AEM 1905 – High-Power Rocketry

Calculating the motion of a rocket for purely vertical flight.

Phase I – Boost phase: motor firing (rocket losing mass), going upwards faster and faster (accelerating upwards)

Phase II – Coast phase: motor burned out so mass is now fixed, still going upwards but slowing down (accelerating downwards)

Phase III – Descent phase: parachute deployed, mass still fixed, going downwards at a constant (terminal) velocity (accel = 0)

Physics refresher (define “up” as positive)

x -- position (height) above the ground
v -- velocity; rate of change of position (positive means moving up)
a -- acceleration; rate of change of velocity (positive means velocity is changing so as to favor upwards-motion)
t -- elapsed time

For constant-acceleration situations, velocity changes as

$$v[t] = v_{init} + a\,t$$

and position changes as

$$x[t] = x_{init} + v_{init}\,t + \frac{1}{2}\,a\,t^2$$

However it turns out that rocket motion is rarely constant-acceleration.

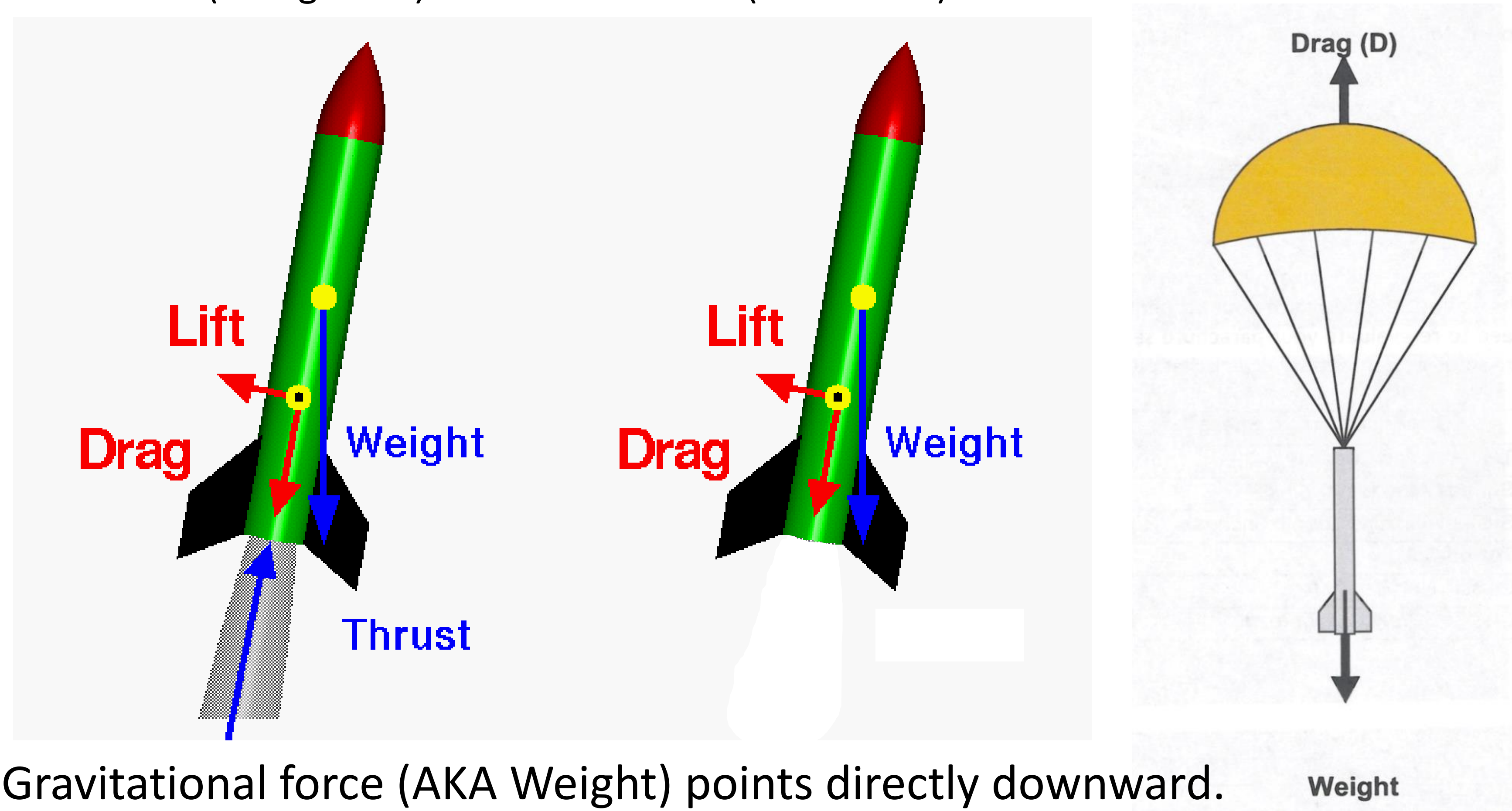
According to Newton’s 3 “Laws of Motion” <u>forces explain motion</u> – in particular, the “net force” on an object explains its acceleration.

The forces acting on a rocket during various parts of a flight.
Weight acts through the CG. Aerodynamic forces act through the CP.

Phase I: Boost
(losing mass)

Phase II: Coast
(fixed mass)

Phase III:
Descent



- Gravitational force (AKA Weight) points directly downward.
- Thrust points forward, along the rocket axis. Boost phase only.
- Drag always points along the rocket axis opposite the direction of travel.
- Any aerodynamic force normal (perpendicular) to the rocket axis is called Lift.

The "net force" on an object, denoted $\sum \vec{F}$, is the vector sum of all the forces. That means add the forces, paying attention to direction.

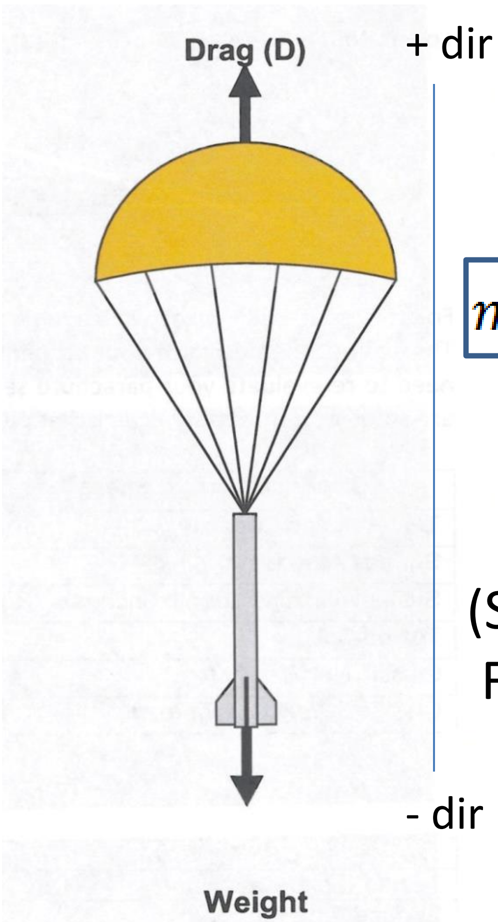
Newton's 1st Law: If $\sum \vec{F} = 0$ (i.e. there is no net force AKA the forces acting on the object "are balanced") then $\vec{v}$ is constant in size and dir. Put another way, if there is no net force the object keeps doing what it is doing (e.g. continues moving in a straight line at a constant speed).

Newton's 2nd Law: (when mass is constant): $\vec{a} = \frac{\sum \vec{F}}{m}$ more-commonly written as $\sum \vec{F} = m\,\vec{a}$. More-generally, as originally stated by Newton, net force explains changes in momentum $\vec{p} = m\,\vec{v}$ where velocity can change (called acceleration) or mass can change or both can change.

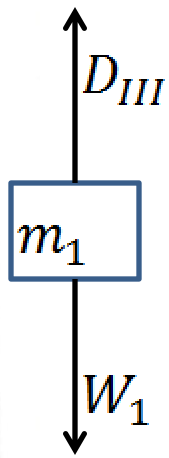
Newton's 3rd Law: When two objects A and B interact, both objects get forced. The size of the force is the same, but the direction is opposite. $\vec{F}_{A\ on\ B} = -\vec{F}_{B\ on\ A}$ (Note: Since the objects may have different masses, these forces may not lead to the same acceleration of the two objects). (Note: These forces act on different objects so they never cancel.)

Analysis of Phase III – descent at a constant (terminal) velocity v_T.

Phase III: Descent



(Realistic) Force Diagram



$a_{III} = 0$

(Simplified) Free-body Diagram

Weight $W_1 = m_1\, g$ where $g = 9.8\ m/s^2 = 32.2\ ft/s^2$ is the accel of grav. free-fall.

Drag $D_{III} = \frac{1}{2}\rho\, {v_T}^2\, A_P\, C_{DP}$. Here v_T is the terminal velocity and A_P is the parachute frontal area and C_{DP} is the chute drag coefficient (between 1 and 3) and

$\rho = 0.0749\, lb_{mass}/ft^3 = 1.20\, kg/m^3$

is the density of air (standard conditions).

Apply Newton's 2nd Law:

$$\sum F = D_{III} - W_1 = m\, a_{III} = 0$$

so $\frac{1}{2}\rho\, {v_T}^2\, A_P\, C_{DP} - W_1 = 0$

Can solve for terminal velocity v_T if A_P and C_{DP} are known.

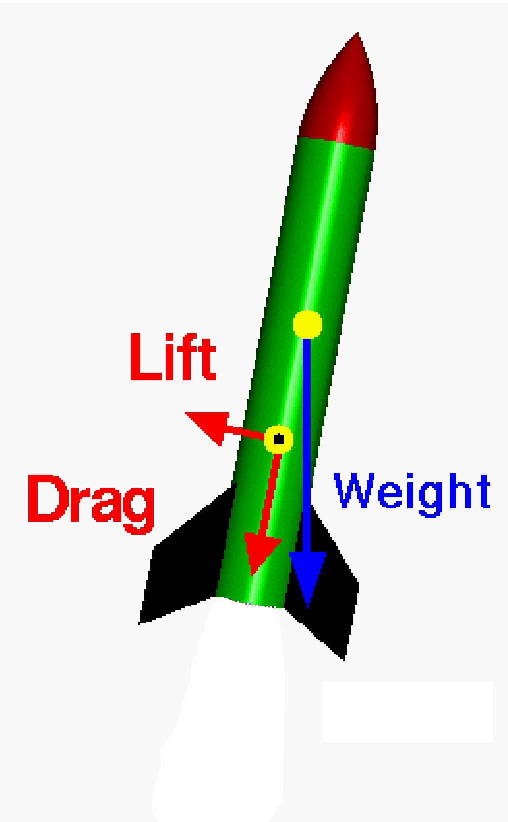
$$v_T = \sqrt{\frac{2\, W_1}{\rho\, A_P\, C_{DP}}}$$

Can estimate v_T and C_{DP} and solve for parachute area A_P ("sizing").

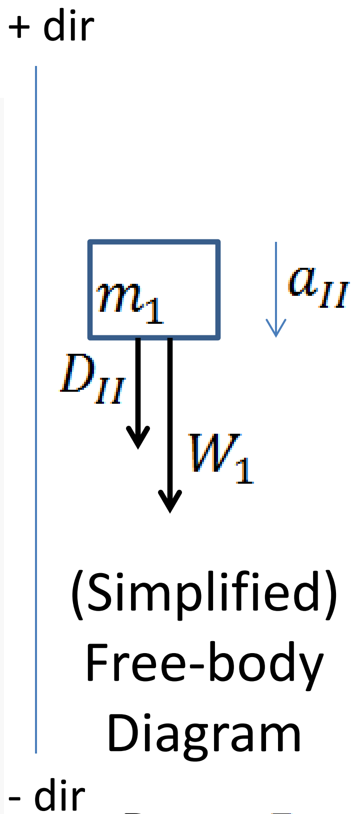
$$A_P = \frac{2\, W_1}{\rho\, {v_T}^2\, C_{DP}}$$

Analysis of Phase II – coasting up after motor burnout from x_1 with speed v_1 .

Phase II: Coast
(fixed mass)



(Realistic)
Force
Diagram



(Simplified)
Free-body
Diagram

Weight $W_1 = m_1\, g$ where $g = 9.8\ m/s^2 = 32.2\ ft/s^2$ is the accel of grav. free-fall.

Drag $D_{II} = \frac{1}{2}\rho\, v^2\, A_R\, C_{DR}$. Here v is the (diminishing) velocity and A_R is the rocket frontal area and C_{DR} is the rocket drag coeff. (streamlined ⇨ small) and $\rho = 0.0749\ lb_{mass}/ft^3 = 1.20\ kg/m^3$ is the density of air (standard conditions).

Apply Newton's 2nd Law:

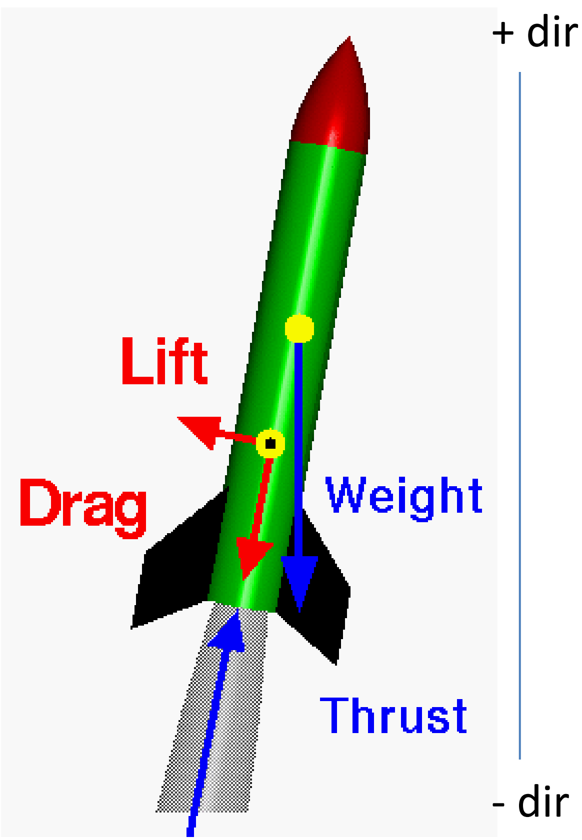
$$\sum F = -D_{II} - W_1 = m\, a_{II}$$

Drag D_{II} is likely to be small due to the streamlined rocket body and it gets even smaller as v drops to 0 at apogee. If we neglect drag altogether we see that $-W_1 = -m_1\, g = m\, a_{II}$ ⇨ $a_{II} = -g$ ("free fall").

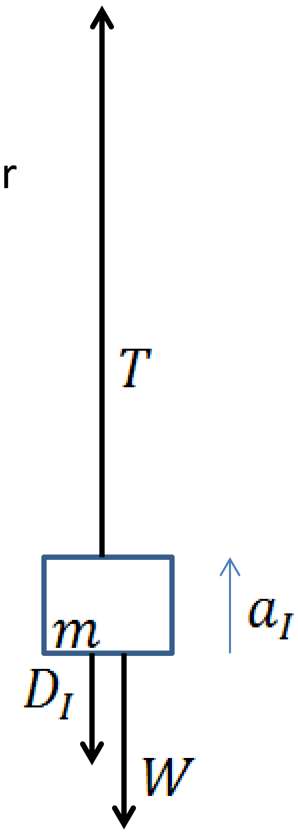
Constant accel. so $x_2 = x_1 + v_1\, t_{coast} - \frac{1}{2} g\, {t_{coast}}^2$ and $v_{apogee} = 0 = v_1 - g\, t_{coast}$ so $t_{coast} = v_1/g$.

Analysis of Phase I – boost phase to height x_1 and velocity v_1 at time t_b.

Phase I: Boost
(losing mass)



(Unrealistic)
Force
Diagram
(The thrust vector should be much bigger than W.)



(Simplified)
Free-body
Diagram

The largest force is now thrust T, which is not constant in time. The rocket mass goes down too, from $m_{full} \equiv m_0$ to $m_{empty} \equiv m_1$ where $m_0 = m_1 + m_{prop}$.

At any moment in time the weight W is related to the mass m by $W = m\,g$ where $g = 9.8\ m/s^2 = 32.2\ ft/s^2$ is the accel. of gravitational free-fall.

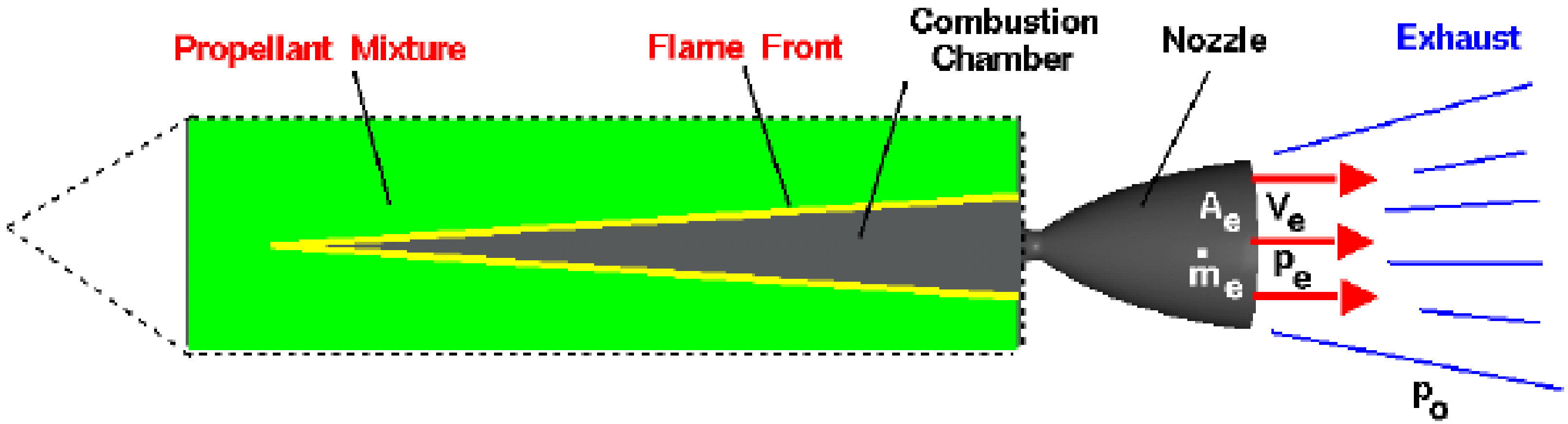
Drag $D_I = \frac{1}{2}\rho\, v^2 A_R\, C_{DR}$. Here v is the (increasing) velocity and A_R is the rocket frontal area and C_{DR} is the rocket drag coeff. (streamlined ⇨ small) and $\rho = 0.0749\ lb_{mass}/ft^3 = 1.20\ kg/m^3$ is the density of air (standard conditions).

Try to apply Newton's 2nd Law:

$$\sum F = T - D_I - W = m\,a_I$$

But... this is NOT constant acceleration motion!

How a solid rocket motor burns (inside) and produces thrust.

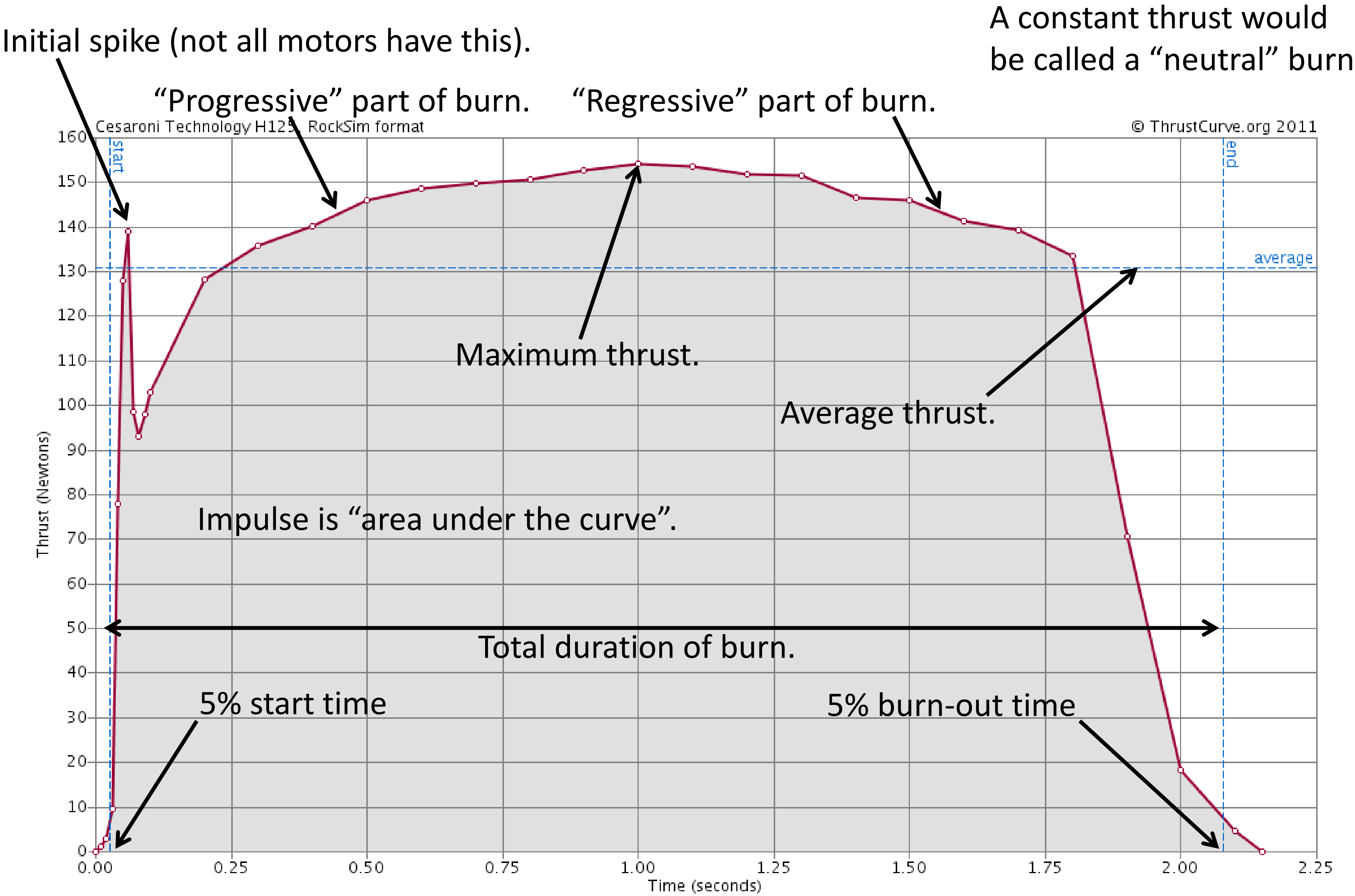


Total thrust T can be written as the sum of two separate forces due to:
(a) the momentum exchange between the rocket and its exhaust and
(b) the pressure imbalance between the exhaust and the environment.

$$T = T_{mom\ exchange} + T_{press\ imbalance}$$
$$T_{mom\ exchange} = \dot{m}_{exhaust}\ v_{exhaust} \equiv \dot{m}_e\ v_e$$
$$T_{press\ imbalance} = (P_{exhaust} - P_{environ})\ A_{exhaust}$$
$$\equiv (P_e - P_0)\ A_e$$

Here $\dot{m}$, pronounced "m-dot," refers to the rate the rocket mass is changing in time (i.e. the mass flow rate out of the rocket).

Thrust curve for Cesaroni Pro38 266H125-12 Classic motor (AKA “H125”).



Letter classifications of rocket motors refer to the range of total impulse, not to the average or maximum thrust.

Impulse Class		Category
H	160.01Ns to 320.01Ns	Level 1
I	320.01Ns to 640.00Ns	
J	640.01Ns to 1280.00Ns	Level 2
K	1280.01Ns to 2560.00Ns	
L	2560.01Ns to 5120.00Ns	
M	5120.01Ns to 10240.00Ns	Level 3
N	10240.01Ns to 20480.00Ns	
O	20480.00Ns to 40960.00Ns	

“Cesaroni Pro38 266H125-12” (AKA a “Cesaroni H125” motor)

manufacturer

diameter (in mm)

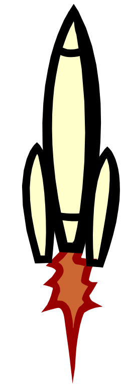
total impulse (in N sec)

impulse class

average thrust (in N)

built in delay time (in sec)

Because every motor is different it is hard to calculate general rocket motion from Newton's Laws for the entire boost time period. Either do it moment by moment (with a computer) or else…



… use "**the rocket equation**" (Tsiolkovsky ~ 1900)!

Impulse is "the area under the thrust vs. time curve": $I_{tot} = \int T\,dt$

Approximate total impulse from the thrust curve as: $I_{tot} = T_{avg} t_b$

Motor efficiency (AKA "specific impulse") characterizes how much impulse is delivered per weight of propellant burned.
I_{SP} varies from 170 to 220 sec for solid motors.

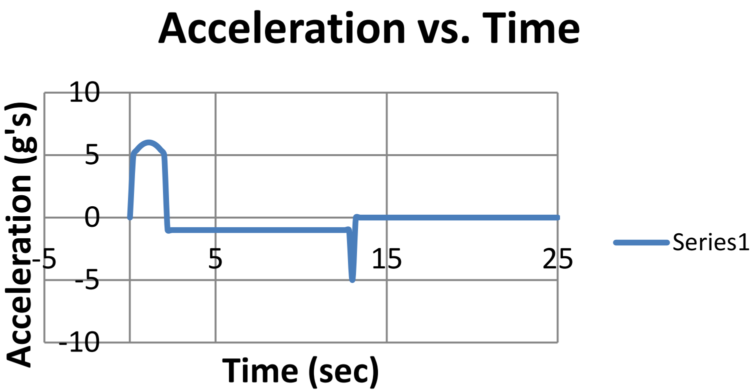
$$I_{SP} \equiv {}^{I_{tot}}\!/_{m_{pp}\,g}$$

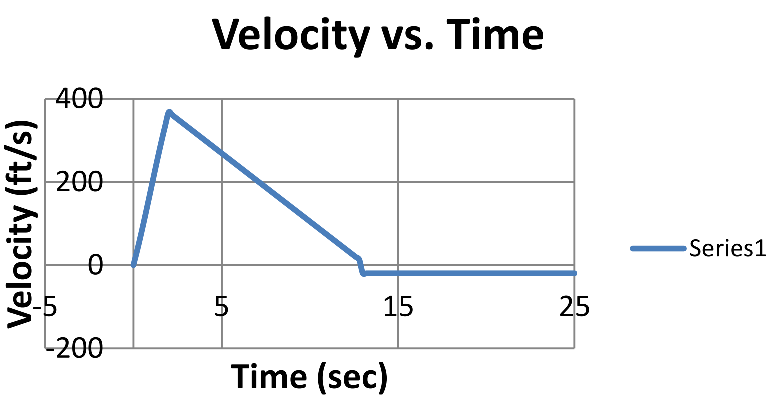
The "ideal rocket equation" (no gravity, no drag) says velocity can change by

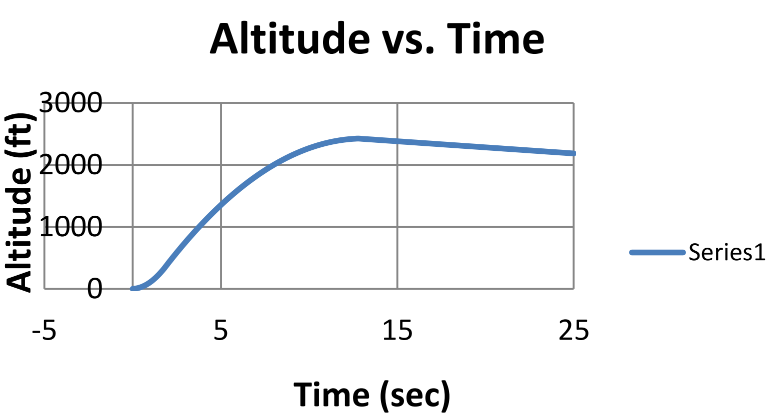
$\Delta v_{ideal} = I_{SP}\,g\,ln\left[{}^{m_{init}}\!/_{m_{final}}\right]$ so if $v_0 = 0$ then $v_1 = I_{SP}\,g\,ln\left[{}^{m_0}\!/_{m_1}\right]$.

Here $ln[x]$ means "the natural logarithm of x". Still neglecting drag, for a vertical boost we pay a "gravity penalty" so $v_1 = I_{SP}\,g\,ln\left[{}^{m_0}\!/_{m_1}\right] - g\,t_b$.

Motion graphs.







Graph interpretation comments:

In these graphs you should be able to spot Phase I – Boost, Phase II – Coast, and Phase III – Descent.

You should also be able to identify and understand the timing of maximum acceleration, maximum velocity, and maximum altitude (AKA apogee).

When does burnout occur? When does parachute deployment occur? What is the delay time? How can you identify these events in the graphs? How would the graphs change if the delay time were increased by 3 sec? Reduced by 3 sec?

What would each of the graphs look like if they were extended beyond 25 sec? How fast does the rocket descend under parachute? When does it touch down?

Programs like RockSim or OpenRocket might show all 3 motion values (and more!) on a single graph, which can be confusing.

(Pay close attention to vertical axes – altitude on the left, velocity and acceleration on the right.)