Lift-off of a single particle in an Oldroyd-B fluid

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Abstract

In this paper we study the lift-off of a single circular particle in an Oldroyd-B fluid by two-dimensional direct numerical simulation. A heavy particle, placed on the bottom of the channel, is driven forward by a plane Poiseuille flow. At a certain critical shear Reynolds number the buoyant weight of the particle just balances the upward thrust from the hydrodynamic force. As the Reynolds number is increased beyond the critical value, the particle attains an equilibrium position away from the wall. The aim of the calculation is to study the critical condition of lift-off. A correlation for the critical condition of lift-off is obtained assuming a surface roughness (or the particle separation from the wall) of 0.001d. It is seen that the fluid elasticity reduces the critical shear Reynolds number for lift-off. The effect of the gap size between the particle and the wall, on the lift force, is also studied. The lift force on the particle is seen to be singular as the gap size approaches zero. We also investigate the presence of multiple steady equilibrium positions, so far observed only for Newtonian fluids, when the Reynolds number is greater than the critical value. We find that multiple equilibrium positions are observed even in Oldroyd-B fluids. This is due to the presence of two turning points of the equilibrium solution. The location of the turning point is shifted due to the elasticity of the fluid as compared to the Newtonian results.

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1. Introduction

Lift on a particle plays a central role in several applications e.g. in the oil industry we can consider the removal of drill cuttings in horizontal drill holes and sand transport in fractured reservoirs. A polymer solution is often used in these applications. Other problems include the cleaning of particles from surfaces, fines mobilization in porous media etc. The theory of lift for these particle applications is undernourished and in most simulators no lift forces are modeled. A force experienced by a particle moving through a fluid with circulation (or shearing motion for a viscous fluid) shall be referred to as the lift force in the present work.

The problem of inertial lift on a moving sphere near a plane wall in shear flow of a Newtonian fluid has been analyzed as a perturbation of Stokes flow with inertia by Leighton & Acrivos (1985), Cherukat & McLaughlin (1994) and Krishnan & Leighton (1995). These studies lead to specific and useful analytic results expressed in terms of the translational and rotational velocities of the sphere and the shear-rate.

The lift force on a sphere in a shear flow of a second-order fluid was studied by Hu & Joseph (1999). Their analysis is valid for slow and slowly varying flows. The sphere was allowed to rotate and translate. They found that, due to the normal stress effect, the flow gives rise to a positive elastic lift force on the sphere when the gap between the sphere and the wall is small. They concluded that smaller particles would be easier to suspend due to the elastic lift in contrast to the inertial lift, which does not suspend small particles. The lift force was found to be singular when the minimum gap between the sphere and the wall approaches zero.

Direct two-dimensional simulations of the motion of circular particles in wall bounded Couette and Poiseuille flows of a Newtonian fluid was done by Feng, Hu & Joseph (1994). Feng, Huang & Joseph (1995) numerically studied the lift force on an elliptic particle in pressure driven flows of Newtonian fluids. Numerical investigation of the motion of circular particles in Couette and Poiseuille flows of an Oldroyd-B fluid was done by Huang, Feng, Hu & Joseph (1997). Zhu (2000) studied the lift-off of particles from the bottom of a horizontal channel by simple shear (Couette) flow.

N. Patankar, Huang, Ko & Joseph (2001) studied the lift-off of a single particle in Newtonian and viscoelastic fluids by direct numerical simulation. They considered a particle heavier than the fluid driven forward on the bottom of a channel by a plane Poiseuille flow. After a certain critical Reynolds number the particle rises from the wall to an equilibrium height at which the buoyant weight just balances the upward thrust from the hydrodynamic force. A correlation for the critical shear Reynolds number for lift-off was obtained for a particle in a Newtonian fluid. In this paper we study the critical condition of lift-off of a circular particle close to the wall in a plane Poiseuille flow of an Oldroyd-B fluid. The results are compared to the corresponding correlation for the Newtonian case reported by N. Patankar *et al.* (2001). Beyond the critical Reynolds number the particle attains an equilibrium height away from the wall. We also investigate the presence of multiple equilibrium positions of the particle in an Oldroyd-B fluid.

2. Governing equations and the parameters of the problem

The two-dimensional computational domain for our simulations is shown in figure 1. We perform simulations in a periodic domain. The applied pressure gradient is given by $-\overline{p}$.

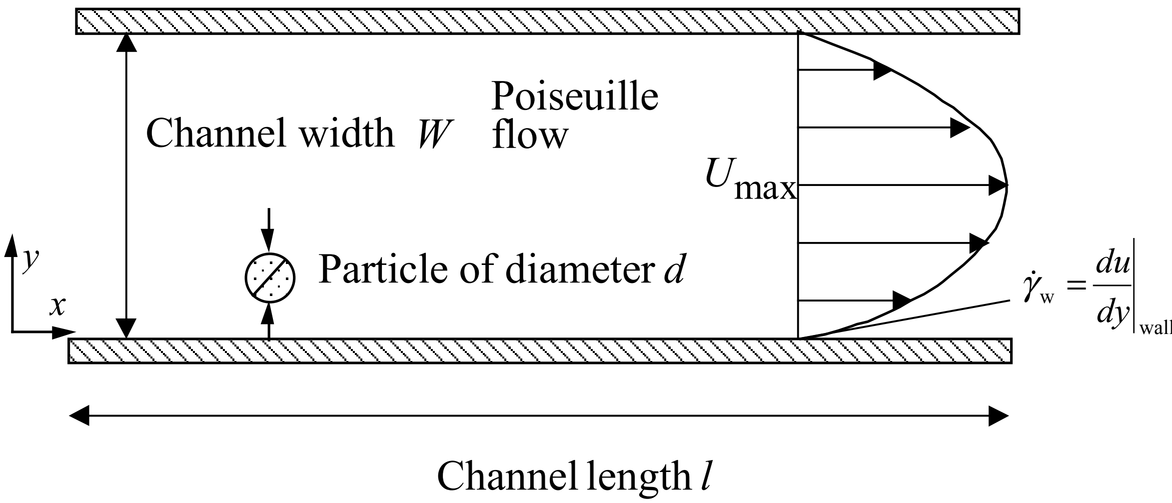

Figure 1. Computational domain for the lift-off of a single particle in plane Poiseuille flow.

We split the pressure as follows:

$$\begin{aligned} P &= p + \rho_f \mathbf{g}\cdot\mathbf{x} - \bar{p}\mathbf{e_x}\cdot\mathbf{x} \\ -\nabla P &= -\nabla p - \rho_f \mathbf{g} + \bar{p}\mathbf{e_x} \end{aligned} \tag{1}$$

where $P(\boldsymbol{x},t)$ is the pressure, ρ_f is the fluid density, $\mathbf{g}$ = -g$\mathbf{e_y}$ is the acceleration due to gravity, $\mathbf{x}$ is the position vector at a point and $\mathbf{e_x}$ and $\mathbf{e_y}$ are the unit vectors in the x– and y- directions, respectively. We solve for the 'dynamic' pressure p in our simulations. The external pressure gradient term then appears as a body force like term in the fluid and particle equations.

During simulations, the particle is free to rotate and translate in the axial (x-) direction. The height, h, of the particle center from the bottom wall of the channel is fixed so that it does not translate in the transverse direction. The gap δ between the particle and the bottom wall is $h-d/2$. There is no external body force in the axial direction and no external torque is applied. The particle is initially at rest and eventually reaches a state of steady motion. At steady state the particle translates in the axial direction at a constant velocity and rotates at a constant angular velocity. At the prescribed height, these velocities are such that there is no net hydrodynamic drag (force along the axial direction) or torque but the particle can experience a hydrodynamic lift force.

The governing equations in non-dimensional form are:

$$
\begin{aligned}
&\nabla \cdot \mathbf{u} = 0\,,\\
&R\left(\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u}\cdot\nabla)\mathbf{u}\right) = -\nabla p + 2\frac{d}{W}\mathbf{e}_{\mathbf{x}} + \nabla\cdot\mathbf{T}\,,\\
&\mathbf{T} = \mathbf{A}\,;\ \text{for a Newtonian fluid}\,,\\
&\mathbf{T} + De\,\overset{\nabla}{\mathbf{T}} = \left(\mathbf{A} + \frac{\lambda_2}{\lambda_1}De\,\overset{\nabla}{\mathbf{A}}\right);\ \text{for an Oldroyd - B fluid}\,,\\
&\frac{\rho_p}{\rho_f}R\frac{d\mathbf{U}_p}{dt} = 2\frac{d}{W}\mathbf{e}_{\mathbf{x}} + \frac{4}{\pi}\circ[-p\mathbf{1}+\mathbf{T}]\cdot\mathbf{n}d\Gamma,\\
&\frac{\rho_p}{\rho_f}R\frac{d\boldsymbol{\Omega}_p}{dt} = \frac{32}{\pi}\circ(\mathbf{x}-\mathbf{X})\times([-p\mathbf{1}+\mathbf{T}]\cdot\mathbf{n})d\Gamma,
\end{aligned}
\tag{2}
$$

where $\mathbf{u}(\boldsymbol{x},t)$ is the fluid velocity, ρ_p is the particle density, $\mathbf{T}$ is the extra-stress tensor, $\mathbf{A} = (\nabla\mathbf{u} + \nabla\mathbf{u}^T)$ is two times the deformation-rate tensor, λ_1 and λ_2 are the constant relaxation and retardation times, respectively, $\mathbf{U}_p$ is the translational velocity of the particle in the axial direction, $\boldsymbol{\Omega}_p$ is the angular velocity of the particle, $\mathbf{X}$ is the coordinate of the center of mass of the particle and we consider circular particles of diameter d. The non-dimensional parameters of the problem are listed below. We assume that gravity acts along the –ve y–direction. Note that only the axial and angular motion equations of the particle are solved in our simulations since the particle height is fixed. Therefore, the y-momentum equation for particle motion is not represented in equation 2. Equation 2 and the corresponding initial and boundary conditions define an initial boundary value problem that can be solved by direct numerical simulation.

The steady state translational and angular velocities as well as the hydrodynamic lift force on a particle freely rotating and translating (along the axial direction), *at a prescribed height*, are obtained from the solution. These values are independent of the particle density used in our simulations (note that the particle acceleration term which has the particle density as its coefficient drops out at steady state, see equation 2). Only the transient solution, which is not the focus of this paper, depends on the choice of the particle density used in the simulations but the final steady state is the same. A hydrodynamic lift force L, obtained from the numerical solution, would balance the buoyant weight of a particle of density ρ_p given by

$$\rho_p = \rho_f + \frac{L}{V_p g}, \tag{3}$$

where V_p is the volume per unit length of the particle. The prescribed height can then be considered as an equilibrium position of a particle of density ρ_p given by equation 3. Therefore, when we present the results we interpret the lift force L to be equal to the buoyant weight of this particle (equation 3) and the prescribed height as its equilibrium height.

The parameters in this problem at steady state are (N. Patankar *et al.* 2001):

$\varepsilon = \dfrac{\delta}{d}$,	non-dimensional gap,
$R = \dfrac{\rho_f V d}{\eta} = \dfrac{\rho_f \dot{\gamma}_w d^2}{\eta} = \dfrac{\rho_f W d^2}{2\eta^2}\overline{p}$,	shear Reynolds number,
$R_G = \dfrac{\rho_f d^3 L}{\eta^2 V_p} = \dfrac{4\rho_f d L}{\pi\eta^2} = \dfrac{4\rho_f d(\rho_p - \rho_f)V_p g}{\pi\eta^2}$,	non-dimensional lift or buoyant weight,
$\dfrac{\overline{p}d^2}{\eta V} = \dfrac{\overline{p}d}{\eta\dot{\gamma}_w} = 2\dfrac{d}{W}$,	aspect ratio,
$E = DeR = \dfrac{\lambda_1 V}{d}R = \dfrac{\lambda_1\eta}{\rho_f d^2}$,	Elasticity number and
$\dfrac{\lambda_2}{\lambda_1}$,	ratio of retardation and relaxation times.

where De is the Deborah number and a dimensionless description of the governing equations is constructed by introducing the following scales: the particle size d for length, V for velocity, d/V for time, $\eta V/d$ for stress and pressure and V/d for angular velocity of the particle where η is the viscosity of the fluid. We have chosen $V = \dot{\gamma}_w d$, where $\dot{\gamma}_w$ is the shear-rate at the wall (in the absence of the particle) as shown in figure 1. The elasticity of the fluid increases with the elasticity number whereas $\lambda_2/\lambda_1 = 1$ corresponds to a Newtonian fluid and $\lambda_2/\lambda_1 = 0$ corresponds to a highly elastic Maxwell fluid.

The channel length l is chosen large enough so that the solution is only weakly dependent on its value. An Arbitrary Lagrangian-Eulerian (ALE) moving mesh technique with the EVSS (Elastic-Viscous-Stress-Split) scheme for the Oldroyd-B constitutive model is used to solve the governing equations. More details of this numerical scheme are given by Hu (1996), Hu & N. Patankar (2001) and Hu, N. Patankar & Zhu (2001).

The dimensionless lift on the particle depends on the parameters listed above:

$$R_G = f\left(R, \frac{W}{d}, \varepsilon, E, \frac{\lambda_2}{\lambda_1} \right). \tag{4}$$

3. Results and discussion

3(a). Critical condition of lift-off

Krishnan & Leighton (1995) calculated the lift force on a smooth sphere rotating and translating in a simple shear flow in contact with a rigid wall. Hu & Joseph (1999) extended their analysis to second-order fluids. Their results were valid at low Reynolds numbers. The non-dimensional lift R_G for a particle in an Oldroyd-B fluid is given by (Hu & Joseph 1999)

$$\begin{aligned} R_G = {} & \frac{3}{2\pi}\left(1.755 R_U^2 + 0.1365 R_\Omega^2 - 0.019 R_U R_\Omega - 4.522 R_U R + 0.303 R_\Omega R + 2.314 R^2\right) \\ & + \frac{24}{5\varepsilon} E \left(1 - \frac{\lambda_2}{\lambda_1} \right)\left(R_U^2 + 0.25 R_\Omega^2 - 0.25 R_U R_\Omega\right), \end{aligned} \tag{5}$$

$$R_U = \frac{\rho_f U_p d}{\eta} \text{ and } R_\Omega = \frac{\rho_f \Omega_p d^2}{\eta}.$$

The above expression is valid in the limit of slow and slowly varying flows so that the second-order fluid expansion is valid.

For a freely translating and rotating sphere, R_U and R_Ω are functions of R and the gap size. The above calculations were performed for semi-infinite domains. Hence W/d is not a parameter of the problem. Equation 5 is in agreement with the functional form in equation 4.

The expression for the Newtonian case is obtained by substituting $E = 0$ in equation 5. The resulting expression is valid in the limit of zero gap size. A heavy particle freely translating and rotating in contact with a plane wall in simple shear flow of a Newtonian fluid is lifted from the wall and suspended in the fluid if the shear Reynolds number R is greater than a critical value. Beyond the critical shear Reynolds number the particle rises from the wall to an equilibrium height at which the buoyant weight just balances the upward thrust from the hydrodynamic force. In a Newtonian fluid the case of zero separation distance corresponds to an infinite drag force due to the logarithmic singularities in the lubrication equations for drag and torque. This results in zero translational and rotational velocities of the particle (Krishnan & Leighton 1995). For a particle in a viscoelastic fluid the elastic component of the lift force is also singular when the gap between the sphere and the wall approaches zero. This is an important qualitative feature that differentiates the lift force in a Newtonian and a viscoelastic fluid.

In practical applications the particle acquires some finite separation distance from the wall due to the presence of surface roughness. This eliminates the lubrication singularity in a Newtonian fluid (Krishnan & Leighton 1995, Smart, Beimfohr & Leighton 1993). The additional non-hydrodynamic frictional force due to the surface roughness does not significantly affect the lift force on the particle within a reasonable range of the coefficient of friction for particles in a Newtonian fluid (Krishnan & Leighton 1995).

N. Patankar *et al.* (2001) investigated the lift-off of a circular particle in a plane Poiseuille flow of a Newtonian fluid. Their results were not restricted to low Reynolds numbers. They performed two-dimensional numerical simulations where the gap between the particle and the wall cannot be zero (Hu & N. Patankar 2001). Following Krishnan & Leighton (1995) the minimum gap size (or surface roughness) was set to 0.001*d*. The additional frictional force was assumed to be zero. The critical Reynolds number was defined as the minimum shear Reynolds number required to lift a particle to an equilibrium height greater than 0.501*d*. N. Patankar *et al.* (2001) reported that there was no effect of the channel width on the critical shear Reynolds number for $W/d > 12$. They obtained a correlation for R_G as a function of R. Here we present results for a particle in an Oldroyd-B fluid. To facilitate comparison we use $W/d = 12$ and $\varepsilon = 0.001$ in our simulations.

Figures 1a and 1b show the plot of R_G vs. the critical shear Reynolds number for lift-off at different values of the elasticity number and λ_2/λ_1, respectively. It is seen that larger R is required to lift a heavier particle. The fluid elasticity enhances the lift on the particle. The data from the simulations can be represented by a power law equation given by $R_G = aR^n$, where the values of a and n are given in the figures. We observe that the slopes, n, for a Newtonian and an Oldroyd-B fluid are different. The prefactor a also changes as E and λ_2/λ_1 changes. This is in agreement with equation 4.

In a three-dimensional low Reynolds number case, equation 5 predicts that the effect of elasticity is represented by $E(1-\lambda_2/\lambda_1)$. Figure 2a shows the plot of a vs. $E(1-\lambda_2/\lambda_1)$. We observe that a varies rapidly near the Newtonian limit ($E(1-\lambda_2/\lambda_1) = 0$) and tends to vary linearly at higher values of $E(1-\lambda_2/\lambda_1)$. Note that equation 5 predicts a linear variation of R_G with respect to $E(1-\lambda_2/\lambda_1)$. Figure 2b shows the variation of n vs. $E(1-\lambda_2/\lambda_1)$. Once again we note that near the Newtonian limit the value of n changes rapidly and eventually attains an almost constant value.

In the above simulations the critical shear Reynolds number for lift-off is defined for a equilibrium height corresponding to $\varepsilon = 0.001$. Equations 4 and 5 predict that in general R_G is also a function of the gap size. Figures 3a and 3b show the plot of R_G vs. the shear

Reynolds number for a Newtonian and an Oldroyd-B fluid, respectively, at different gap sizes. The parameters are as defined in the figure. The slope n does not significantly change with the gap size whereas the value of a does depend on ε. Figure 4 compares the variation of a vs. ε for the given parameters for a Newtonian and an Oldroyd-B fluid. It is seen that the value of a increases rapidly as the gap size tends to zero in an Oldroyd-B fluid; in qualitative agreement with the theoretical predictions of equation 5. Such behavior is not observed for the Newtonian case.

3(b). Turning point bifurcation of the equilibrium position

Beyond the critical shear Reynolds number the particle acquires an equilibrium position away from the wall. We investigate the presence of multiple steady equilibrium positions, so far observed only for Newtonian fluids (N. Patankar *et al*. 2001), when the Reynolds number is greater than the critical value. As before we perform simulations with fixed height of the particle and calculate the hydrodynamic lift on it.

Figure 5 shows the plot of L as a function of the height of the particle center from the bottom wall at different values of shear Reynolds number. We have W/d = 12, E = 0.05 and λ_2/λ_1 = 0.125. This plot can be used to find the equilibrium height of a particle of given density at different values of R. As an example we consider a particle of density 1.01 g/cc. This particle will be in equilibrium when L = 7.705 dyne/cm (from equation 3, ρ_f = 1 g/cc and d = 1 cm). The equilibrium heights at a given shear Reynolds number are identified as the points of intersection between the curve of L vs. h/d and L = 7.705 in figure 6. The intersection points where the slope of the L vs. h/d curve is positive are unstable equilibrium points whereas a negative slope represents a stable equilibrium point (figure 6). Figure 7 shows the plot of equilibrium height of the particle of density 1.01 g/cc vs. R. We observe that multiple equilibrium positions are obtained for particles in Oldroyd-B fluids. This is due to the presence of two turning points in equilibrium solution. The equilibrium height plot for Oldroyd-B fluids is compared to the corresponding graph for particles in Newtonian fluids. We see that the fluid elasticity increases the equilibrium height for the same Reynolds number. We also observe that the fluid elasticity shifts the turning points towards lower Reynolds numbers. Equilibrium height plots at higher elasticity numbers, that exhibited multiple solutions, were not computed due to numerical convergence issues. Implications of multiple steady states for single particle lifting on the flow of dilute suspensions in pipes and on models of lift-off in slurries should be a subject of future investigation. To our knowledge multiple equilibrium positions have not be reported experimentally.

4. Conclusions

In this note we study the critical condition for lift-off of a circular particle from a wall in a plane Poiseuille flow of an Oldroyd-B fluid. Two-dimensional numerical simulations are performed. A correlation for the critical condition for lift-off is obtained assuming a surface roughness (or the particle separation from the wall) of 0.001d. It is seen that the fluid elasticity reduces the critical shear Reynolds number for lift-off. The effect of the gap size, between the particle and the wall, on the lift force is also studied. The lift force on the particle is seen to be singular as the gap size approaches zero; in qualitative agreement with previous theoretical predictions. Beyond the critical Reynolds number we observe multiple equilibrium positions for particles in Oldroyd-B fluids. This is due to two turning points in equilibrium solution. The location of the turning point is shifted due to the elasticity of the fluid as compared to the Newtonian results.

Acknowledgment

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Figure 1 (a). The plot of R_G vs. the shear Reynolds number R for lift-off on a logarithmic scale for a Newtonian and an Oldroyd-B fluid at different elasticity numbers.
(W/d =12, ε = 0.001, λ_2/λ_1 = 0.125)

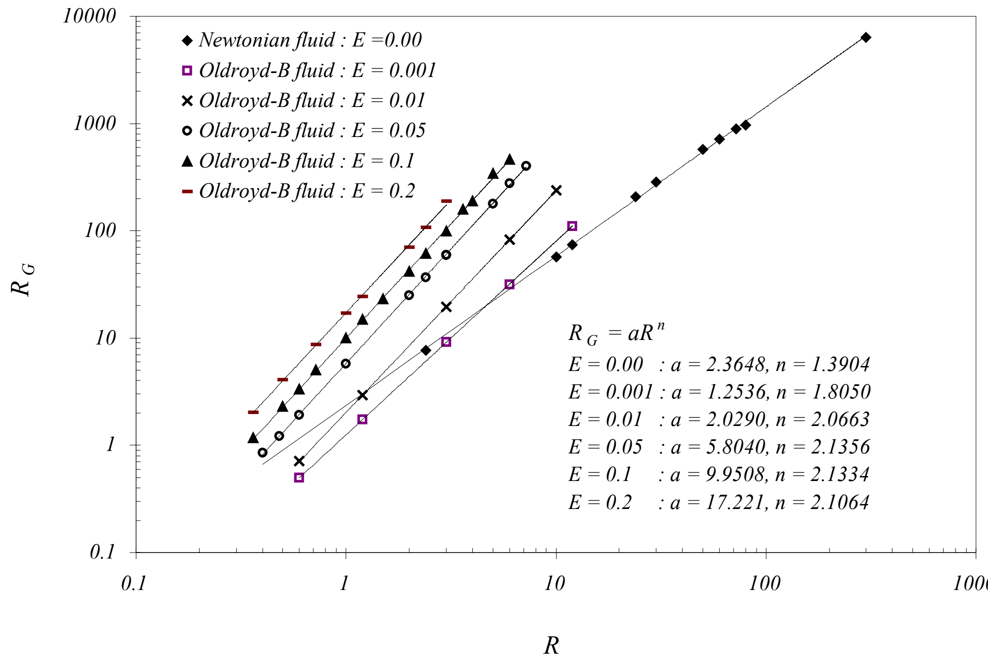

Figure 1 (b). The plot of R_G vs. the shear Reynolds number R for lift-off on a logarithmic scale for a Newtonian and an Oldroyd-B fluid at different values of λ_2/λ_1.
(W/d = 12, ε = 0.001, E = 0.05)

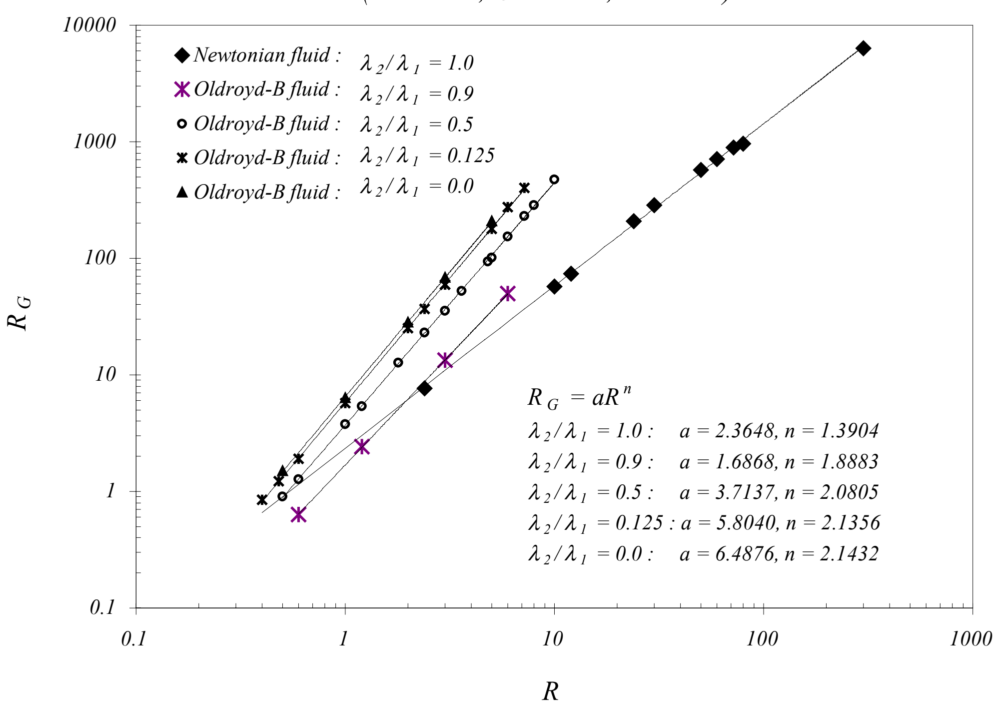

Figure 2 (a). The plot of a vs. $E(1-\lambda_2/\lambda_1)$.
$(W/d = 12,\ \varepsilon = 0.001)$

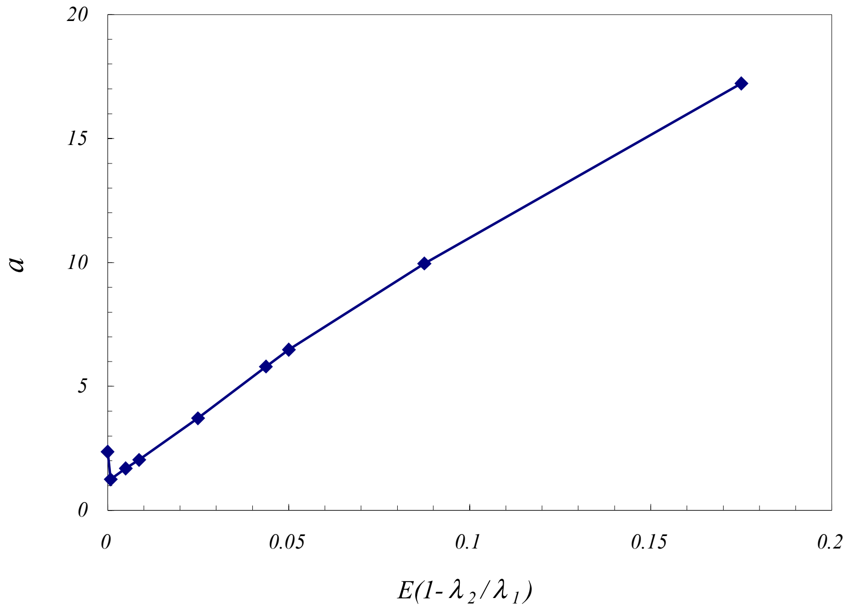

Figure 2 (b). The plot of n vs. $E(1-\lambda_2/\lambda_1)$.
$(W/d = 12,\ \varepsilon = 0.001)$

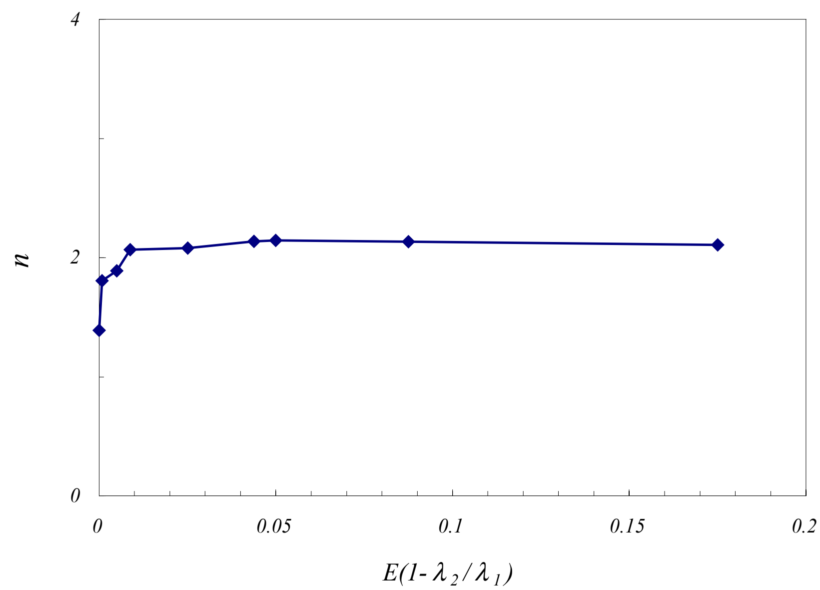

Figure 3 (a). The plot of R_G vs. the critical shear Reynolds number R for lift-off on a logarithmic scale for a Newtonian fluid for various gap sizes.
(W/d =12)

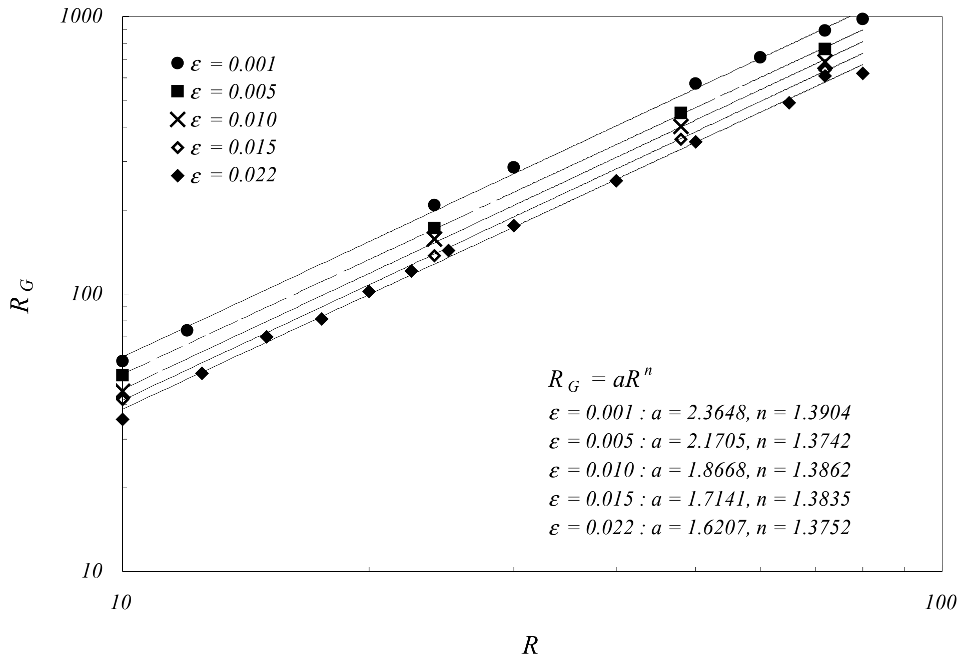

Figure 3 (b). The plot of R_G vs. the critical shear Reynolds number R for lift-off on a logarithmic scale at Oldroyd-B fluid for various gap sizes.
(W/d = 12, E = 0.05, λ_2/λ_1 = 0.125)

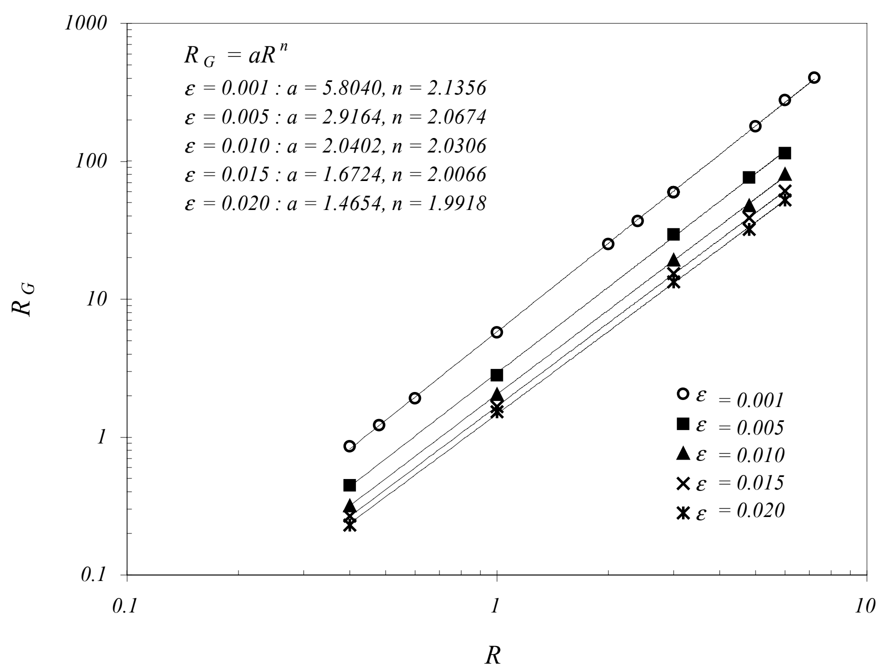

Figure 4. The comparison of the variation of a vs. ε for a Newtonian and an Oldroyd-B fluid with $\lambda_2/\lambda_1 = 0.125$ and $E=0.05$.
(W/d = 12)

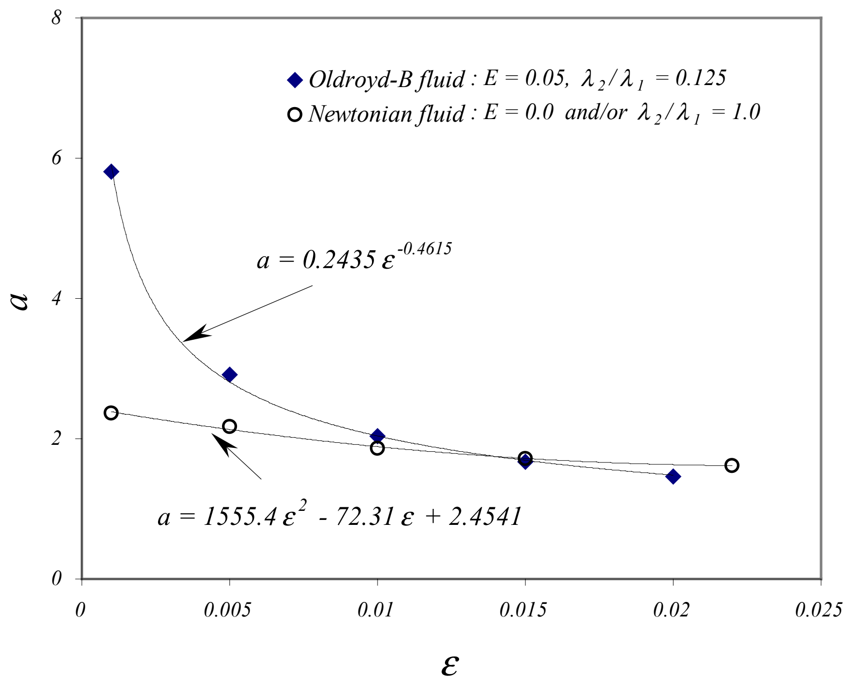

Figure 5. The hydrodynamic lift force on the particle as a function of the height of its center from the bottom wall at different shear Reynolds numbers. The bottom wall is $h/d = 0$ and the channel centerline is $h/d = 6$.

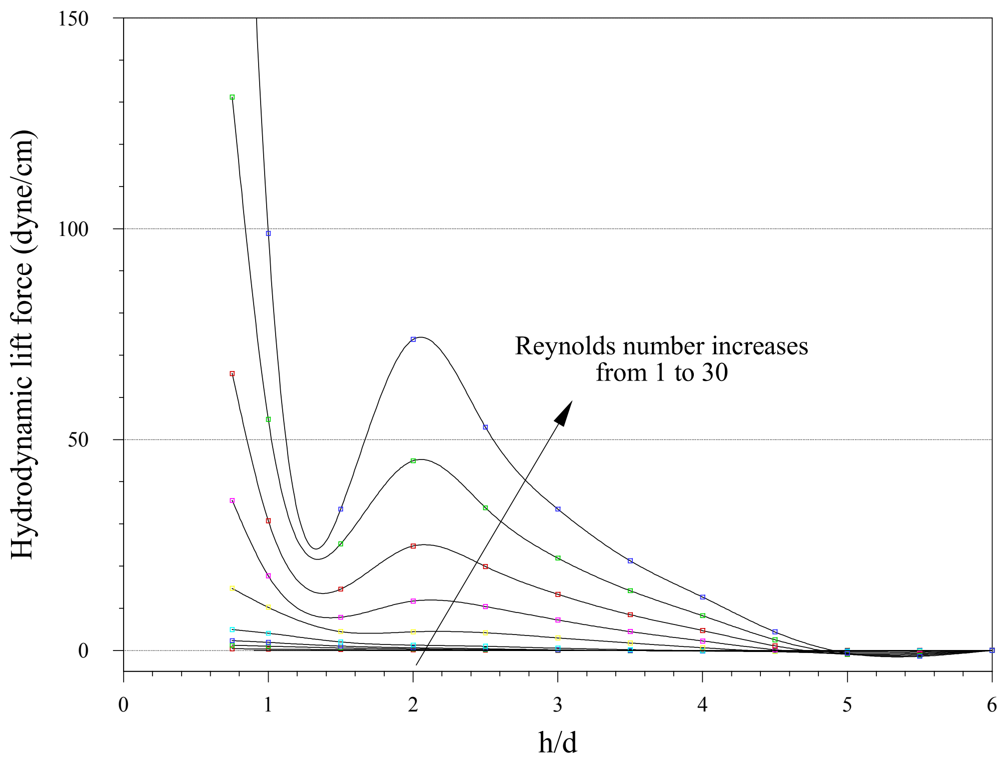

Figure 6. Finding the equilibrium height of a particle of a given density at different values of shear Reynolds number.

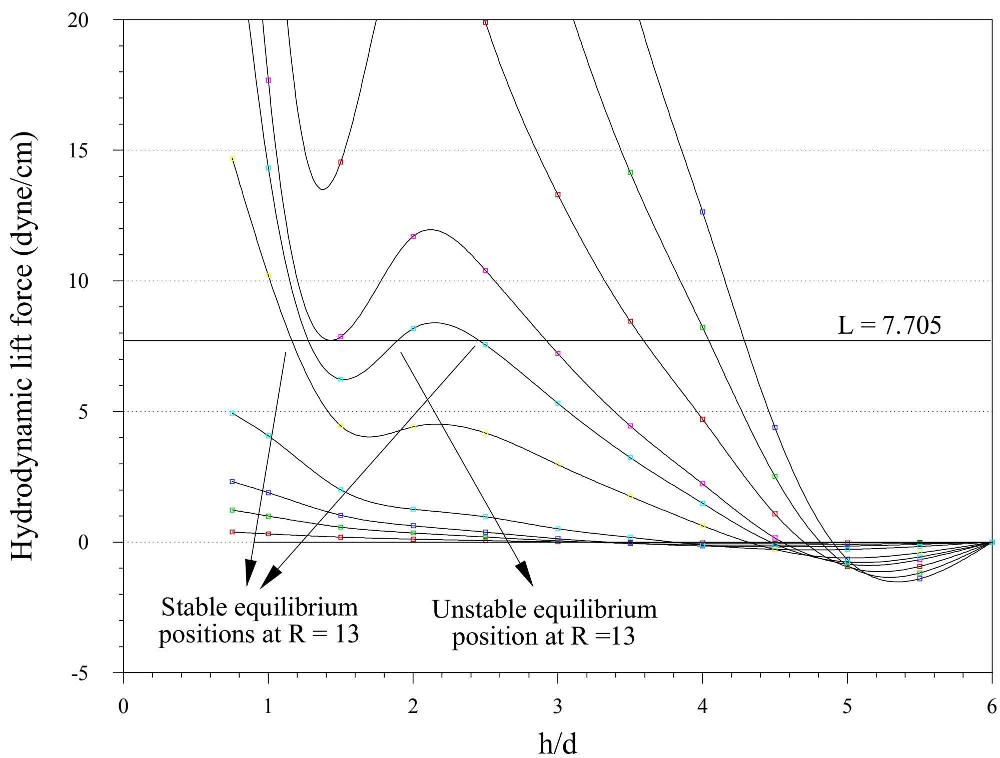

Figure 7. Equilibrium height as a function of shear Reynolds number for a particle of density 1.01 g/cm^3.