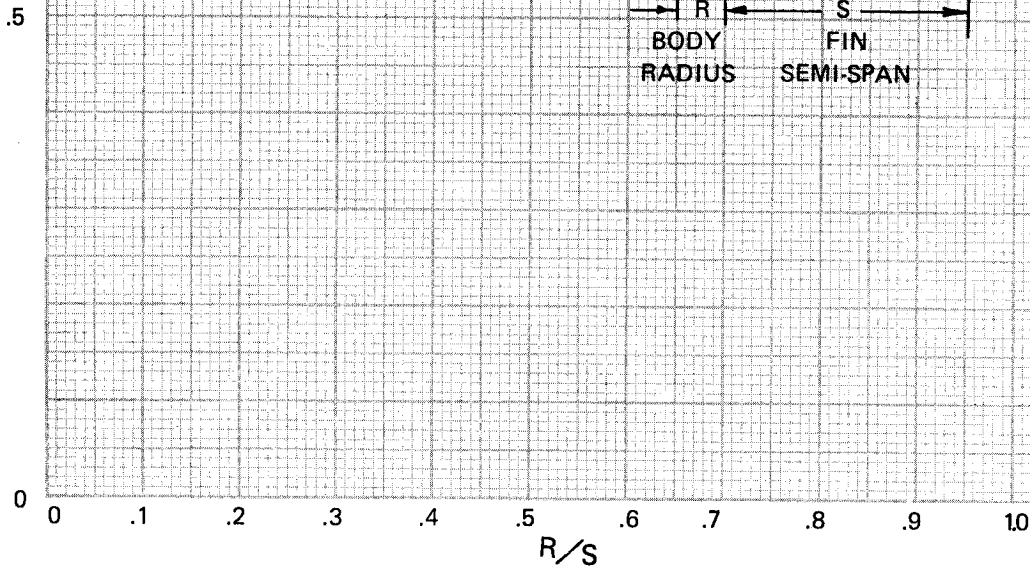
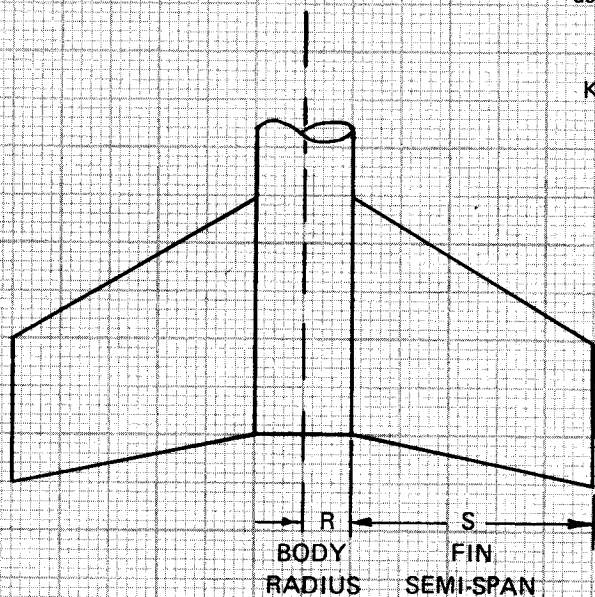


CHART 6 FIN CENTER OF PRESSURE

$$\bar{X}_f = X_f + \left(\frac{\Delta X_f}{a}\right) a$$

Use the above equation with the $\Delta X_f/a$ value from this chart to find the fin center of pressure \bar{X}_f .

NOTE: The fin center of pressure is the same for any number of fins.

$\frac{\Delta X_f}{a}$

1.2
1.1
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0

$m/a = 2.0$

$m/a = 1.9$

$m/a = 1.8$

$m/a = 1.7$

$m/a = 1.6$

$m/a = 1.5$

$m/a = 1.4$

$m/a = 1.3$

$m/a = 1.2$

$m/a = 1.1$

$m/a = 1.0$

$m/a = .9$

$m/a = .8$

$m/a = .7$

$m/a = .6$

$m/a = .5$

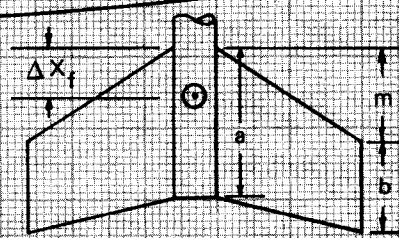
$m/a = .4$

$m/a = .3$

$m/a = .2$

$m/a = .1$

$m/a = 0$



Basic ΔX_f equation rearranged for plotting as a function of b/a and m/a .

$$\frac{\Delta X_f}{a} = \frac{1}{3} \frac{m}{a} \left[\frac{1 + 2 \frac{b}{a}}{1 + \frac{b}{a}} \right] + \frac{1}{8} \left[\frac{b}{a} + \frac{1}{1 + \frac{b}{a}} \right]$$

0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0

b/a

6. PROCEDURE FOR USING THE CHARTS

FINS

NOSE

The nose $(C_{N\alpha})_n$ and \bar{X}_n are calculated exactly as outlined in Section 4.

1. $(C_{N\alpha})_n = 2$

2. There are three basic shapes:

a. Cone $\bar{X}_n = \frac{2}{3}L$

b. Ogive $\bar{X}_n = .466 L$

c. Parabola $\bar{X}_n = \frac{1}{2}L$

CONICAL SHOULDER

1. Compute the ratios $\frac{d_1}{d}$ and $\frac{d_2}{d_1}$, then

use Chart 1 to find $(C_{N\alpha})_{cs}$, and

use Chart 3 to find $\left(\frac{\Delta X_{cs}}{L}\right)$

2. Compute \bar{X}_{cs} using the equation

$$\bar{X}_{cs} = X_{cs} + \left(\frac{\Delta X_{cs}}{L}\right)L$$

CONICAL BOATTAIL

1. Compute $\frac{d_1}{d}$ and $\frac{d_2}{d_1}$, then

use Chart 2 to find $(C_{N\alpha})_{cb}$, and

use Chart 3 to find $\frac{\Delta X_{cb}}{L}$

2. Compute \bar{X}_{cb} using the equation

$$\bar{X}_{cb} = X_{cb} + \left(\frac{\Delta X_{cb}}{L}\right)L$$

1. Compute $\frac{S}{d}$ and $\frac{l}{a+b}$, then

use Chart 4 to get $(C_{N\alpha})_f$ for four (4) fins

a. To convert to a three (3) fin value multiply by .75

b. To convert to a six (6) fin value multiply by 1.5

2. Compute $\frac{R}{S}$, then

use Chart 5 to get the interference factor K_{fb} . Be sure to use the K_{fb} line which is correct for the number of fins on the model.

Next, compute the total force on the fins in the presence of the body

$$(C_{N\alpha})_{fb} = (C_{N\alpha})_f K_{fb}$$

3. Compute $\frac{m}{a}$ and $\frac{b}{a}$, then

use Chart 6 to get $\frac{\Delta X_f}{a}$

Compute \bar{X}_f using the equation

$$\bar{X}_f = X_f + \left(\frac{\Delta X_f}{a}\right)a$$

COMBINATION CALCULATIONS

The center of pressure location of the entire rocket is calculated exactly as presented in Section 4.

$$C_{N\alpha} = (C_{N\alpha})_n + (C_{N\alpha})_{cs} + (C_{N\alpha})_{cb} + (C_{N\alpha})_{fb}$$

$$\bar{X} = \frac{(C_{N\alpha})_n \bar{X}_n + (C_{N\alpha})_{cs} \bar{X}_{cs} + (C_{N\alpha})_{cb} \bar{X}_{cb} + (C_{N\alpha})_{fb} \bar{X}_f}{C_{N\alpha}}$$

7. DESIGNING STABLE MODEL ROCKETS

Designing a new rocket is basically a problem of designing the fins. First, determine an initial rocket design that fits such requirements as the desired body tube size, payload compartment, nose-cone shape, engine type and size, and any other special features desired. Second, calculate the center of gravity (C.G.) of the design using a technique such as the one given in Appendix A. Third, calculate the center of pressure (C.P.) location of the design. Fourth, compare the result with the center of gravity to see if the design is stable and has the proper static margin. Again, a good value for the static margin is the largest body diameter of the rocket.

If the proper static margin has not been obtained, then alter the fin design and re-analyze the rocket. Changing the fin will not appreciably change the center of gravity, so only the change in center of pressure need be considered. By changing only the fin shape, only the fin terms in the combination calculations have to be changed each time.

Once the rocket is re-analyzed, check the static margin again. Keep changing the fins until the proper static margin is obtained. The changes that should be made each time will be indicated by the previous result. This is essentially a trial and error method. The more experience you have doing it, the better and faster you'll become. There are no hard and fast rules for designing anything. You must use your own engineering judgment. The center of pressure equations are just a tool to help you make judgments in deciding on a final design which will fly safely.

A few helpful hints for organizing your thoughts prior to starting calculations are given below.

1. Preliminary Work

- Determine the needed dimensions of the rocket or proposed design.
- Determine the nose shape (cone, ogive, parabola, or special shape).
- Idealize the fin shape, if necessary.
- If the nose is one of the special shapes discussed, make any drawings and/or special measurements required.

2. Calculations

- This essentially involves plugging appropriate model dimensions into the equations and performing the mathematical operations indicated by the equations. If you are taking dimensions from a drawing rather than directly from a model, it helps to label each dimension with its appropriate terminology as done in all the examples of Section 8. It is suggested that you analyze the different regions of the rocket in the order that they are given in this report, that is:

Nose
Conical shoulder(s)
Conical boattail(s)
Fins
Combination Calculations

- Label each set of calculations with the name of the region being analyzed. Also, circle and label the answers you get for each region. This will allow you to easily find the answers for each region and use them in the combination calculations.
- If you can operate a slide rule, use it. Slide rule accuracy is adequate, but be sure to maintain the full 3 or 4 place accuracy available. If you are not able to operate a slide rule, learn how! It can save you very much time and effort in the long run. Like everything else worth while though, it takes some initial effort on your part to become familiar with it.
- Above all, be neat in writing down the computations.

MULTISTAGE MODELS

If the rocket being analyzed has two or three stages, make absolutely sure that each stage is stable. For a three-stage model, you must essentially analyze three separate rockets:

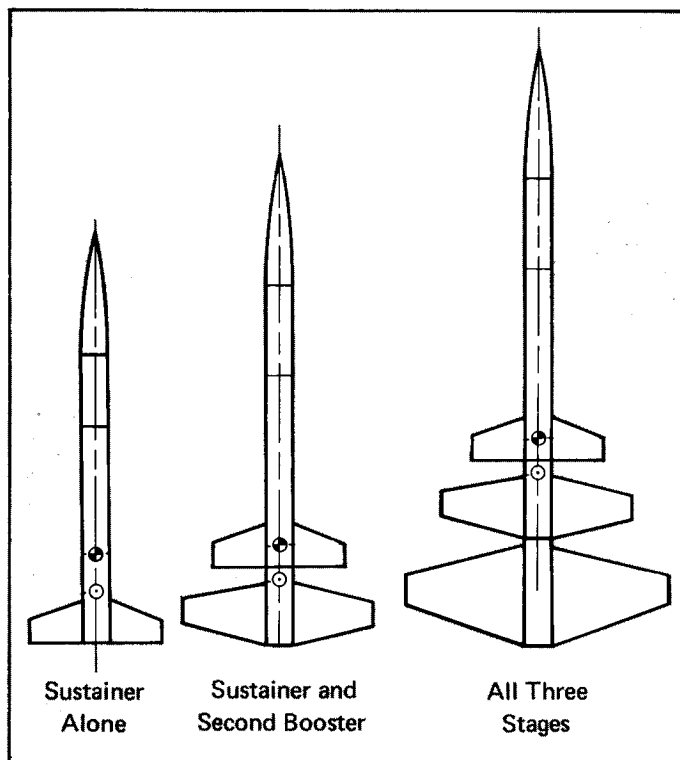


FIGURE 12

Fortunately, by analyzing the three-stage combination first, the C.P. of the other two combinations can be found simply by dropping the fin terms of the burnt-out stage from the combination calculations. An example of this technique is given for CENTURI's ARCON-HI model in Section 8. Remember! You must also find the C.G. of each stage and have the static margin of each stage equal to at least the largest body diameter.

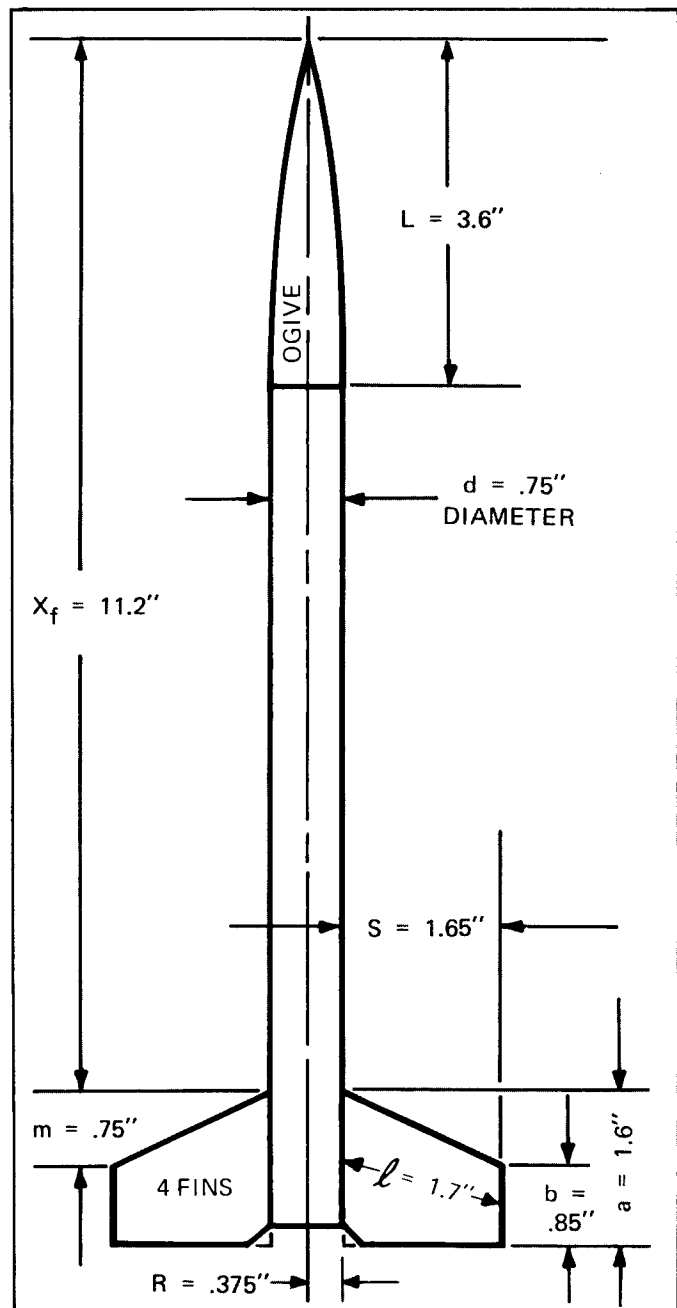
8. EXAMPLES

In order to illustrate the calculation of the center of pressure, three CENTURI model rockets are analyzed using both the equations and the charts. The three rockets that are analyzed are: the JAVELIN, a fairly simple bird with an ogive nose and one set of four fins; the RECRUITER, a more complex-shaped rocket with a complex nose shape and six fins; and the ARCON-HI, a two-stage model.

All of the mathematical operations in the following examples were done using a 10" slide rule. The numbers computed reflect the three or four significant figure accuracy obtainable with a slide rule.

JAVELIN EXAMPLE

In diagram below, the fin is idealized by including the corner at the back of the fin root (see dotted lines).



JAVELIN ANALYSIS BY USING THE EQUATIONS

Nose

normal force

$$(C_{N\alpha})_n = 2$$

center of pressure

$$\bar{X}_n = .466 L$$

$$= .466 (3.6)$$

$$\bar{X}_n = 1.68 \text{ inches}$$

Fins

normal force on four fins

$$(C_{N\alpha})_f = \frac{4n\left(\frac{S}{d}\right)^2}{1 + \sqrt{1 + \left(\frac{2l}{a+b}\right)^2}}$$

$$= \frac{4(4)\left(\frac{1.65}{.75}\right)^2}{1 + \sqrt{1 + \left[\frac{2(1.7)}{1.6 + .85}\right]^2}} = \frac{16(2.2)^2}{1 + \sqrt{1 + \left[\frac{3.4}{2.45}\right]^2}}$$

$$= \frac{16(4.84)}{1 + \sqrt{1 + (1.38)^2}} = \frac{77.4}{1 + \sqrt{2.904}} = \frac{77.4}{2.704}$$

$$(C_{N\alpha})_f = 28.6$$

interference factor for four fins

$$K_{fb} = 1 + \frac{R}{S + R}$$

$$= 1 + \frac{.375}{1.65 + .375} = 1 + \frac{.375}{2.025}$$

$$K_{fb} = 1.185$$

normal force on fins in presence of body

$$(C_{N\alpha})_{fb} = K_{fb} (C_{N\alpha})_f$$

$$= 1.185 (28.6)$$

$$(C_{N\alpha})_{fb} = 33.9$$

center of pressure

$$\begin{aligned}\bar{X}_f &= X_f + \Delta X_f \\ &= X_f + \frac{m(a + 2b)}{3(a + b)} + \frac{1}{6} \left(a + b - \frac{ab}{a + b} \right)\end{aligned}$$

$$= 11.2 + \frac{.75 [1.6 + 2(.85)]}{3(1.6 + .85)} + \frac{1}{6} \left[1.6 + .85 - \frac{1.6(.85)}{1.6 + .85} \right]$$

$$= 11.2 + \frac{.25(1.6 + 1.7)}{2.45} + \frac{1}{6} \left[2.45 - \frac{1.36}{2.45} \right]$$

$$= 11.2 + .102(3.3) + \frac{1}{6}(2.45 - .55) = 11.2 + .34 + \frac{1.9}{6}$$

$$= 11.54 + .32$$

$$\boxed{\bar{X}_f = 11.86 \text{ inches}}$$

Javelin Combination Calculations

total normal force

$$\begin{aligned}C_{N\alpha} &= (C_{N\alpha})_n + (C_{N\alpha})_{fb} \\ &= 2 + 33.9\end{aligned}$$

$$\boxed{C_{N\alpha} = 35.9}$$

center of pressure of the entire rocket

$$\begin{aligned}\bar{X} &= \frac{(C_{N\alpha})_n \bar{X}_n + (C_{N\alpha})_{fb} \bar{X}_f}{C_{N\alpha}} \\ &= \frac{2(1.68) + 33.9(11.86)}{35.9} = \frac{3.36 + 402}{35.9} = \frac{405}{35.9}\end{aligned}$$

$$\boxed{\bar{X} = 11.3 \text{ inches}}$$

JAVELIN ANALYSIS BY USING THE CHARTS

Nose

(uses the equations as before)

normal force

$$\boxed{C_{N\alpha} = 2}$$

center of pressure

$$\bar{X}_n = .466 L$$

$$= .466(3.6)$$

$$\boxed{\bar{X}_n = 1.68 \text{ inches}}$$

Fins

normal force

$$\frac{S}{d} = \frac{1.65}{.75} = 2.2, \quad \frac{l}{a+b} = \frac{1.7}{1.6 + .85} = \frac{1.7}{2.45} = .69$$

$$(C_{N\alpha})_f = 28.5 \text{ (from Chart 4)}$$

interference factor

$$\frac{R}{S} = \frac{.375}{1.65} = .227$$

$$K_{fb} = 1.185 \text{ (from Chart 5)}$$

total normal force on fins in presence of body

$$\begin{aligned}(C_{N\alpha})_{fb} &= K_{fb} (C_{N\alpha})_f \\ &= 1.185(28.6)\end{aligned}$$

$$\boxed{(C_{N\alpha})_{fb} = 33.8}$$

center of pressure

$$\frac{m}{a} = \frac{.75}{1.6} = .47, \quad \frac{b}{a} = \frac{.85}{1.6} = .53$$

$$\frac{X_f}{a} = .41 \text{ (from Chart 6)}$$

$$\bar{X}_f = X_f + \left(\frac{X_f}{a} \right) a$$

$$= 11.2 + (.41) 1.6 = 11.2 + .66$$

$$\boxed{\bar{X}_f = 11.86}$$

Javelin Combination Calculations

(uses the equations as before)

total normal force

$$\begin{aligned}C_{N\alpha} &= (C_{N\alpha})_n + (C_{N\alpha})_{fb} \\ &= 2 + 33.8\end{aligned}$$

$$\boxed{C_{N\alpha} = 35.8}$$

center of pressure of entire rocket

$$\begin{aligned}\bar{X} &= \frac{(C_{N\alpha})_n \bar{X}_n + (C_{N\alpha})_{fb} \bar{X}_f}{C_{N\alpha}} \\ &= \frac{2(1.68) + 33.8(11.86)}{35.8} = \frac{3.36 + 401}{35.8} = \frac{404}{35.8}\end{aligned}$$

$$\boxed{\bar{X} = 11.3 \text{ inches}}$$

JAVELIN STABILITY CHECK

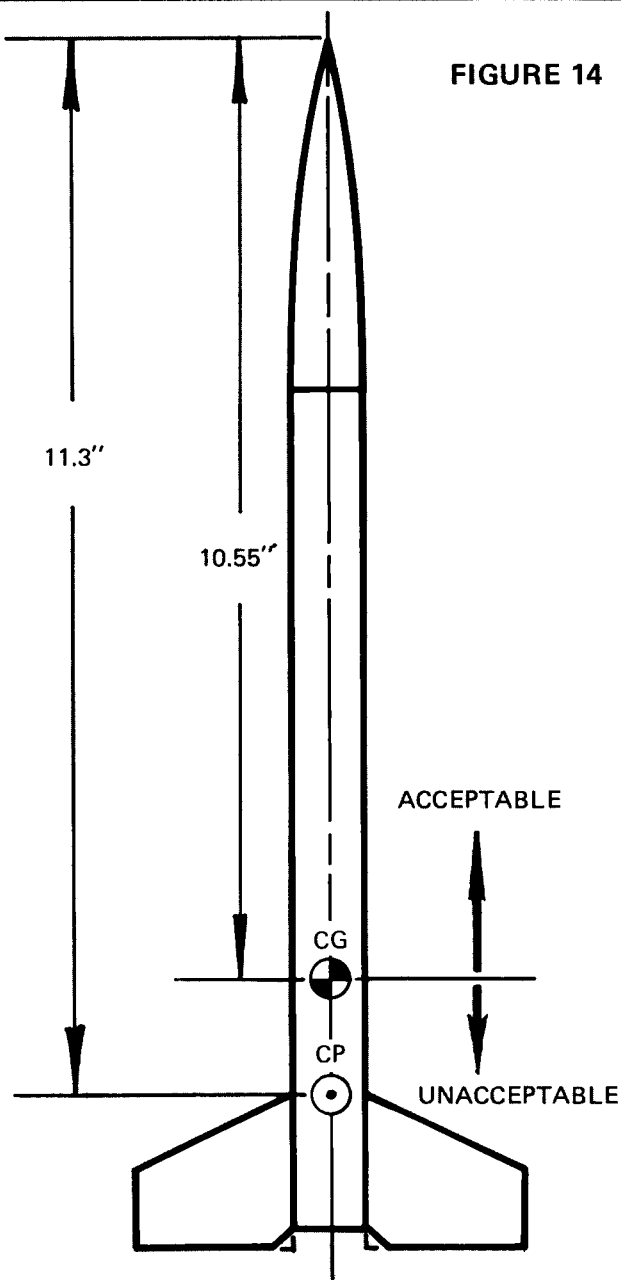
Now that the center of pressure location has been determined, a location for the center of gravity (C.G.) can be established which will guarantee safe stable flights. For one caliber stability, the C.G. (including engine and parachute) should be one body diameter (D) ahead of the center of pressure.

$$X_{CG} = \bar{X} - D$$

$$= 11.3 - .75$$

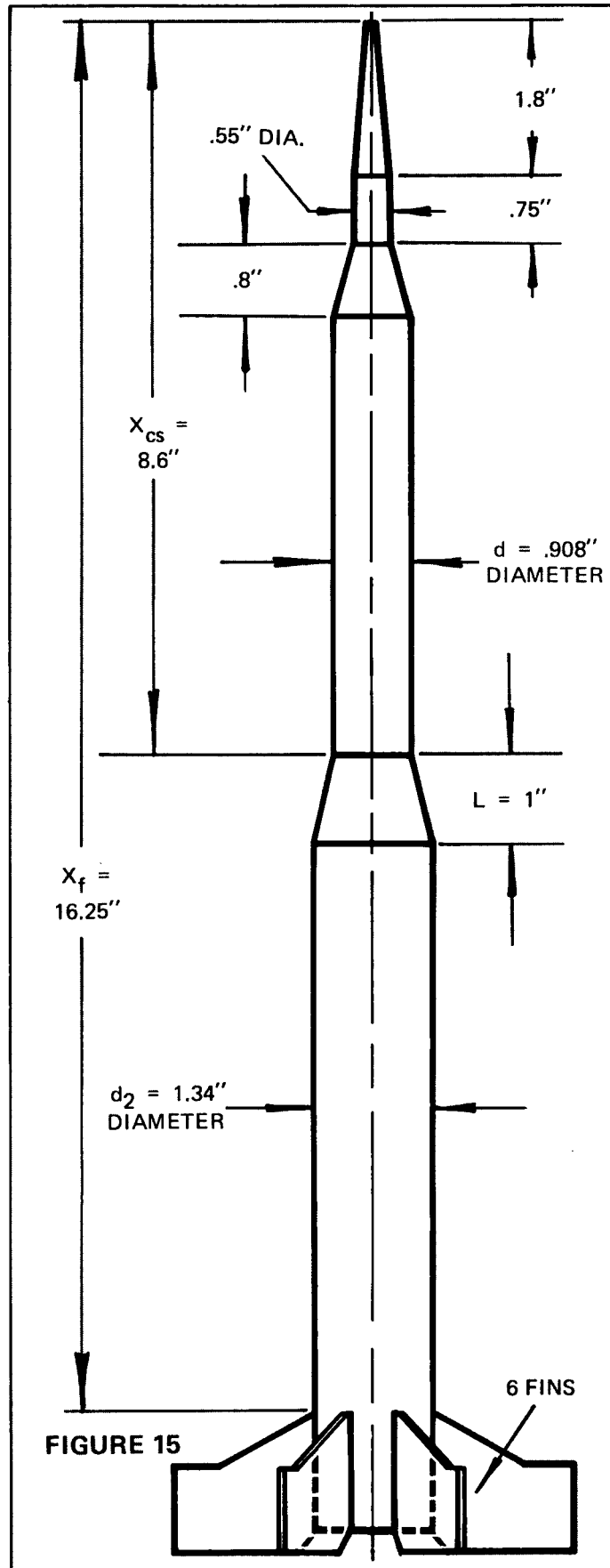
$$X_{CG} = 10.55 \text{ inches from the nose tip}$$

This C.G. value represents the furthest acceptable balance point from the nose. Balance points closer to the nose make the rocket proportionally more stable.



RECRUITER EXAMPLE

A diagram of the RECRUITER and its required dimensions is shown below.



RECRUITER ANALYSIS BY USING THE EQUATIONS

Nose

normal force

$$(C_N \alpha)_n = 2$$

center of pressure

$$\begin{aligned} \bar{X}_n &= \frac{2}{3} L \\ &= \frac{2}{3} (4.15) \end{aligned}$$

$$\bar{X}_n = 2.77 \text{ inches}$$

But, the above C.P. is measured from the imaginary tip of the idealized cone. Correcting it back to the actual nose tip yields:

$$\bar{X}_n = 2.77 - .8$$

$$\bar{X}_n = 1.97 \text{ inches}$$

Conical Shoulder

normal force

$$\begin{aligned} (C_N \alpha)_{cs} &= 2 \left[\left(\frac{d_2}{d} \right)^2 - \left(\frac{d_1}{d} \right)^2 \right] \\ &= 2 \left[\left(\frac{1.34}{.908} \right)^2 - \left(\frac{.908}{.908} \right)^2 \right] \\ &= 2 \left[(1.476)^2 - 1 \right] = 2 (2.18 - 1) = 2 (1.18) \end{aligned}$$

$$(C_N \alpha)_{cs} = 2.36$$

center of pressure location

$$\bar{X}_{cs} = X_{cs} + \Delta X_{cs} = X_{cs} + \frac{L}{3} \left[1 + \frac{1 - \frac{d_1}{d_2}}{1 - \left(\frac{d_1}{d_2} \right)^2} \right]$$

$$= 8.6 + \frac{1.0}{3} \left[1 + \frac{1 - \frac{.908}{1.34}}{1 - \left(\frac{.908}{1.34} \right)^2} \right]$$

$$= 8.6 + \frac{1}{3} \left(1 + \frac{.322}{1 - (.678)^2} \right)$$

$$= 8.6 + \frac{1}{3} \left(1 + \frac{.322}{.541} \right) = 8.6 + \frac{1.595}{3} = 8.6 + .532$$

$$\bar{X}_{cs} = 9.13 \text{ inches}$$

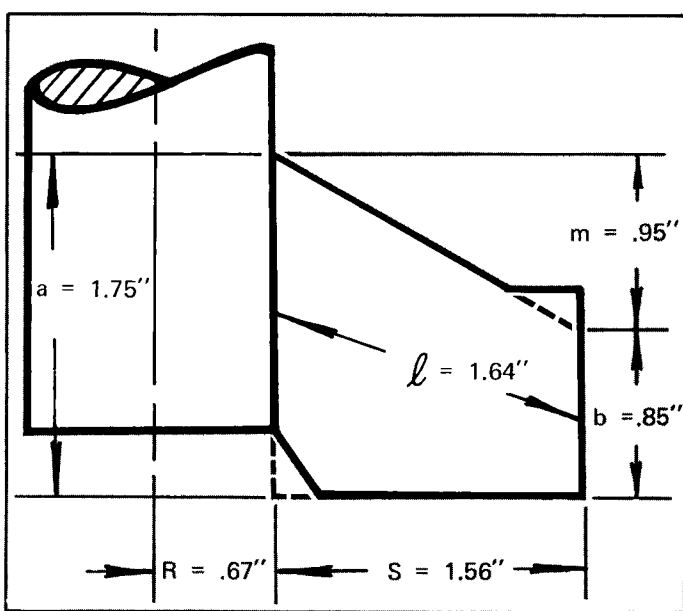
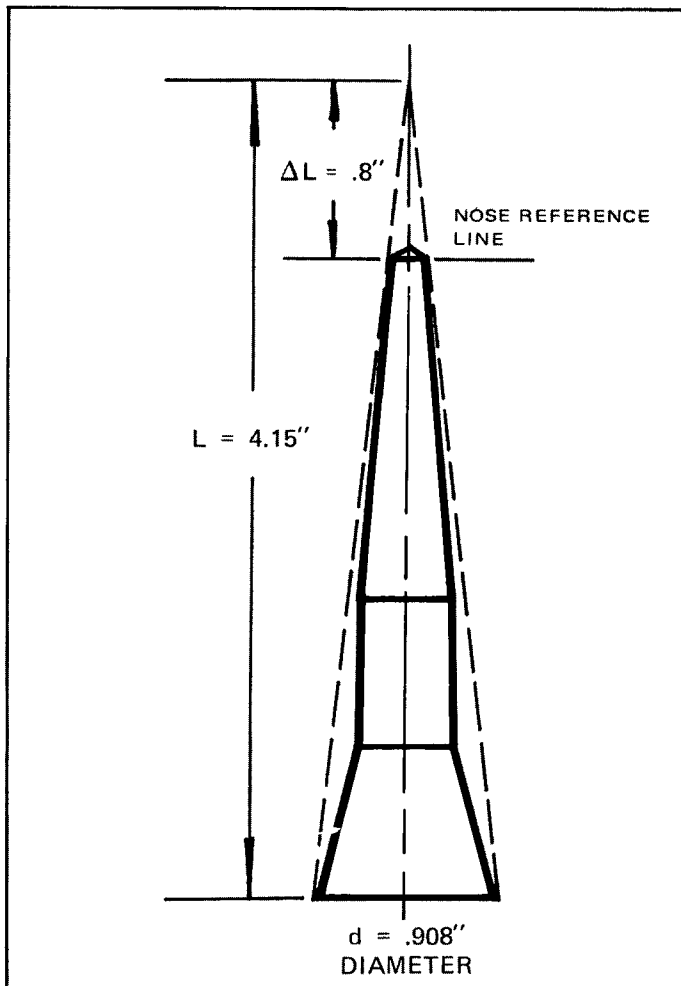


FIGURE 16

The RECRUITER's fins are idealized as shown above (actual size).

The RECRUITER's nose shape is idealized as shown below (actual size).

FIGURE 17



Fins

normal force on six fins

$$(C_{N\alpha})_f = \frac{4n\left(\frac{S}{d}\right)^2}{1 + \sqrt{1 + \left(\frac{2\ell}{a+b}\right)^2}}$$

$$= \frac{4(6)\left(\frac{1.56}{.908}\right)^2}{1 + \sqrt{1 + \left[\frac{2(1.64)}{1.75 + .8}\right]^2}} = \frac{24(1.717)^2}{1 + \sqrt{1 + \left(\frac{3.28}{2.55}\right)^2}}$$

$$= \frac{24(2.95)}{1 + \sqrt{1 + (1.286)^2}} = \frac{70.8}{1 + \sqrt{2.65}} = \frac{70.8}{2.63}$$

$$(C_{N\alpha})_f = 26.9$$

interference factor for six fins

Since the RECRUITER has six fins, the interference factor is calculated using the equation having the .5 factor in the numerator.

$$K_{fb} = 1 + \frac{.5R}{S + R}$$

$$= 1 + \frac{.5(.67)}{1.56 + .67} = 1 + \frac{.335}{2.23}$$

$$K_{fb} = 1.15$$

normal force on fins in presence of body

$$(C_{N\alpha})_{fb} = K_{fb} (C_{N\alpha})_f$$

$$= 1.15 (26.9)$$

$$(C_{N\alpha})_{fb} = 31.0$$

center of pressure

$$\bar{X}_f = X_f + \Delta X_f$$

$$= X_f + \frac{m(a+2b)}{3(a+b)} + \frac{1}{6}\left(a+b - \frac{ab}{a+b}\right)$$

$$= 16.25 + \frac{.95[1.75 + 2(.8)]}{3(1.75 + .8)}$$

$$+ \frac{1}{6}\left[1.75 + .8 - \frac{1.75(.8)}{1.75 + .8}\right]$$

$$= 16.25 + \frac{.317(1.75 + 1.6)}{2.55} + \frac{1}{6}\left(2.55 - \frac{1.4}{2.55}\right)$$

$$= 16.25 + .124(3.35) + \frac{1}{6}(2.55 - .55)$$

$$= 16.25 + .42 + \frac{2}{6} = 16.67 + .33$$

$$\bar{X}_f = 17.0 \text{ inches}$$

Recruiter Combination Calculations

total normal force

$$C_{N\alpha} = (C_{N\alpha})_n + (C_{N\alpha})_{cs} + (C_{N\alpha})_{fb}$$

$$= 2 + 2.36 + 31.0$$

$$C_{N\alpha} = 35.4$$

center of pressure of the entire rocket

$$\bar{X} = \frac{(C_{N\alpha})_n \bar{X}_n + (C_{N\alpha})_{cs} \bar{X}_{cs} + (C_{N\alpha})_{fb} \bar{X}_f}{C_{N\alpha}}$$

$$= \frac{2(1.97) + 2.36(9.13) + 31(17)}{35.4}$$

$$= \frac{3.94 + 21.5 + 527}{35.4} = \frac{552}{35.4}$$

$$\bar{X} = 15.6 \text{ inches}$$

RECRUITER ANALYSIS BY USING THE CHARTS

Nose

(uses the equations as before)

normal force

$$(C_{N\alpha})_n = 2$$

center of pressure

$$\bar{X}_n = \frac{2}{3}L$$

$$= \frac{2}{3}(4.15)$$

$$\bar{X}_n = 2.77 \text{ inches}$$

But the above C.P. is measured from the imaginary tip of the idealized cone. Correcting it back to the actual nose tip yields:

$$\bar{X}_n = 2.77 - .8$$

$$\bar{X}_n = 1.97 \text{ inches}$$

Conical Shoulder

normal force

$$\frac{d_1}{d} = \frac{.908}{.908} = 1, \frac{d_2}{d_1} = \frac{1.34}{.908} = 1.48$$

$$(C_{N\alpha})_{cs} = 2.45 \text{ (from Chart 1)}$$

center of pressure

$$\frac{\Delta X_{cs}}{L} = .532 \text{ (from Chart 3)}$$

$$\bar{X}_{cs} = X_{cs} + \left(\frac{\Delta X_{cs}}{L}\right) L$$

$$= 8.6 + (.532)(1.0) = 8.6 + .532$$

$$X_{cs} = 9.13 \text{ inches}$$

Fins

normal force

$$\frac{S}{d} = \frac{1.56}{.908} = 1.72, \quad \frac{l}{a+b} = \frac{1.64}{1.75 + .8} = \frac{1.64}{2.55} = .64$$

$$(C_{N\alpha})_f = 18 \text{ (four fin value from Chart 4)}$$

Since the RECRUITER has six fins, the value of $(C_{N\alpha})_f$ from the four fin chart must be multiplied by 1.5.

$$(C_{N\alpha})_f = 1.5(18) = 27$$

interference factor

$$\frac{R}{S} = \frac{.67}{1.56} = .429$$

$$K_{fb} = 1.15 \text{ (six fin value from Chart 5)}$$

total normal force on the fins in the presence of the body

$$(C_{N\alpha})_{fb} = K_{fb} (C_{N\alpha})_f$$

$$= 1.15(27)$$

$$(C_{N\alpha})_{fb} = 31.0$$

center of pressure

$$\frac{m}{a} = \frac{.95}{1.75} = .54, \quad \frac{b}{a} = \frac{.8}{1.75} = .46$$

$$\frac{\Delta X_f}{a} = .42 \text{ (from Chart 6)}$$

$$\bar{X}_f = X_f + \left(\frac{\Delta X_f}{a}\right) a$$

$$= 16.25 + (.42) 1.75 = 16.25 + .735$$

$$\bar{X}_f = 16.99 \text{ inches}$$

Recruiter Combination Calculations

(uses the equations as before)

total normal force

$$C_{N\alpha} = (C_{N\alpha})_n + (C_{N\alpha})_{cs} + (C_{N\alpha})_{fb}$$

$$= 2 + 2.45 + 31.0$$

$$C_{N\alpha} = 35.4$$

center of pressure of the entire rocket

$$\bar{X} = \frac{(C_{N\alpha})_n \bar{X}_n + (C_{N\alpha})_{cs} \bar{X}_{cs} + (C_{N\alpha})_{fb} \bar{X}_f}{C_{N\alpha}}$$

$$= \frac{2(1.97) + 2.45(9.13) + 31(16.99)}{35.4}$$

$$= \frac{3.94 + 22.4 + 527.}{35.4} = \frac{553.}{35.4}$$

$$\bar{X} = 15.6 \text{ inches}$$

RECRUITER STABILITY CHECK

The C.G. location from the nose tip which will give one caliber stability is

$$X_{CG} = \bar{X} - D$$

$$= 15.6 - 1.34$$

$$X_{CG} = 14.26 \text{ inches from the nose}$$

Note that the diameter D is the largest diameter tube used in the construction of the model ($D = d_2$ in this instance).

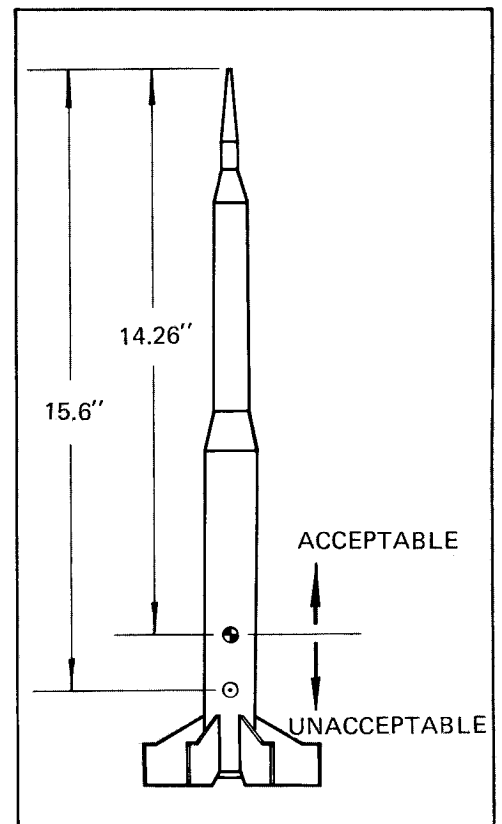


FIGURE 18

ARCON-HI EXAMPLE

A diagram of the ARCON-HI and its required dimensions is shown below.

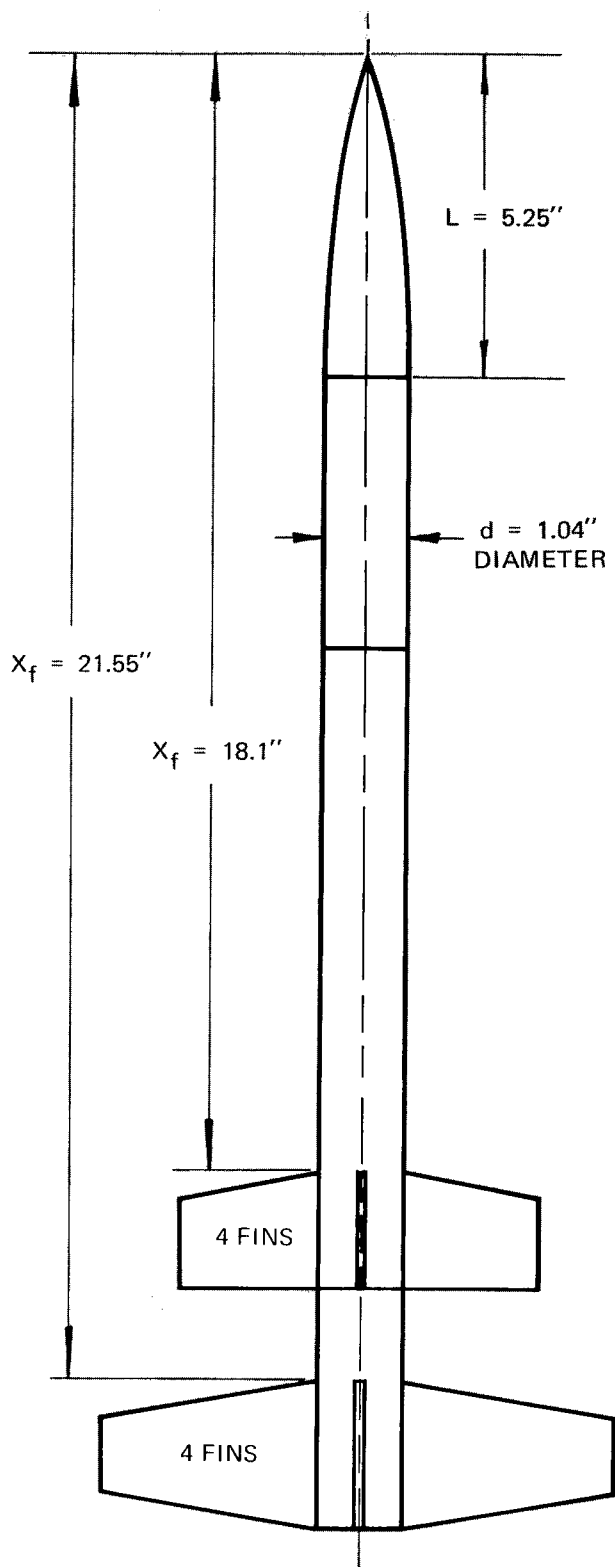


FIGURE 19

SUSTAINER FIN DIMENSIONS

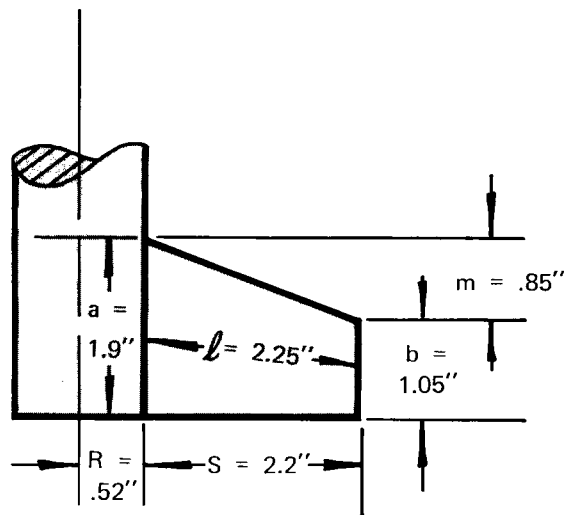


FIGURE 20

BOOSTER FIN DIMENSIONS

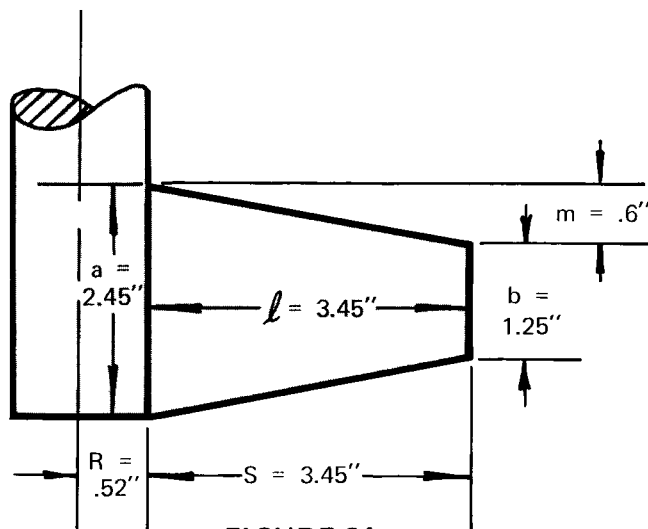


FIGURE 21

ANALYSIS OF THE TWO STAGE ARCON-HI USING THE EQUATIONS

Nose

normal force

$$(C_N \alpha)_n = 2$$

center of pressure

$$\bar{X}_n = .466 L$$

$$= .466 (5.25)$$

$$\bar{X}_n = 2.45 \text{ inches}$$

Sustainer Fins

normal force on four fins

$$\begin{aligned} (C_N \alpha)_f &= \frac{4n \left(\frac{S}{d}\right)^2}{1 + \sqrt{1 + \left(\frac{2l}{a+b}\right)^2}} \\ &= \frac{4(4) \left(\frac{2.2}{1.04}\right)^2}{1 + \sqrt{1 + \left[\frac{2(2.25)}{1.9 + 1.05}\right]^2}} \\ &= \frac{16(2.12)^2}{1 + \sqrt{1 + \left(\frac{4.5}{2.95}\right)^2}} = \frac{16(4.48)}{1 + \sqrt{1 + (1.525)^2}} \\ &= \frac{71.7}{1 + \sqrt{3.32}} = \frac{71.7}{2.82} \end{aligned}$$

$$(C_N \alpha)_f = 25.4$$

interference factor

$$\begin{aligned} K_{fb} &= 1 + \frac{R}{S + R} \\ &= 1 + \frac{.52}{2.2 + .52} = 1 + \frac{.52}{2.72} \end{aligned}$$

$$K_{fb} = 1.191$$

normal force on fins in presence of body

$$\begin{aligned} (C_N \alpha)_{fb} &= K_{fb} (C_N \alpha)_f \\ &= 1.191 (25.4) \end{aligned}$$

$$\boxed{(C_N \alpha)_{fb} = 30.2} \text{ Sustainer}$$

center of pressure

$$\begin{aligned} \bar{X}_f &= X_f + \Delta X_f \\ &= X_f + \frac{m(a+2b)}{3(a+b)} + \frac{1}{6} \left(a + b - \frac{ab}{a+b} \right) \\ &= 18.1 + \frac{.85 [1.9 + 2(1.05)]}{3(1.9 + 1.05)} \\ &\quad + \frac{1}{6} \left[1.9 + 1.05 - \frac{1.9(1.05)}{1.9 + 1.05} \right] \\ &= 18.1 + \frac{.283(1.9 + 2.1)}{2.95} + \frac{1}{6} \left(2.95 - \frac{1.995}{2.95} \right) \\ &= 18.1 + .0959(4.0) + \frac{1}{6} (2.95 - .68) \\ &= 18.1 + .38 + \frac{2.27}{6} = 18.48 + .38 \end{aligned}$$

$$\boxed{\bar{X}_f = 18.86 \text{ inches}} \text{ Sustainer}$$

Booster Fins

normal force on four fins

$$\begin{aligned} (C_N \alpha)_f &= \frac{4n \left(\frac{S}{d}\right)^2}{1 + \sqrt{1 + \left(\frac{2l}{a+b}\right)^2}} \\ &= \frac{4(4) \left(\frac{3.45}{1.04}\right)^2}{1 + \sqrt{1 + \left[\frac{2(3.45)}{2.45 + 1.25}\right]^2}} \\ &= \frac{16(3.32)^2}{1 + \sqrt{1 + \left(\frac{6.9}{3.7}\right)^2}} = \frac{16(11.02)}{1 + \sqrt{1 + (1.865)^2}} \\ &= \frac{176.4}{1 + \sqrt{4.48}} = \frac{176.4}{1 + 2.12} = \frac{176.4}{3.12} \end{aligned}$$

$$(C_N \alpha)_f = 56.6$$

Interference factor

$$\begin{aligned} K_{fb} &= 1 + \frac{R}{S + R} \\ &= 1 + \frac{.52}{3.45 + .52} = 1 + \frac{.52}{3.97} = 1 + .131 \end{aligned}$$

$$K_{fb} = 1.131$$

normal force on fins in presence of body

$$\begin{aligned} (C_N \alpha)_{fb} &= K_{fb} (C_N \alpha)_f \\ &= 1.131 (56.6) \end{aligned}$$

$$\boxed{(C_N \alpha)_{fb} = 64.0} \text{ Booster}$$

center of pressure

$$\begin{aligned} \bar{X}_f &= X_f + \Delta X_f \\ &= X_f + \frac{m(a+2b)}{3(a+b)} + \frac{1}{6} \left(a + b - \frac{ab}{a+b} \right) \\ &= 21.55 + \frac{.6 [2.45 + 2(1.25)]}{3(2.45 + 1.25)} \\ &\quad + \frac{1}{6} \left[2.45 + 1.25 - \frac{2.45(1.25)}{2.45 + 1.25} \right] \\ &= 21.55 + \frac{.2(2.45 + 2.5)}{3.7} + \frac{1}{6} \left(3.7 - \frac{3.06}{3.7} \right) \\ &= 21.55 + .0541(4.95) + \frac{1}{6} (3.7 - .83) \\ &= 21.55 + .27 + \frac{2.87}{6} = 21.82 + .48 \end{aligned}$$

$$\boxed{\bar{X}_f = 22.3 \text{ inches}} \text{ Booster}$$

Arcon-Hi Combination Calculations (Booster Plus Sustainer)

total normal force

$$C_{N\alpha} = (C_{N\alpha})_n + (C_{N\alpha})_{fb} + (C_{N\alpha})_{fb}$$

$$= 2 + 30.2 + 64$$

$$C_{N\alpha} = 96.2 \text{ Booster Plus Sustainer}$$

center of pressure of the entire rocket

$$\bar{X} = \frac{(C_{N\alpha})_n \bar{X}_n + (C_{N\alpha})_{fb} \bar{X}_f + (C_{N\alpha})_{fb} \bar{X}_f}{C_{N\alpha}}$$

$$= \frac{2(2.45) + 30.2(18.86) + 64(22.3)}{96.2}$$

$$= \frac{4.90 + 570 + 1428}{96.2} = \frac{2000}{96.2}$$

$$\bar{X} = 20.8 \text{ inches Booster Plus Sustainer}$$

ANALYSIS OF THE ARCON-HI SUSTAINER ALONE USING THE EQUATIONS

Once the two stages have been analyzed, the sustainer can be analyzed quite simply since all the normal forces and centers of pressure have already been calculated. Essentially, the booster fin terms are dropped from the Combination Calculations.

Arcon-Hi Combination Calculations (Sustainer Alone)

total normal force

$$C_{N\alpha} = (C_{N\alpha})_n + (C_{N\alpha})_{fb}$$

$$= 2 + 30.2$$

$$C_{N\alpha} = 32.2 \text{ Sustainer Alone}$$

center of pressure

$$\bar{X} = \frac{(C_{N\alpha})_n \bar{X}_n + (C_{N\alpha})_{fb} \bar{X}_f}{C_{N\alpha}}$$

$$= \frac{4.9 + 570}{32.2} = \frac{575}{32.2}$$

$$\bar{X} = 17.9 \text{ inches Sustainer Alone}$$

ANALYSIS OF THE TWO STAGE ARCON-HI USING THE CHARTS

Nose

(uses the equations as before)

normal force

$$(C_{N\alpha})_n = 2$$

center of pressure

$$\bar{X}_n = .466 L$$

$$= .466 (5.25'')$$

$$\bar{X}_n = 2.45 \text{ inches}$$

Sustainer Fins

normal force

$$\frac{S}{d} = \frac{2.2}{1.04} = 2.12, \quad \frac{l}{a+b} = \frac{2.25}{1.9+1.05} = \frac{2.25}{2.95} = .76$$

$$(C_{N\alpha})_f = 25.5 \text{ (from Chart 4)}$$

interference factor

$$\frac{R}{S} = \frac{.52}{2.2} = .236$$

$$K_{fb} = 1.191 \text{ (from Chart 5)}$$

normal force on fins in presence of body

$$(C_{N\alpha})_{fb} = K_{fb} (C_{N\alpha})_f$$

$$= 1.191 (25.5)$$

$$(C_{N\alpha})_{fb} = 30.4 \text{ Sustainer}$$

center of pressure

$$\frac{m}{a} = \frac{.85}{1.9} = .45, \quad \frac{b}{a} = \frac{1.05}{1.9} = .55$$

$$\frac{\Delta X_f}{a} = .405 \text{ (from Chart 6)}$$

$$\bar{X}_f = X_f + \left(\frac{\Delta X_f}{a} \right) a$$

$$= 18.1 + .405 (1.9) = 18.1 + .76$$

$$\bar{X}_f = 18.86 \text{ inches Sustainer}$$

Booster Fins

normal force

$$\frac{S}{d} = \frac{3.45}{1.04} = 3.32, \quad \frac{l}{a+b} = \frac{3.45}{2.45+1.25} = \frac{3.45}{3.7} = .93$$

$$(C_{N\alpha})_f = 57. \text{ (from Chart 4)}$$

interference factor

$$\frac{R}{S} = \frac{.52}{3.45} = .151$$

$$K_{fb} = 1.131 \text{ (from Chart 5)}$$

normal force on fins in presence of body

$$\begin{aligned} (C_{N\alpha})_{fb} &= K_{fb} (C_{N\alpha})_f \\ &= 1.131 (57) \end{aligned}$$

$$(C_{N\alpha})_{fb} = 64.4 \text{ Booster}$$

center of pressure

$$\frac{m}{a} = \frac{.6}{2.45} = .245, \quad \frac{b}{a} = \frac{1.25}{2.45} = .51$$

$$\frac{\Delta X_f}{a} = .305 \text{ (from Chart 6)}$$

$$\bar{X}_f = X_f + \left(\frac{\Delta X_f}{a} \right) a$$

$$= 21.55 + .305 (2.45) = 21.55 + .75$$

$$\bar{X}_f = 22.3 \text{ inches Booster}$$

Arcon-Hi Combination Calculations

(Booster Plus Sustainer)
uses equations as before

total normal force

$$\begin{aligned} C_{N\alpha} &= (C_{N\alpha})_n + (C_{N\alpha})_{fb} + (C_{N\alpha})_{fb} \\ &= 2 + 30.4 + 64.4 \end{aligned}$$

$$C_{N\alpha} = 96.8 \text{ Booster Plus Sustainer}$$

center of pressure of the entire rocket

$$\bar{X} = \frac{(C_{N\alpha})_n \bar{X}_n + (C_{N\alpha})_{fb} \bar{X}_f + (C_{N\alpha})_{fb} \bar{X}_f}{C_{N\alpha}}$$

$$= \frac{2(2.45) + 30.4(18.86) + 64.4(22.3)}{96.8}$$

$$= \frac{4.9 + 574 + 1437}{96.8} = \frac{2016}{96.8}$$

$$\bar{X} = 20.8 \text{ inches Booster Plus Sustainer}$$

ANALYSIS OF THE ARCON-HI SUSTAINER USING THE CHARTS

The procedure simply involves dropping the booster terms from the Combination Calculations.

Arcon-Hi Combination Calculations (Sustainer Alone)

total normal force

$$\begin{aligned} C_{N\alpha} &= (C_{N\alpha})_n + (C_{N\alpha})_{fb} \\ &= 2 + 30.4 \end{aligned}$$

$$C_{N\alpha} = 32.4 \text{ Sustainer Alone}$$

center of pressure of the entire rocket

$$\begin{aligned} \bar{X} &= \frac{(C_{N\alpha})_n \bar{X}_n + (C_{N\alpha})_{fb} \bar{X}_f}{C_{N\alpha}} \\ &= \frac{4.9 + 574}{32.4} = \frac{579}{32.4} \end{aligned}$$

$$\bar{X} = 17.9 \text{ inches Sustainer Alone}$$

ARCON-HI STABILITY CHECK

The C.P. and C.G. relations for both flight configurations (booster plus sustainer and sustainer alone) must be established since the rocket must be stable for both conditions. One caliber stability will in both cases be based on the body diameter (D).

$$D = 1.04''$$

At lift-off the C.G. can be no farther back than

$$\begin{aligned} X_{CG} &= \bar{X} - D \\ &= 20.8 - 1.04 \end{aligned}$$

$$X_{CG} = 19.76 \text{ inches from the nose}$$

For stable second-stage flight, the C.G. can be no farther back than

$$\begin{aligned} X_{CG} &= \bar{X} - D \\ &= 17.9 - 1.04 \end{aligned}$$

$$X_{CG} = 16.86 \text{ inches from the nose}$$

The location of acceptable balance points for both configurations is shown in the illustration on the next page.

9. APPENDIX A — ESTIMATING THE CENTER OF GRAVITY OF MODEL ROCKET DESIGN

Until a model rocket design is completely built, painted and ready to fly, its center of gravity (C.G.) can only be estimated. The way a model is glued, sanded, and finished can strongly affect its final C.G. location. Because of this, a new design should always be balance tested before it is flown. If the static margin isn't adequate, the C.G. location can then be changed and the rocket re-balanced by the proper addition or removal of weight. However, if the C.G. is estimated carefully and the construction and finishing of the rocket is well done, not much re-balancing will be needed.

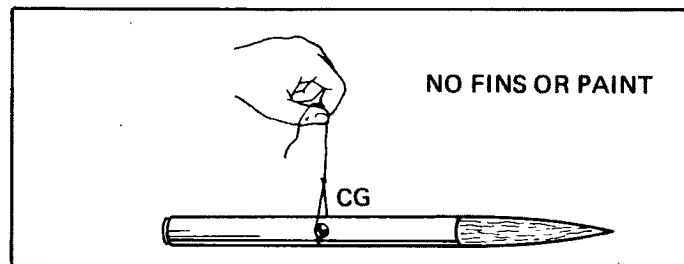
There are two basic techniques that can be used to estimate the C.G. location of a new rocket design. Both of them concentrate on the body of the rocket where most of the rocket's weight is concentrated. Remember, when you are finding the weight and C.G. location of a rocket, you must always include the largest engine expected to be used for a flight.

BALANCE TEST TECHNIQUE

When you are designing a model rocket, you will usually have on hand all the parts of the model you are going to build. Since the body design must be known before the fins can be designed, the C.G. location of the body is easily found by a simple balance test. The steps for doing this are similar to the normal procedure for building a model.

1. Decide what the design of the body will be.
2. Fit the body parts together or actually build the body.
3. Insert the engine(s) and parachute.
4. Determine C.G. location of the body by the string balance test.

FIGURE 23



If the rocket design fits any of the following descriptions, then the fins and painting will not change the C.G. location appreciably.

1. The overall length of the body is greater than twelve times its largest diameter.
2. The design has more than one engine (two or three stages or clusters).

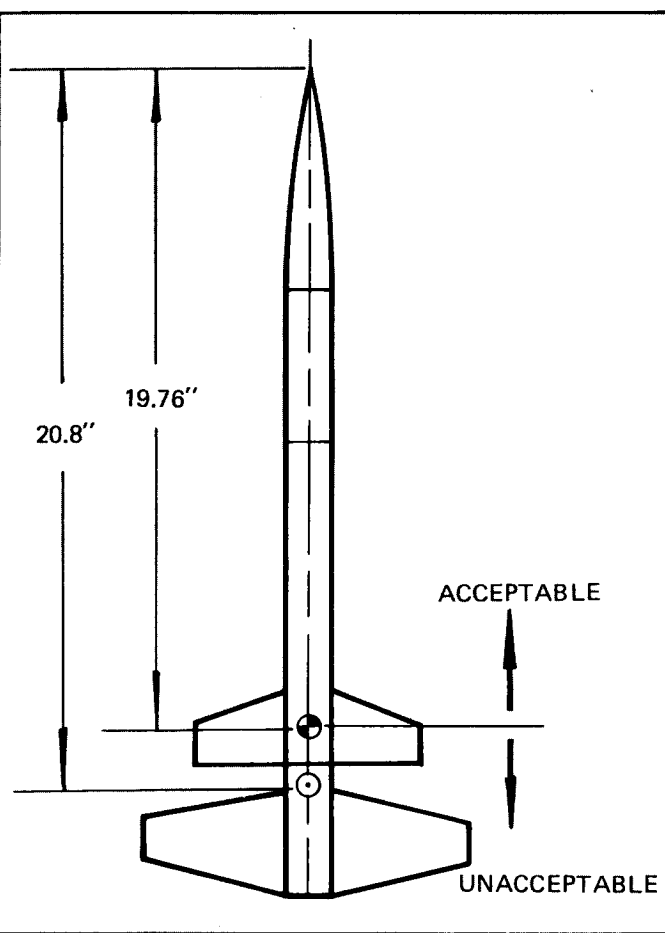
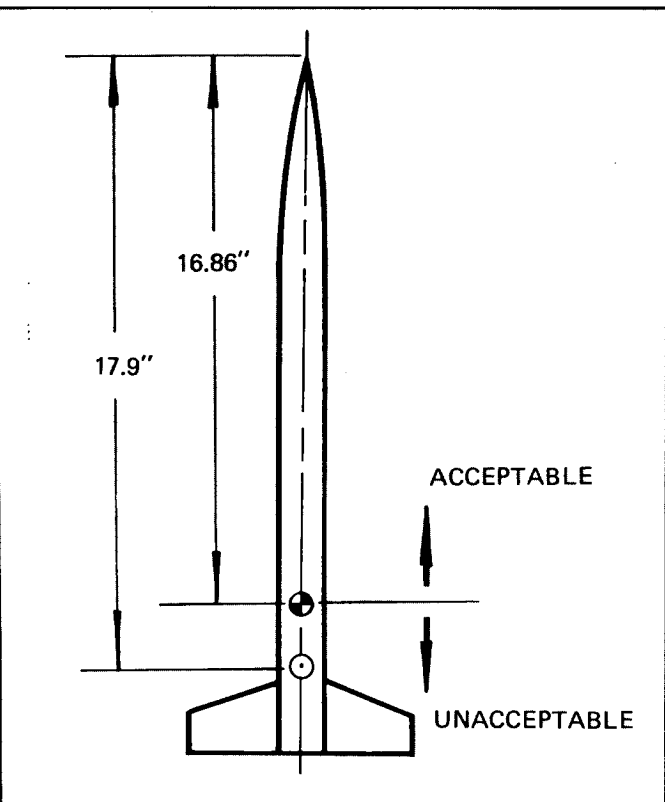


FIGURE 22



- The design has a payload that weighs at least as much as an engine.

Thus, if any of the above conditions is satisfied, the C.G. location of the design will be at the point determined by the balance test alone.

If none of the above conditions are satisfied, the effect of the fins can be estimated by assuming that the fins will weigh 5% of the total body weight and that the fin C.G. will be at the base of the rocket. Then the overall C.G. location can be estimated using the moment balance equation.

$$X_{CG} = \frac{(X_{CG})_B + .05L}{1.05}$$

where:

X_{CG} = C.G. location of body plus the fins measured from the nose tip.

$(X_{CG})_B$ = C.G. location, measured from the nose tip, of the body alone determined by the balance test.

L = Total length of the body.

For example, if the balance test showed that the C.G. of the body alone was located at 6.3 inches from the nose; and the body was 10 inches long over-all; then the estimated C.G. location would be,

$$X_{CG} = \frac{6.3 + .05(10)}{1.05} = \frac{6.3 + .5}{1.05} = \frac{6.8}{1.05}$$

$X_{CG} = 6.48$ inches from the nose.

As expected, the C.G. has shifted somewhat to the rear. Alternatively, the same answer would be obtained by adding 5% of the total body weight in the form of modeling clay to the base of the rocket and simply finding the new balance point. This is, after all, exactly what the above equation is physically simulating.

CALCULATION TECHNIQUE

Of course, it is not always possible to fit together or build your body design. If this is the case, you must rely on the theoretical calculations to estimate the body C.G. location. An outline of how to do this is given below.

- Determine the weight of each individual component of the body design (nose cone, body tube, engine, etc.) either by weighing it or by using the net weight given in the CENTURI catalog. Small light parts such as launch lugs and detailing don't have to be considered.
- Determine the C.G. location of each individual component. Some reasonable approximations of the C.G. locations of different components are given below.

a. Cylindrical-shaped components, such as body tubes, balsa plugs, engine mounts, thrust rings, tubing couplers, rolled streamers, and engines have their C.G.'s at their midpoints.

b. Nose cones and reduction fittings made of solid balsa will have their C.G.'s at about 2/3 their total length from the narrow end, including the parts that fit inside the body tube.

c. The parachute, shock cord, and lines can be considered as a single package which will have its C.G. at the middle of its length when packed into the body tube.

- Make an accurate full scale or scaled drawing of the body design that shows the placement of each component. Mark the appropriate C.G. location on each piece.
- Measure the distance between the nose tip and the C.G. location of each component. If the drawing is scaled, make sure you take this into account when you measure.
- Add the weights of the individual components to get the total weight of the body. In equation form,

$$W_B = W_1 + W_2 + W_3 + W_4 + \dots$$

where:

W_B is the total weight of the body.

Each W with a numbered subscript (W_1, W_2, W_3, W_4) represents the weight of an individual component.

The dots (...) at the end indicate that the numbers can go as high as is necessary, depending on the total number of different components.

- Multiply the weight of each individual component by the distance between its C.G. and the nose tip.
- Add together all the numbers resulting from Step 6.
- Divide the result of the addition in Step 7 by the total body weight from Step 5. The result of this division is the body C.G. location, measured from the nose tip.

The last three Steps 6, 7, and 8 can be represented by the equation:

$$(X_{CG})_B = \frac{W_1(X_{CG})_1 + W_2(X_{CG})_2 + W_3(X_{CG})_3 + \dots}{W_B}$$

where:

$(X_{CG})_B$ is the body C.G. location, measured from the nose tip.

Each X_{CG} with a numbered subscript ($X_{CG}1$, $X_{CG}2$, $X_{CG}3$ ) represents the distance between the nose tip and the C.G. of an individual component.

Notice that the above equation is a moment balance, just like the combination calculation equation used in determining the C.P.

Once the body C.G. location has been calculated, the effect of the fins can be estimated by the same method given in the Balance Test Technique section.

10. APPENDIX B — THEORY OF MOMENTS

The tendency of a force to rotate a body about a certain point is known as the moment of the force about this point. A mathematical formula for this tendency can be written as follows:

$$\text{(Moment)} = \text{(Force)} \times \text{(Moment Arm)}$$

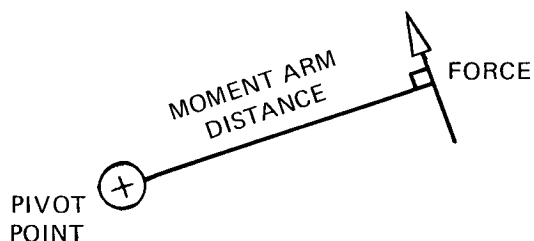


FIGURE 24

It can easily be seen that this formula physically represents the rotation tendency of interest. If the force is made larger, the moment and the tendency to rotate become proportionally larger. Similarly, if the moment arm (or lever arm) is increased, the same force will produce a corresponding larger moment.

Notice that we used an arrow to represent the force. It is very useful to do this on drawings since the arrow can represent the properties of magnitude and direction which are associated with "forces".

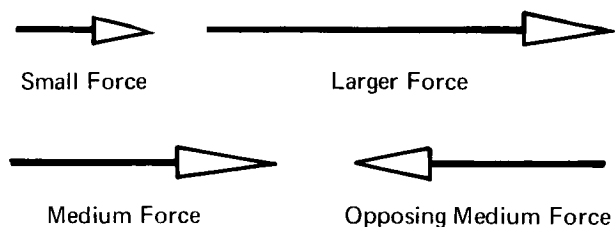


FIGURE 25

If the first three forces were considered to be acting in a positive (+) direction then the opposing force is acting in a negative (-) direction.

When the total normal force at a rocket's center of pressure is represented as an arrow, we can immediately see which direction the rocket would tend to rotate.

In the case of a stable rocket, the normal force produces a moment about the pivot point (the C.G.) which tends to bring the rocket back to zero degree angle-of-attack. Simultaneously, if the C.P. is ahead of the C.G., using an arrow to represent the normal force should help visualizing that the rocket has a tendency to keep increasing its angle-of-attack resulting in an unstable flight condition.

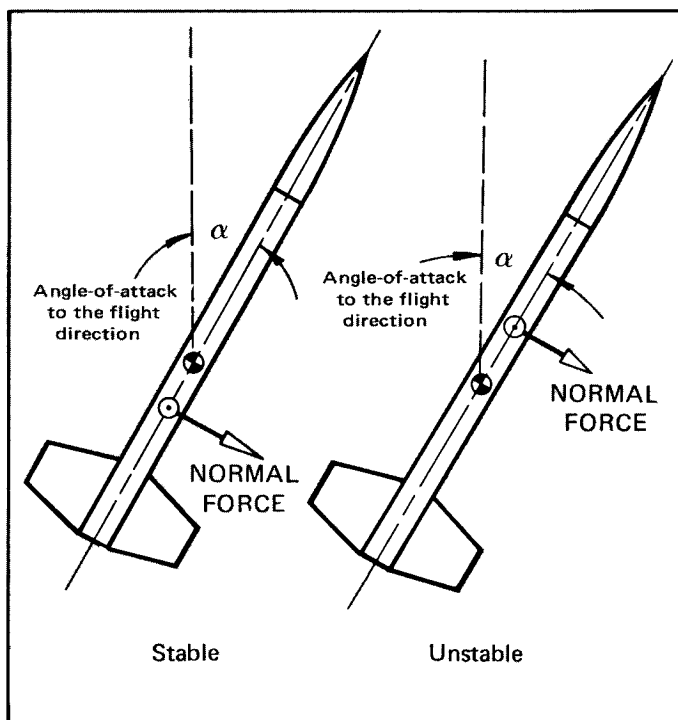


FIGURE 26

Physical quantities that possess both magnitude and a specified direction are referred to in mathematics as vectors. Some specific vectors model rocketeers will come across in their studies are velocity, acceleration, thrust, and aerodynamic drag.

Vector forces are common in everyday living; so are moments. Every time you open a door you are applying a force in a specific direction which produces a moment.

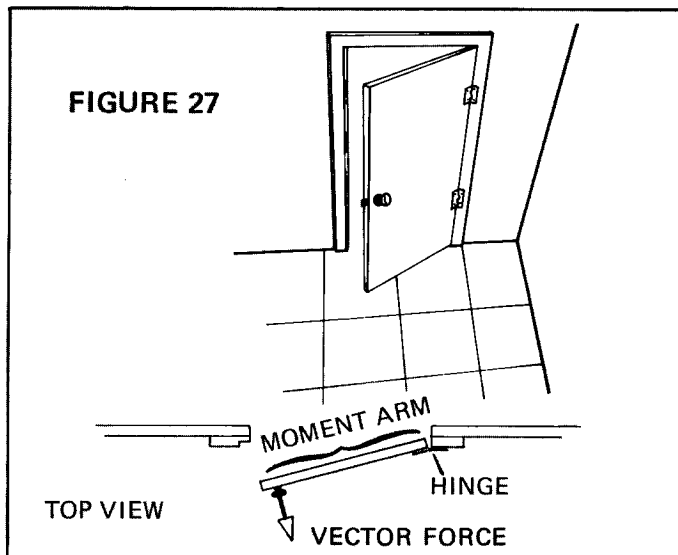


FIGURE 27

TOP VIEW

If you pull harder, the moment is larger and the door opens faster. We may ask ourselves why aren't door knobs placed as shown below?

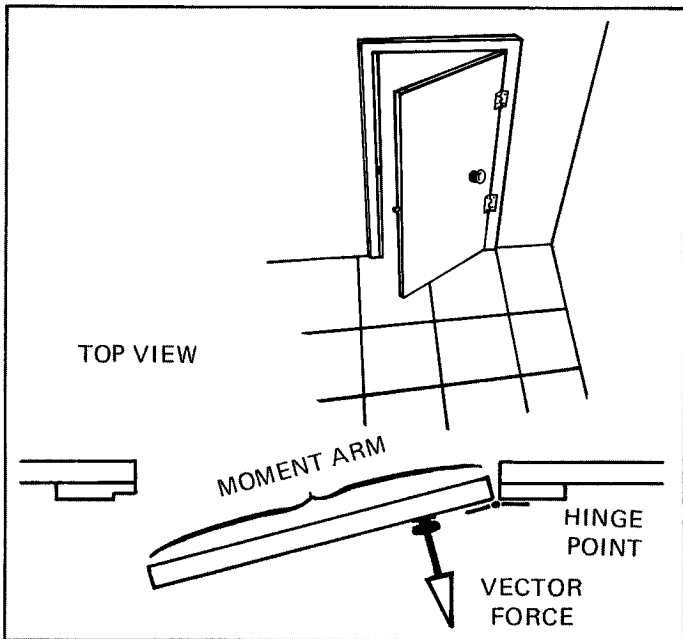


FIGURE 28

That's right! To get the door to respond in the usual manner you have to reproduce the usual moment. Since the moment arm is so short, you have to greatly increase the applied force in order to open the door. If you were having a strength contest using a door to push against as shown below, which of the two positions would you choose?

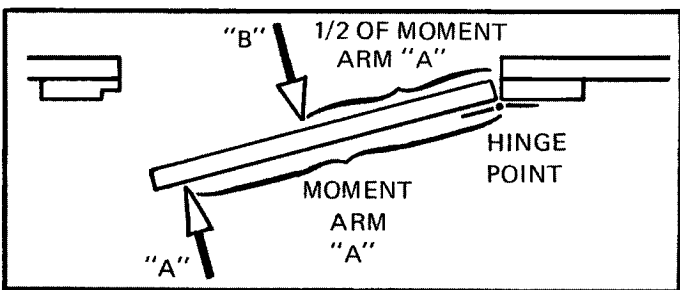


FIGURE 29

Intuitively, most everyone would pick position "A". If you think about it a while you'll mathematically understand why "A" has an advantage. The moment about the hinge point produced by "B" is his total pushing force times the moment arm. To match that moment, "A" only has to push half as hard as "B", since "A"'s moment arm is twice as long. If "A" pushes anything slightly over half as hard, he will cause "B" to lose.

You may want to experimentally verify the theory of moments using your dad's help. It is an excellent way to also verify if he is still twice as strong as you are.

We mentioned earlier that continual corrections to disturbances are being applied when riding a bicycle or steering a car. In both cases these corrections are also due to moments produced by vector forces.

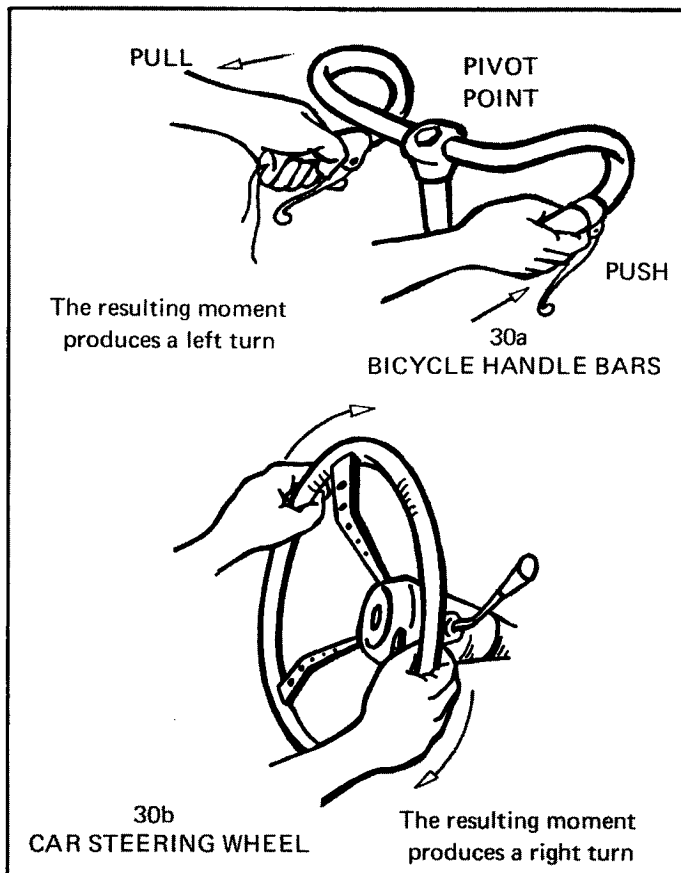


FIGURE 30

11. APPENDIX C—RESOLUTION OF FORCES

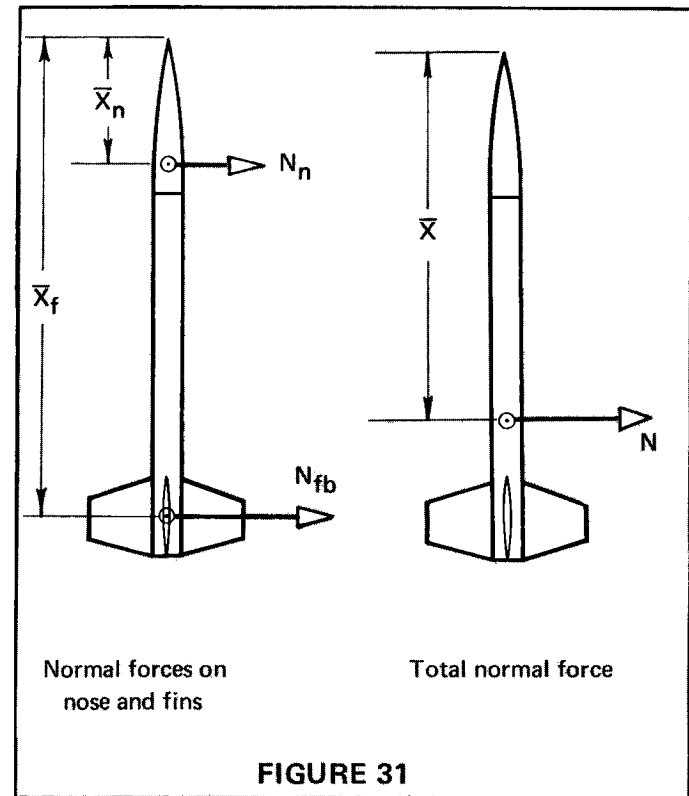
Scientists and engineers gain understanding of complex problems by simplifying them as much as possible. It would be very difficult and time consuming for us to draw general conclusions concerning stability if we could only work with the actual pressure force distribution over the rocket as shown previously in Figure 3. Knowing this distribution meant nothing until it was discovered that by simplifying the model into separate regions, general equations could be developed that would give the total normal force on these regions.

However, even knowing the force on each region, in itself, doesn't help in deciding if a rocket will be stable or not. Only by completing one more simplification does the stability question reduce to one that can be understood easily. This last reduction in complexity involves replacing all the forces acting separately on each region by a single force which would physically cause the same effect on the rocket in free flight. This is called resolving the forces. In effect what has been termed the "total normal force" throughout the report doesn't really exist. Applied to the rocket though, this fictional force would produce the exact same effect on the rocket's motion as the actual distributed air pressure forces.

In other words, the single force has a magnitude equal to the sum of all the actual distributed forces and most important it produces the same moment about the pivot point that the actual distributed forces produce.

12. APPENDIX D ----- WHY $C_{N\alpha}$ CAN BE USED TO REPRESENT THE TOTAL NORMAL FORCE (N)

The following derivation, using the principle of resolution of forces, demonstrates why it is mathematically acceptable to replace the normal forces by their associated dimensionless coefficients ($C_{N\alpha}$'s) in the moment balance equations.



The single force must have the same value as the total of the two separate forces

$$N = N_n + N_{fb}$$

The total moment about the reference point due to the two actual normal forces is

$$M_1 = N_n \bar{X}_n + N_{fb} \bar{X}_f$$

and the moment about the reference point due to the single replacement force will be

$$M_2 = N \bar{X} = (N_n + N_{fb}) \bar{X}$$

where \bar{X} is the presently unknown location for the replacement force.

Next, by writing a balance equation we make the moment produced by the replacement force identical to the actual total moment.

$$M_2 = M_1$$

$$N \bar{X} = N_n \bar{X}_n + N_{fb} \bar{X}_f$$

Dividing both sides by the single force N, we obtain an equation for the unknown location of the single replacement force

$$\frac{N \bar{X}}{N} = \bar{X} = \frac{N_n \bar{X}_n + N_{fb} \bar{X}_f}{N}$$

or

$$\bar{X} = \frac{N_n \bar{X}_n + N_{fb} \bar{X}_f}{N + N_{fb}}$$

Now we write the exact equation for each normal force

$$N_n = (C_{N\alpha})_n \frac{1}{2} \rho V^2 \alpha A_r$$

$$N_{fb} = (C_{N\alpha})_{fb} \frac{1}{2} \rho V^2 \alpha A_r$$

Substitution of the exact values into the \bar{X} equation we get

$$\bar{X} = \frac{[(C_{N\alpha})_n \frac{1}{2} \rho V^2 \alpha A_r] \bar{X}_n + [(C_{N\alpha})_{fb} \frac{1}{2} \rho V^2 \alpha A_r] \bar{X}_f}{(C_{N\alpha})_n \frac{1}{2} \rho V^2 \alpha A_r + (C_{N\alpha})_{fb} \frac{1}{2} \rho V^2 \alpha A_r}$$

notice that the $\frac{1}{2} \rho V^2 \alpha A_r$ can be withdrawn from the equations. This gives

$$\bar{X} = \frac{[(C_{N\alpha})_n \bar{X}_n + (C_{N\alpha})_{fb} \bar{X}_f] \frac{1}{2} \rho V^2 \alpha A_r}{[(C_{N\alpha})_n + (C_{N\alpha})_{fb}] \frac{1}{2} \rho V^2 \alpha A_r}$$

and it is easy to see that these terms cancel, leaving

$$\bar{X} = \frac{(C_{N\alpha})_n \bar{X}_n + (C_{N\alpha})_{fb} \bar{X}_f}{(C_{N\alpha})_n + (C_{N\alpha})_{fb}}$$

which is the equation we've been using throughout the report to find the overall center of pressure.

Thus, even the $C_{N\alpha}$ is just one of the factors affecting the normal force acting on the rocket, it is the only factor which varies for each region.

13. APPENDIX E -- ROCKET FLEXIBILITY

An interesting point can be made at this time regarding cancelling the angle-of-attack from the previous equation since they were all identical. Assumption 6 specified that the rocket must be an axially symmetric rigid body. If a model rocket is not a rigid body and can bend, various parts of it could have different angles-of-attack.

The angle-of-attack of each region would have to be accounted for in the overall center-of-pressure equation. If the rocket was flexible enough to bend then the distributed normal pressure forces could deflect the rocket so that the fins are at say $\frac{1}{2}$ degree angle-of-attack and the nose would be at say 2 degrees angle-of-attack as shown.

AIR FLOW

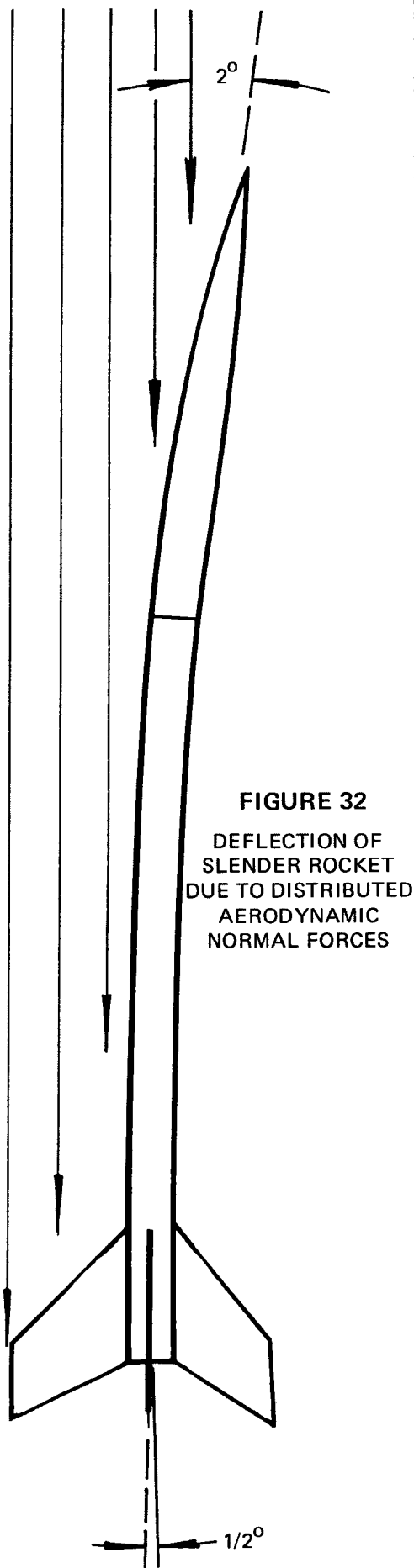


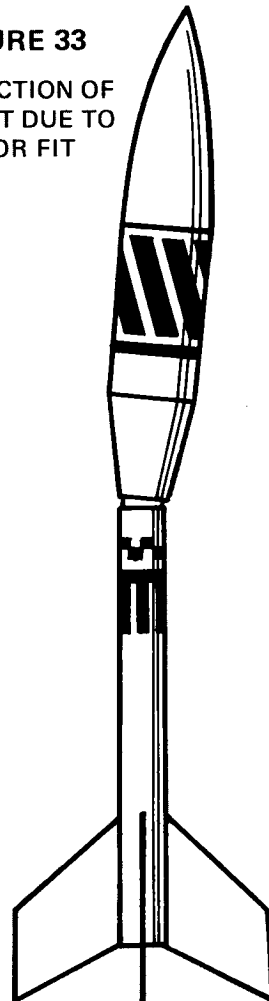
FIGURE 32
DEFLECTION OF
SLENDER ROCKET
DUE TO DISTRIBUTED
AERODYNAMIC
NORMAL FORCES

This kind of problem exists with all real missiles to some degree and is carefully accounted for by the engineers doing center-of-pressure calculations.

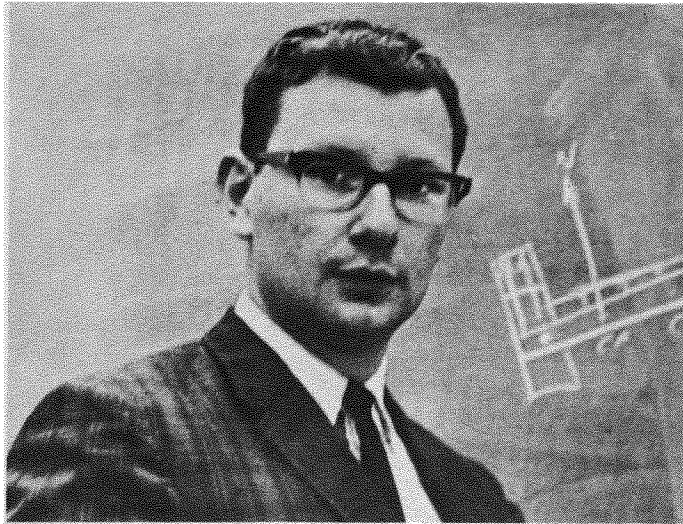
One of our country's earliest Satellite launch vehicles, the Vanguard lacked adequate lengthwise stiffness, (it was very long and slender) and that section of the rocket containing the gyro control sensed angles-of-attack which had nothing to do with where the nose of the vehicle was pointed. Needless to say, it caused problems. Fortunately, the model rocket kits of today are generally more than stiff enough and the assumption of a rigid body is quite realistic.

Watch out though if you design a long rocket that has a break in the middle. If it is not a good fit and has some wiggle to it, the nose could quite easily get to a higher angle-of-attack than the fins. This would create proportionally larger normal force ahead of the C.G.; possibly more than the fins produce behind the C.G. and could bring the new resultant center of pressure forward — ahead of the C.G. This example does not actually represent a flexible body which bends gradually over its entire length, but instead is really two rigid bodies connected by a sloppy mechanical joint. Either way, this still violates assumption 6 and, as such, even careful C.P. analysis may not produce useable results.

FIGURE 33
DEFLECTION OF
ROCKET DUE TO
POOR FIT



ABOUT THE AUTHOR



JIM BARROWMAN

JIM BARROWMAN is presently employed by NASA's Goddard Space Flight Center in Greenbelt, Maryland as an Aerospace Engineer in Fluid and Flight Dynamics. Jim was born in Toledo, Ohio 25 years ago and graduated from the University of Cincinnati in 1965 with a Bachelor of Science degree in Aerospace Engineering. Jim, together with his wife Judy and their two year old daughter Julie Ann, presently reside in Hyattsville, Maryland where he continues graduate level studies at nearby Catholic University of America.

He has been employed by NASA since 1961, and worked as a co-op student trainee during the first four years. Here he performed magnetometer data reduction for the Vanguard III Satellite, was a member of a Mars Atmospheric Entry Capsule design team, assisted in the thermal design of the IMP (Interplanetary Monitoring Probe) Spacecraft, performed dynamic motion studies of the Aerobee 150A Sounding Rocket, and wrote a computer program for his aerodynamic analysis of the Tomahawk, Nike-Tomahawk, and Black Brant IIB Sounding Rockets.

Jim's interest in Model Rocketry dates back to 1964. He enjoys working with young people, and in addition to occasional lectures to Junior and Senior High School groups on aerospace careers, he has become the senior advisor for the NARHAMS Section of the National Association of Rocketry. The method Jim developed for calculating the exact center of pressure of a model rocket earned him a First Place Senior Research and Development Award at NARAM-8 in August, 1966. He is also an active NAR Trustee and has been appointed Chairman of the NAR Publications Committee and Contest Director for NARAM-10. Jim's obviously few leisure moments are spent sailing and experimenting with his favorite model rocket - - - the boost glider.

TECHNICAL INFORMATION REPORTS

STABILITY OF A MODEL ROCKET IN FLIGHT TIR-30

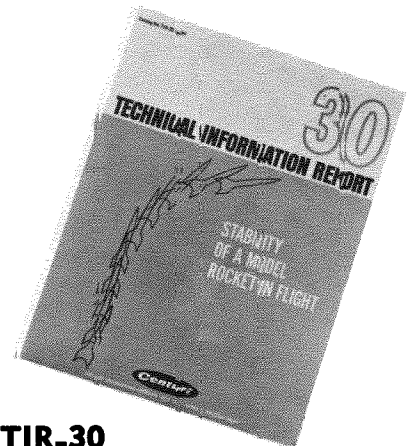
BY JAMES BARROWMAN

Highly recommended for a non-theoretical explanation of stability. TIR-30 does not delve into any of the mathematics of calculating the center of pressure covered here in TIR-33 and, as such, provides an easy-to-follow introduction to the subject. The report provides valuable information to those who want to gain insight and a true understanding of basic stability concepts.

Soon after being introduced to model rocketry, most rocketeers hear the statement that a stable rocket flight requires that the center of pressure must lie behind the center of gravity.

What is center of pressure? Why is the rocket's balance point called center of gravity? What does the word "stability" really mean? Are there any simple tests which tell you whether or not a new rocket design will be "stable"? How come rockets arc over and head into the wind (weathercock) during thrusting and coasting instead of being blown along with the wind as a feather or piece of paper would?

We think that these and other important questions are carefully and clearly answered in CENTURI's TIR-30 with the assistance of a total of 42 illustrations. In addition, a section has also been included on how the amount of stability can be adjusted to improve altitude performance.



TIR-30
\$.75 Postpaid

MODEL ROCKET ALTITUDE PERFORMANCE TIR-100

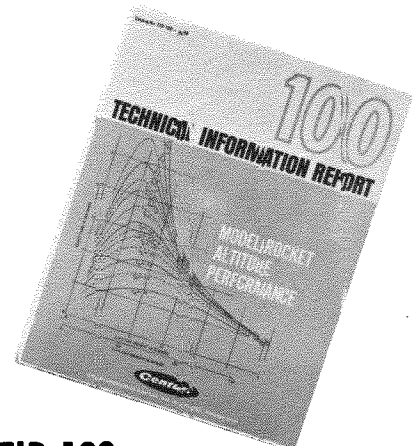
BY DOUGLAS MALEWICKI

Presents easy-to-use graphs for accurately predicting the peak altitudes which can be reached by single-stage rockets using 1/4A thru F type engines. Also included are graphs for selecting the best delay time to use. No mathematical calculations, whatever, are involved in finding altitudes or engine delay times. These graphs, along with the numerous discussion sections of this report, should be most useful in helping the rocketeer towards a real understanding of how engine power, rocket weight, and aerodynamic drag on various nose and body shapes are interrelated in their effects on performance.

All the altitude data in this report is based entirely on Centuri's latest model rocket engines. The National Association of Rocketry

(NAR), the Federation Aeronautique Internationale (FAI), and the United States Model Rocket Manufacturers Association have all recently adopted the Metric System of measurement. As a result, Centuri model rocket engines were redesigned to give the maximum total impulse allowed in each new Metric category. These modifications mean that the new engines have slightly different average thrust levels and thrust duration characteristics than the old engines and TIR-100 properly reflects these changes.

Also note that altitude performance graphs for the new "C" type engines with time delays are included. These new engines have 50 per cent more total impulse than the old C.8-0 booster engines and twice the total impulse of the old "B" type engines.



TIR-100
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