A DYNAMIC WALL MODEL CONSTRAINED BY EXTERNAL REYNOLDS STRESS

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Summary A new wall model for LES is proposed. Unlike conventional zonal approaches, given Reynolds stresses are not imposed as the solution, but used as constraints on the SGS stress so that the given Reynolds stress closely matches the computed one only in the mean sense. Also, since LES in general outperforms RANS even at coarse resolution except very near the wall, RANS constraints are limited to the points where the LES solution is expected to be erroneous. We use the Germano–identity error as an indicator of LES quality so that the RANS constraints are activated only where the Germano–identity error exceeds a certain bound. The proposed model is applied to LES of turbulent channel flow at various Reynolds numbers and grid resolutions to obtain significant improvement over the dynamic Smagorinsky model, especially at coarse resolutions.

INTRODUCTION: IDEAL ZONAL SIMULATION

In order to highlight the main concern of RANS–LES zonal simulations, an idealized version of such simulations for turbulent channel flow at $Re_\tau = 590$ is performed to obtain statistics shown in Fig. 1. In this simulation, mean velocity from DNS is enforced in an arbitrarily chosen RANS region $y^+ \leq 60$ with all turbulence fluctuations suppressed, while DNS is performed in the outer region $y^+ > 60$ so that a perfect LES is obtained by filtering the solution in this region. As shown in Fig. 1, the mean velocity profile in the outer layer rises above the log law significantly, consistent with previous observations in DES computations (Nikitin *et al.* 2000). Also, this approach creates false wall–turbulence starting at the zonal interface that has striking similarity with true wall–turbulence (Fig. 1(b)).

CONSTRAINED DYNAMIC SMAGORINSKY MODEL

From the ensemble average of the spatially filtered Navier–Stokes equations, we obtain the following equality for the SGS stress $\tau_{ij} = \overline{u_i u_j} - \overline{u}_i \overline{u}_j$:

$$\langle \tau_{ij} \rangle + r_{ij} = \mathcal{R}_{ij} \tag{1}$$

under the assumption that $\langle \overline{\phi} \rangle = \langle \phi \rangle$, where $r_{ij} = \langle \overline{u}_i \overline{u}_j \rangle - \langle \overline{u}_i \rangle \langle \overline{u}_j \rangle$ and $\mathcal{R}_{ij} = \langle u_i u_j \rangle - \langle u_i \rangle \langle u_j \rangle$ are respectively, resolved and exact Reynolds stress and $\langle \cdot \rangle$ denotes the ensemble average. Invoking the SGS model $\tau_{ij}^M - \frac{1}{3}\tau_{kk}\delta_{ij} = -2C_s\Delta^2|\overline{S}|\overline{S}_{ij}$, one can define an error $\epsilon_{ij}^{\mathcal{R}}$ as $\epsilon_{ij}^{\mathcal{R}} = \langle \tau_{ij}^M \rangle + r_{ij} - \mathcal{R}_{ij}$. On the other hand, the well known Germano identity error is defined as $\epsilon_{ij}^G = T_{ij}^M - \hat{\tau}_{ij}^M - L_{ij}$, where T_{ij} is the subtest scale stress model, L_{ij} is resolved scale stress and hat denotes test filtering.
Although the dynamic model coefficient C_s is obtained by the minimization of $\epsilon_{ij}^G \epsilon_{ij}^G$, such a procedure does not guarantee small absolute error. Fig. 2(b) shows normalized Germano–identity error from LES of channel flow with dynamic Smagorinsky model at $Re_\tau = 1000$. A 32^3 uniform grid is used on $\pi\delta(x) \times 2\delta(y) \times 0.5\pi\delta(z)$ domain. Fourier expansion and the fourth-order compact difference scheme are used for discretization of homogeneous directions and the wall–normal direction, respectively. It is surprising to see that the error is orders of magnitude larger than SGS term especially near the wall. This shows the failure of the scale–similarity assumption that single model coefficient is valid across different scales. This suggests that we discard the Germano–identity where the error is unacceptably large, and use

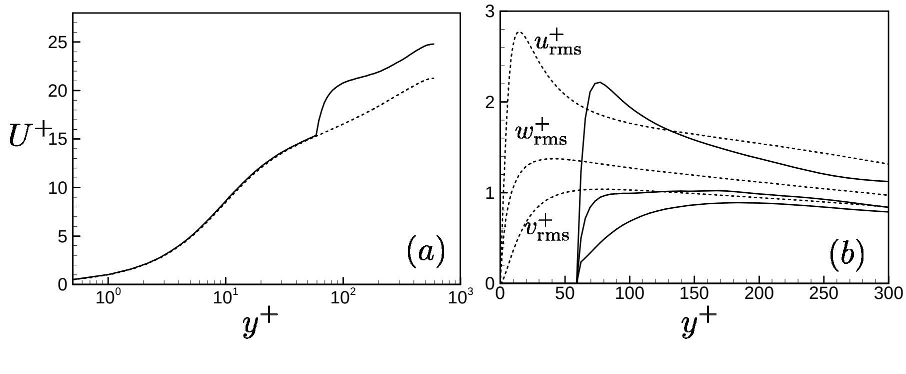

Figure 1. Mean statistics from turbulent channel flow at $Re_\tau = 590$: (a) mean velocity, (b) RMS velocity fluctuations. Dotted line, Moser *et al.* (1999); Solid line, ideal RANS–LES zonal simulation.

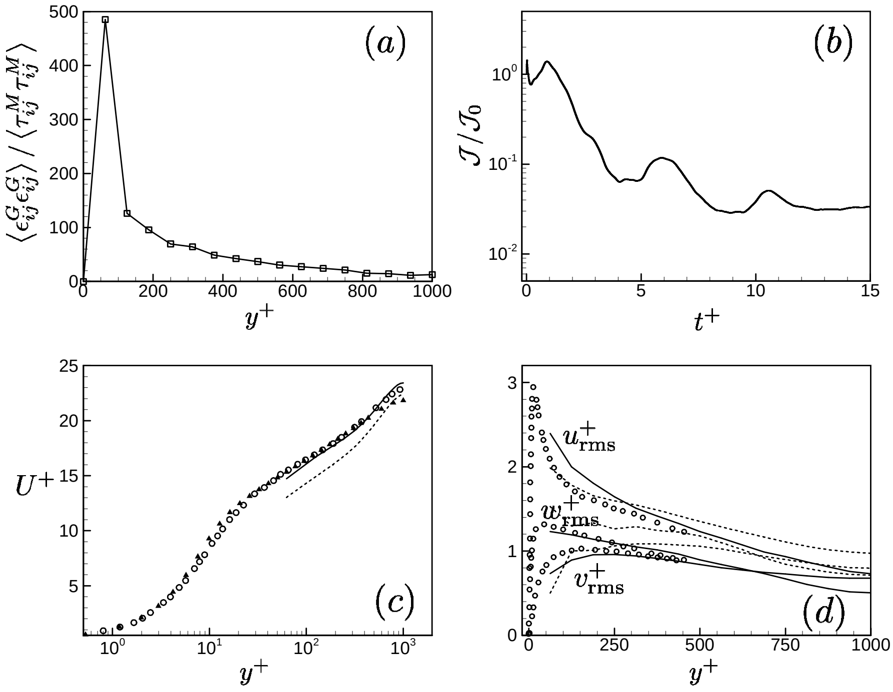

Figure 2. LES of turbulent channel flow at $Re_\tau = 1000$: (a) Germao–identity error with dynamic Smagorinsky model; (b) time evolution of cost function (2); (c) mean velocity profiles; (d) RMS velocity fluctuations. Solid lines in (c) & (d), the proposed model; dotted lines in (c) & (d), dynamic Smagorinsky model; open circle, Kravchenko and Moin (1997); solid triangle, Spalart–Allmaras model simulation.

RANS–based condition (1) to determine SGS eddy viscosity. We consider the following cost function:

$$\mathcal{J} = \int_{\Omega_0} \left[\left\langle \epsilon_{ij}^G \epsilon_{ij}^G \right\rangle_h + \omega^{\mathcal{R}} \epsilon_{ij}^{\mathcal{R}} \epsilon_{ij}^{\mathcal{R}} \right] d\mathbf{x}, \tag{2}$$

where Ω_0 is the domain without homogeneous direction(s), and $\langle \cdot \rangle_h$ denotes average over homogeneous direction(s). $\omega^{\mathcal{R}}$ denotes the weight for RANS–based condition, and can be a function of local Germano–identity error. In this study, we use $\omega^{\mathcal{R}} = \max(\mathcal{E} - \mathcal{E}_t, 0)$, where $\mathcal{E} = \left\langle \epsilon_{ij}^G \epsilon_{ij}^G \right\rangle / \left\langle \tau_{ij}^M \tau_{ij}^M \right\rangle$, and $\mathcal{E}_t$ is the threshold value. $\mathcal{E}_t = 100$ is chosen from EDQNM analysis on isotropic turbulence.

For a practical model implementation, we use a simple procedure to obtain C_s that minimizes $\mathcal{J}$. We introduce C_s^g which is obtained instantaneously from $\partial \left\langle \epsilon_{ij}^G \epsilon_{ij}^G \right\rangle_h / \partial C_s^g = 0$. Then at each time step, C_s is obtained by the following penalty-like correction:

$$C_s = C_s^g + \mathcal{S} \lambda \omega^{\mathcal{R}} \epsilon_{ij}^{\mathcal{R}} \epsilon_{ij}^{\mathcal{R}}, \tag{3}$$

where $\mathcal{S}$ is a sign factor to determine whether to add or subtract dissipation. $\mathcal{S} = 1$ when the absolute value of the production of mean kinetic energy, $(\langle \tau_{ij} \rangle + r_{ij}) \langle S_{ij} \rangle$ is greater than that from prescribed Reynolds stress $\mathcal{R}_{ij} \langle S_{ij} \rangle$, and $\mathcal{S} = -1$ when the opposite is true. $\lambda = \lambda_0 / \left(\frac{1}{V} \int_\Omega \epsilon_{ij}^{\mathcal{R}} \epsilon_{ij}^{\mathcal{R}} d\mathbf{x} \right)$ is a dimensional relaxation factor, where λ_0 is an adjustable relaxation constant chosen to be the computational time step, and V is the volume of the entire computational domain Ω.

The proposed model is applied to LES of turbulent channel flow at $Re_\tau = 1000$ with 32^3 uniform grid as described above. The Spalart–Allmaras model (1994) is used to provide the external Reynolds stress $\mathcal{R}_{ij}$. As shown in Fig. 2(b) the proposed procedure actually reduces the cost function $\mathcal{J}$, and a steady value of $\partial \mathcal{J} / \partial C_s \approx 0$ is reached at $t^+ > 12$. Mean statistics from the proposed model and the dynamic Smagorinsky model are shown in Figs. 2(c) and (d). Results from the proposed model compare well with RANS model prediction and LES data of Kravchenko & Moin (1997), while those from the dynamic Smagorinsky model underestimate the mean velocity in wall units and overpredict RMS velocity fluctuations.

References

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