

# Cavitation and the state of stress in a flowing liquid

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## Abstract

The problem of the inception of cavitation is formulated in terms of a comparison of the breaking strength or cavitation threshold at each point in a liquid sample with the principle stresses there. A criterion of maximum tension is proposed which unifies the theory of cavitation, the theory of maximum tensile strength of liquid filaments and the theory of fracture of amorphous solids. Liquids at atmospheric pressure which cannot withstand tension will cavitate when and where tensile stresses due to motion exceed one atmosphere. Cavitation may be considered in pure shear and in pure extension. A cavity will open in the direction of the maximum tensile stress which is 45 degrees from the plane of shearing in pure shear of a Newtonian fluid. The formation of cavities in pure shear has been observed in several experiments. Cavitation will occur in pure extension when the extensional stress is large enough, at high rates of extension. Tom Lundgren and I did a heuristic analysis of the breakup of a liquid capillary filament using viscous potential flow near a stagnation point on the centerline of the filament towards which the surface collapses under the action of surface tension forces. We find that the neck is of parabolic shape and its radius collapses to zero in a finite time. During the collapse the tensile stress due to viscosity increases in value until at a certain finite radius, which is about 1.5 microns for water in air, the stress in the throat passes into tension, presumably inducing a cavitation there. The predicted linear in time collapse of the neck is predicted by other theories (with different collapse rates) and is observed in experiments. The collapse of the filament in finite time could lead to unbounded viscous extensional stresses inducing rupture by cavitation before total collapse.

## 1 Introduction

This paper updates results given by Joseph [1998]. In previous papers (Joseph [1995], Joseph, Huang and Candler [1996]) I drew attention to the fact that the pressure in a flowing incompressible liquid is not a fundamental dynamic variable; at each point of the liquid the state of stress is determined by three principal stresses. In Newtonian fluids the pressure is the negative of the mean of these stresses (6); in non-Newtonian fluids the pressure is an unknown field variable whose relation to the principal stresses depends on the choice of a constitutive equation.

We may generally express the stress  $\mathbf{T}$  by a constitutive equation of the form

$$\mathbf{T} = -p\mathbf{1} + \boldsymbol{\tau}[\mathbf{u}] \quad (1)$$

where the part  $\boldsymbol{\tau}$  of  $\mathbf{T}$  which is characterized by a constitutive equation can be regarded as functional of the velocity  $\mathbf{u}$ . For incompressible liquids, the conservation of mass is expressed by

$$\operatorname{div} \mathbf{u} = 0 \quad (2)$$

and the conservation of momentum by

$$\rho \left[ \frac{\partial \mathbf{u}}{\partial \tau} + \mathbf{u} \cdot \nabla \mathbf{u} \right] = -\nabla p + \operatorname{div} \boldsymbol{\tau}[\mathbf{u}]. \quad (3)$$

Equations (2) and (3) are four equations for three components of velocity and the pressure  $p$  is an additional unknown which we need to close the system.

For Newtonian liquids

$$\boldsymbol{\tau}[\mathbf{u}] = 2\eta \mathbf{D}[\mathbf{u}] \quad (4)$$

where  $\mathbf{D}[\mathbf{u}]$ , the rate of strain, is the symmetric part of the gradient of velocity,  $\eta$  is the viscosity, and

$$\operatorname{Trace} \mathbf{D}[u] = \operatorname{div} \mathbf{u} = 0 \quad (5)$$

As a consequence of (5),

$$p = -\frac{1}{3} \operatorname{Trace} \mathbf{T} \quad (6)$$

More generally,  $\operatorname{Trace} \boldsymbol{\tau} \neq 0$  and

$$p = -\frac{1}{3} \operatorname{trace} (\mathbf{T} - \boldsymbol{\tau}) \quad (7)$$

depends on the constitutive equation, the choice of the functional relating  $\boldsymbol{\tau}$  to  $\mathbf{u}$ .

Though it is true that a liquid at rest, in which all the stresses are all equal to  $-p$ , can make sense of (6), a moving liquid cannot average the principal stresses as is required by (6), and (7) is even more a consequence how we choose to define  $\boldsymbol{\tau}$  than a fundamental quantity which can be felt at a point by the liquid.

## 2 Principal stresses and cavitation

The state of stress rather than its average value is fundamental for all the motions of an incompressible fluid. Here, however we focus on the inception of cavitation and not on the shape and motion of an open cavity. Even though criteria for cavitation ought to be based on the principal stresses and not the pressure, it is useful to introduce a pressure as the mean normal stress as in a Newtonian liquid and to define it that way for Non-Newtonian liquids. If we write

$$\mathbf{T} = -\pi \mathbf{1} + \mathbf{S} = -p \mathbf{1} + \boldsymbol{\tau} \quad (8)$$

where  $p$  is given by (7) and  $\mathbf{S}$  is the stress deviator

$$\pi = -\frac{1}{3} \text{Trace } \mathbf{T}, \quad \text{Trace } \mathbf{S} = S_{11} + S_{22} + S_{33} = 0. \quad (9)$$

Since  $S_{11} \geq S_{33} \geq S_{22}$  we have

$$S_{11} > 0 \text{ and } S_{22} < 0 \quad (10)$$

where

$$S_{11} - S_{22} > 0 \quad (11)$$

is largest in the coordinate system in which  $\mathbf{T}$  is diagonal.

Consider now the opening of a small cavity. It is hard to imagine very large differences in the pressure of the vapor in the cavity so that the cavity should open in the direction where the tension is greatest. The idea that vapor cavities open to tension is endemic in the cavitation community, but it seems not to have been noticed before that this idea requires one to consider the state of stress at a point and, at the very least, to determine the special principal axes coordinates in which the tension is maximum. To remind us of this important point we shall call  $\zeta(\theta)$  the special coordinate system in which the orthogonal transformation  $\mathbf{Q}$  diagonalizes  $\mathbf{T}$  (and  $\mathbf{S}$ ):

$$\mathbf{Q}^T \mathbf{T} \mathbf{Q} = \text{diag}(T_{11}, T_{22}, T_{33}) \quad (12)$$

Here  $\theta$  in  $\zeta(\theta)$  stands for the direction cosines in the diagonalizing transformation, and  $\theta$  is the diagonalizing angle for the two-dimensional rotation. The rotation of  $\mathbf{T}$  is an important part of the theory of cavitation which has not been considered before.

In two dimensions the components of the stress deviator in  $\zeta(0)$  are given by

$$[\mathbf{S}] = \begin{bmatrix} S_{11} & S_{12} \\ S_{12} & -S_{11} \end{bmatrix}; \quad (13)$$

The angle  $\theta$  that diagonalizes  $\mathbf{S}$  is given by

$$\begin{cases} \sin 2\theta = S_{12}/\sqrt{S_{12}^2 + S_{11}^2}, \\ \cos 2\theta = S_{11}/\sqrt{S_{12}^2 + S_{11}^2} \end{cases} \quad (14)$$

and

$$[\mathbf{S}] = \sqrt{S_{12}^2 + S_{11}^2} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \quad (15)$$

The largest stress component in the principal value coordinate system is

$$T_{11} + \frac{1}{2}(T_{11} + T_{22}) = S_{11}; \quad (16)$$

the smallest component is

$$T_{22} + \frac{1}{2}(T_{11} + T_{22}) = -S_{11} \quad (17)$$

and

$$T_{11} - T_{22} = 2S_{11} \quad (18)$$

We call  $T_{11}$  the maximum tension and  $T_{22}$  is the minimum tension. If the maximum tension is negative, it is compressive; the minimum tension is even more compressive.

If the cavitation (outgassing) threshold  $p_c$  is above  $\pi - S_{11}$  but below  $\pi$  the cavity will appear when and where the tension due to motion is large enough; if this threshold is greater than  $\pi - S_{22}$  ( $S_{22} < 0$ ) then the cavity will open only at those points where no component of the total stress is larger than the cavitation threshold; this is the minimum tension criterion and in neither case is the criterion framed in terms of the pressure  $\pi$  alone.

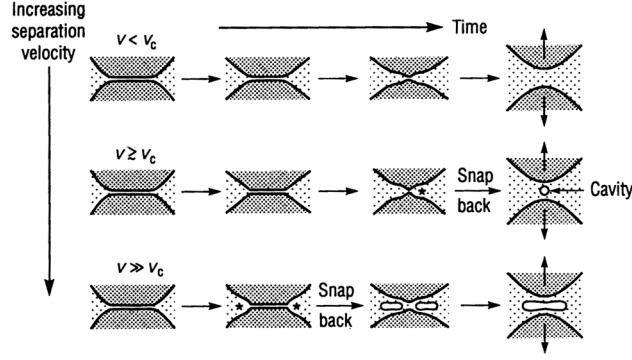


Figure 1:

**If a cavitation bubble opens up, it will open in the direction of maximum tension. Since this tension is found in the particular coordinate system in which the stress is diagonal, the opening direction is in the direction of maximum extension, even if the motion is a pure shear.** It may open initially as an ellipsoid before flow vorticity rotates the major axis of ellipsoid away from the principal tension axis of stress, or it may open abruptly into a “slit” vacuum cavity perpendicular to the tension axis before vapor fills the cavity as in the experiments of Kuhl et al. [1994] (see figures 1 and 2).

The features in the two dimensional problem which were just discussed have an immediate and obvious extension to three dimensions.

### 3 Cavitation criteria

In what follows I am going to assume that the breaking stress is a given parameter which can be defined at each point of a liquid; we then compare the state of stress in a moving liquid at the point with  $p_c$  to form a cavitation criteria.

The cavitation threshold used in the prior literature is framed in terms of a mean stress

$$\pi = -\frac{1}{3}(T_{11} + T_{22} + T_{33}) \quad (19)$$

Cavitation will occur when  $\pi - p_c < 0$  and not when  $\pi - p_c > 0$ . The mean stress may be a good estimate for breaking thresholds, but it has no physical meaning in a moving fluid since the fluid cannot average its stresses.

Two cavitation thresholds based on the maximum tension  $T_{11}$  and minimum tension  $T_{22}$  in three dimensions can be considered, recall that the deviatoric stresses are such that

$$S_{11} > 0, S_{22} < 0 \quad (20)$$

so that  $T_{22} = S_{22} - \pi$  is the minimum tension.

The **maximum tension** criterion is given by

$$B_{11} \stackrel{\text{def}}{=} T_{11} + p_c = S_{11} - \pi + p_c > 0 \quad (21)$$

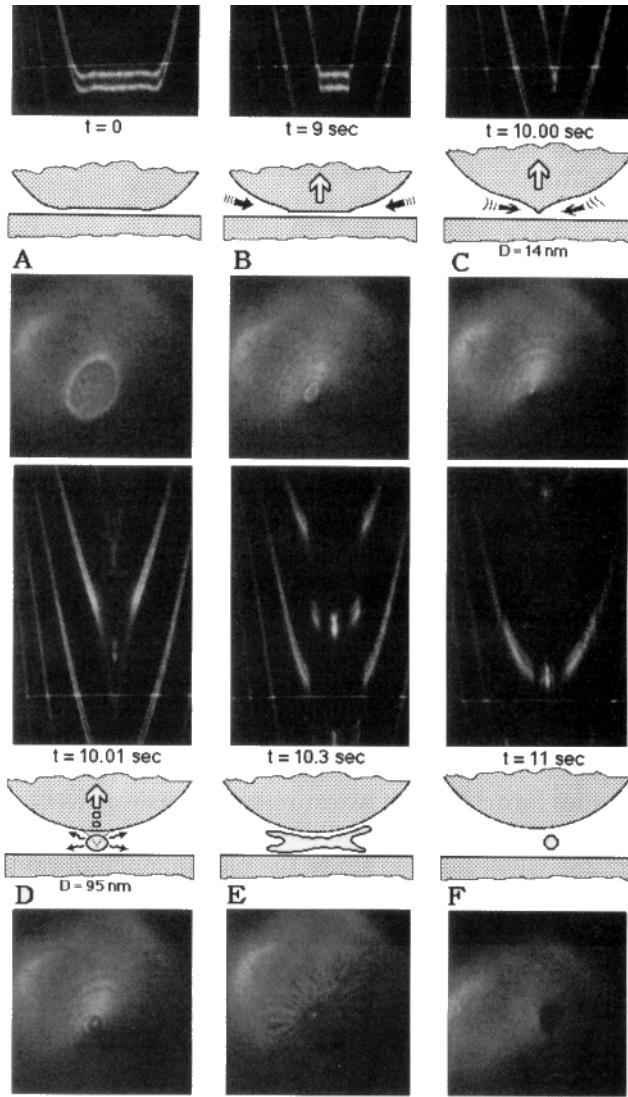


Figure 2:

In this case

$$\pi - p_c < S_{11} \quad (22)$$

and since the tension  $S_{11} > 0$ ,  $\pi - p_c$  could be larger than zero and the liquid would still cavitate. If (21) holds and

$$B_{22} \stackrel{\text{def}}{=} T_{22} + p_c < 0 \quad (23)$$

then relative to the threshold  $p_c$ , the stress  $B_{11}$  is in “tension” and  $B_{22} < 0$  is a “compression”.

If (21) holds and

$$B_{22} > 0 \quad (24)$$

then all three of the relative principal stresses  $B_{11}$ ,  $B_{22}$ ,  $B_{33}$  are positive and a cavity will open. This is the **minimum tension criterion**. This criterion for cavitation is more severe than the classical one which requires that the average value of these relative stresses be positive.

The archival literature on cavitation allows only for breaking in tension, though the state of stress at a point which ought to be considered, has not been considered. Typically the discussion of cavitation is framed in the context of the breaking strength of liquids; the main conclusion is that liquids may withstand very large tensions if impurities and nucleation sites are suppressed. There is a vast literature on the tensile strength of liquids some of which may be found in the book by Knapp, Daily & Hammitt [1970] who say that “. . . Measurements have been made by several different methods and are too numerous to report completely” and in other books on cavitation.

Knapp et al. [1970] have considered whether the cavitation threshold ought to be framed in terms of the vapor pressure or the tensile strength of liquids, concluding for the latter. They say that

. . . the elementary concept of inception is the formation of cavities at the instant the local pressure drops to the vapor pressure of the liquid. However, the problem is not so simple. Although experiments show inception to occur near the vapor pressure, there are deviations of various degrees with both water and other liquids that are not reconcilable with the vapor-pressure concept. We define the vapor pressure as the equilibrium pressure, at a specified temperature, of the liquid’s vapor which is in contact with an *existing* free surface. If a cavity is to be created in a homogeneous liquid, the liquid must be ruptured, and the stress required to do this is not measured by the vapor pressure but is the *tensile strength* of the liquid at that temperature. The question naturally arises then as to the magnitudes of tensile strengths and the relation these have to experimental findings about inception.

A similar point of view was expressed by Plesset [1969]

. . . A central problem in cavitation and boiling is how macroscopic vapor cavities can form when moderate tensions are applied to the liquid. The theory of the tensile strength of pure liquids predicts that a vapor cavity will form only when the liquid is under extremely large tensions; as an equivalent effect the theory also predicts that vapor bubbles appear in boiling only when the liquid has very large superheats. Since these large tensile strengths and superheats are not observed, the idea of nuclei has been introduced. These nuclei are in some sense holes in the liquid which are already beyond molecular dimensions and which may therefore grow into macroscopic bubbles under moderate liquid tensions.

Brennen [1995] notes that “. . . This ability of liquids to withstand tension is very similar to the more familiar property exhibited by solids and is a manifestation of the elasticity of a liquid.” Of course the elasticity of liquids, solid-like behavior, could occur only in time so short that the configurations of molecules is not changed by flow, as could be expected in a cavitation event. Fisher [1948] notes that “. . . Glass and other undercooled liquids may fail by the nucleation and propagation of cracks, rather than of bubbles as do more mobile liquids.” Nucleation and propagation of cracks have been realized in the experiments of Kuhl et al. [1994] discussed in Joseph [1998].

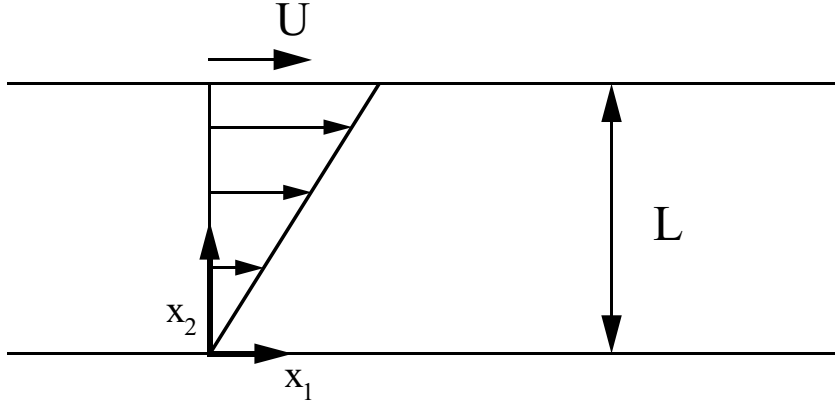


Figure 3: Plane Couette flow between walls

The theory of cavitation, the tensile strength of liquids and the fracture of amorphous solids may be framed in a unified manner in which the breaking strength of the material is defined in terms of tensile stresses along the principal axes of stress. Glass at different temperatures is a perfect material for these considerations. At high temperatures the molten glass flows and under the right conditions, flow bubbles ought to open at a weak spot in the direction of the principal tension. Low temperature glass is an amorphous solid and we can imagine a crack to be initiated under tension at the same weak spot. Glass at intermediate temperatures may exhibit as yet unknown properties between cavity formation and crack propagation.

The nucleation of a cavity can occur as a sudden and not a continuous event. The fluid must first rupture; then it fills with vapor or gas and flows as in the experiments of Israelachvili and his collaborators described by Joseph [1998]. To open a cavity, the liquid must be supersaturated; practically this supersaturation can be achieved by lowering the pressure or by tensions created by flow. If the mean normal stress in a liquid is of the order of one atmosphere, the liquid will be put into tension when and where the tensile component of the flow-induced extra stress is larger than  $10^5$  Pa. Tap water might be expected to nucleate vapor or gas bubbles at points at which the flow-induced tensions exceed 1 atmosphere  $\approx 10^5$  Pa. On the other hand, for flows with large capillary pressures or for fluids, which can withstand tension, larger flow induced tensions, say  $10^6$  Pa, are required.

## 4 Cavitation in shear

Consider plane shear flow between parallel plates as in figure 3.

The stress in this flow is given by

$$\begin{bmatrix} T_{11} & T_{12} & 0 \\ T_{12} & T_{22} & 0 \\ 0 & 0 & T_{33} \end{bmatrix} = -\pi \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + \eta \begin{bmatrix} 0 & \frac{U}{L} & 0 \\ \frac{U}{L} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (25)$$

where  $\pi = \frac{1}{3}(T_{11} + T_{22} + T_{33})$  is determined by the “pressurization” of the apparatus. The angle which diagonalizes



$\mathbf{T}$  is given by (14) as  $S'_{12} = 0$  or

$$\cos 2\theta = 0, \quad \theta = 45^\circ$$

(In the break-up of viscous drop experiments in plane shear flow done by G.I. Taylor [1934], the drops first extend at  $45^\circ$  from the direction of shearing.)

Then, using (25), in principal coordinates, we have

$$\begin{bmatrix} T_{11} + \pi & 0 & 0 \\ & T_{22} + \pi & 0 \\ 0 & 0 & T_{33} + \pi \end{bmatrix} = \eta \frac{U}{L} \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (26)$$

and

$$B_{11} = p_c - \pi + \eta \frac{U}{L}, \quad (27)$$

$$B_{22} = p_c - \pi - \eta \frac{U}{L}, \quad (28)$$

The difference between the largest and smallest stresses is

$$B_{11} - B_{22} = 2\eta \frac{U}{L}. \quad (29)$$

This difference is of the order of one atmosphere of pressure if

$$2\eta \frac{U}{L} = 10^6 \frac{\text{dynes}}{\text{cm}^2} \quad (30)$$

If  $\eta = 1000$  poise,  $U = 10$  cm/sec and  $L = 10^{-1}$  cm, we may achieve such a stress. It is possible to imagine such a shearing motion between concentric rotating cylinders filled with silicon oil, though the conditions are severe. If we could depressurize the system so that a threshold of pressure less than one atmosphere were required, we might see cavities appear in shear flow when  $B_{11} > 0$  and  $B_{22} < 0$ . In Joseph [1998] I wrote that, "I am not aware of reports of cavities forming in shear flows, but the conditions required are at the border of realistic experiments and may have escaped detection. Experiments of this kind ought to be tried."

After putting forward this idea several experiments in which cavity formation in pure shear flows were reported have come to my attention. The first of these was in a paper by Archer, Ternet, and Larson [1997]: "Fracture" phenomena in shearing flow of viscous liquids. They note that "... the shear stress catastrophically collapses if the shear rate is raised above a value corresponding to a critical initial shear stress of around 0.1-0.3 Mpa. ... in polystyrene, bubbles open up within the sample; as occurs in cavitation. Some similarities are pointed out between these phenomena and that of 'lubrication failure' reported in the tribology literature." The critical stress 0.1-0.3 Mpa = 1-3 atmospheres is just what might have been guessed for cavitation under shear.

A very excellent and early report of cavity formation between rotating cylinders has only recently been brought to my attention by S. Bair. Winer and Bair [1987] did experiments with high viscosity liquids between concentric rotating [cylinders.....??] and they looked at rheology high shear stresses. Winer and Bair [1987] concluded that

. . . The tentative conclusion of this work is that the pseudoplastic behavior of some liquids is apparently the result of the reduction of the principal normal stress at high shear stress causing void formation and the reduction of apparent viscosity.

However, for some high shear rate viscosity data at atmospheric pressure the principal normal stress may approach quite low values relative to one atmosphere suggesting the possibility of cavitation or fracture of the material resulting in a reduced shear stress.

. . . Because this type of stress field exists in elastohydrodynamic inlets and classical hydrodynamic configuration, such as high speed journal bearings, it should be further investigated, because it potentially represents a lubricant limitation to feeding bearings. It also may be an influence on the location of the cavitation boundary at the exit of hydrodynamic films. This mechanism should be explored further as a possible cavitation boundary condition to hydrodynamic lubrication.

Pereira, McGrath and Joseph [2001] have looked at the flow and stress induced cavitation in a journal bearing with axial throughput. The system models a device used to test the stability of emulsions against changes in drop size distribution. The analysis looks for the major variation in flow properties, such as cavitation that could put an emulsion at risk to coalescence or breakage.

## 5 Cavitation in extension

We have argued that cavities always appear in the extensional flows defined in principal axes coordinates even when the flow is pure shear. However, the direct creation of a pulling flow without rotation (vorticity) may lead to a higher level of dynamic stresses than could be otherwise achieved. Let us suppose that a small diameter thread open to the atmosphere is anchored at a solid wall at  $x = 0$  and is being pulled out at a constant rapid rate  $\dot{S}$  in the direction  $x$ .

$$u = \dot{S}x, \quad v = -\frac{1}{2}\dot{S}y, \quad \omega = -\frac{1}{2}\dot{S}z. \quad (31)$$

The thread is in tension when  $\dot{S}$  is large enough

$$T_{11} = -\pi + 2\eta \frac{\partial u}{\partial x} \approx -p_a + 2\eta\dot{S} \quad (32)$$

where, for very thin threads  $\pi \approx -p_a + \frac{\gamma}{R}$  where  $p_a$  is atmospheric pressure,  $\gamma$  is surface tension and  $R$  is the radius. According to the maximum tension criterion (19) cavities will form in the thread, and the thread may actually break, when

$$B_{11} \approx p_a + \frac{\gamma}{R} - p_c - 2\eta\dot{S} < 0 \quad (33)$$

If we neglect surface tension the stretch rate  $\dot{S}$  for breaking can be estimated assuming that the thread cannot sustain a tension, by  $p_c = 0$ ; then

$$\dot{S} > 10^6/2\eta(\text{sec}^{-1})$$

For very viscous threads, say  $\eta = 500$  poise, the stretch rate for breaking

$$\dot{S} > 10^5 (\text{sec}^{-1})$$

is rather large.

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The criteria just given for the breaking of viscous threads in tension is in broad agreement with experiments on breaking tension of polymer strands discussed by Joseph [1998]. An interesting topic related to cavitation in extension is the rupture of liquid threads under capillary collapse, which is discussed next.

## 6 Conclusions

A summary of the main points in this paper is listed below.

- The pressure in incompressible Newtonian fluids is the mean normal stress. The stress is decomposed into a pressure and stress deviator with a zero trace. The pressure in incompressible Non-Newtonian liquids is given by the constitutive equation and has no intrinsic significance. Cavitation criteria for liquids in motion must be based on the stress and not on the pressure. The liquid cannot average its stresses or recognize the non-unique quantity called pressure in non-Newtonian fluids.
- It is convenient for the study of cavitation of flowing liquids to decompose the stress into a deviator and mean normal stress. The deviator has positive and negative normal stresses, deviating from the average. The most positive value of principal stresses is the maximum tension. The stress in non-Newtonian liquids should also decompose the stress into average and deviator.
- A cavitation bubble will open in the direction of maximum tension in principal coordinates. The angles defining the principal axis determine how a cavity will open; angles are important.
- A liquid can cavitate in shear. However, it is pulled open by tension in the direction defined by principal stresses; Newtonian liquids in pure plane shear will open  $45^\circ$  from the direction.
- Cavitation in a flowing liquid will occur at a nucleation site when the maximum tensile stress in principal axes coordinates is smaller than the cavitation pressure.
- Cavitation can be a fast, non-equilibrium event resembling fracture in which the cavity first opens and then fills with vapor and/or gas.
- Outgassing is cavitation of liquid gas in solution.

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