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Modal Analysis of Fluid Flow: Introduction to the Virtual Collection

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Modal Analysis of Fluid Flows Virtual Collection: https://arc.aiaa.org/vc/modalanalysis

AT ANY recent bustling fluid dynamics conference, it has become commonplace to hear the terms *POD*, *DMD*, *Koopman*, *global stability*, and *resolvent* in conversations. In fact, it is not surprising to overhear these terms at cafes, restaurants, and bars in the neighborhood of the conference venue, even outside of working hours. These terms, or the modal analysis techniques they represent, are now woven into many of our studies and serve as indispensable tools. The acronyms of POD and DMD are practically known by any fluid mechanician. In our research community, the use of these acronyms has become so widespread that they are rubbing shoulders with other acronyms like DNS, LES, RANS, LDV, and PIV.

There have been a number of extensive papers and reference books on modal analysis techniques [1–14]. With the enhancement in computational and experimental capabilities, there is need for the use of these modal analysis techniques to systematically extract physical insights from massive complex flowfield data. Furthermore, the wealth of information in these large data sets has inspired the development of reduced-order models founded on modal bases [15,16]. What has been very exciting over the past few years are the developments in data science [17] that inspire novel extensions of these analysis techniques. Given these vast and growing selections of modal analysis techniques and their applications, it can be overwhelming for an aspiring fluid mechanician to take a dive into the vast literature. In addition, there are many nuances in their applications that are yet to be fully understood.

To address these issues, a discussion group entitled *Modal Analysis of Aerodynamic Flows* was created by Douglas Smith and the first author under the Fluid Dynamics Technical Committee at AIAA. This discussion group was established with two objectives: 1) to provide an educational service to a nonspecialist who seeks to gain greater insight from a data set with modal decomposition/analysis methods, and 2) to minimize the barriers to implementation of these methods. This discussion group has benefited from a large group of attendees who have supported the technical activities, including the contributions to the invited sessions on modal analysis at AIAA Aviation 2016[§] (Washington, D.C.), Aviation 2017 (Denver), and Aviation 2018 (Atlanta). These efforts have also permeated outside of AIAA, resulting in the organization of a minisymposium at the 2017 APS Division of Fluid Dynamics Meeting (Modal Analysis Methods for Fluid Flows) by Mitul Luhar with the first and second authors. Each and every one of these sessions was attended by a large audience, which is a testament to how widely modal analysis techniques are used and continue to attract attention. Furthermore, as archival products of these efforts from the discussion group, we have compiled this Virtual Collection on *Modal Analysis of Fluid Flows*, comprising of two overview papers [18,19] and eight contributed papers [20–27].

The first overview paper [18] provides an overarching survey of modal decomposition and analysis techniques to serve as an educational guide. Included in the survey are the proper orthogonal decomposition (POD) [1,2,28,29], balanced proper orthogonal decomposition (BPOD) [30], dynamic mode decomposition (DMD) [3,4], Koopman analysis [4,7], global stability analysis [10], and resolvent analysis [11–13]. In the overview paper, the modal analysis techniques are broadly categorized into data-based techniques (POD, BPOD, and DMD) and operator-based techniques (Koopman, global stability, and resolvent analysis), with discussions on the relations among them. The methods covered in the first paper are the most commonly used modal analysis techniques in fluid mechanics and are established on the theories of eigenvalue decomposition and singular value decomposition.

As a companion to the first overview paper, a second overview paper [19] was compiled to demonstrate how the outputs of these modal analysis techniques can be interpreted to extract physical insights. The second overview paper presents a collection of examples and applications of modal analyses for the canonical flows of cylinder wakes, channel flows, airfoil wakes, and open cavity flows. Also included in the second overview paper is an outlook on modal analysis techniques. As data science and applied mathematics make advancements in handling large data sets and operators, it is an exciting time to expand the envelope of modal analysis techniques to analyze high-dimensional high-Reynolds-number turbulent flows and multiphysics fluid flow problems. In fact, there are already promising advances on performing modal analysis with randomized techniques [31–34] and machine learning [35–37], as well as development of sparse models [38].

The remainder of the Virtual Collection consists of eight technical papers, each providing an in-depth demonstration of the modal decomposition methods for analysis, modeling, and control of fluid flows. The paper by Schmidt and Colonius [20] presents some new guidelines on the use of POD in the spectral domain [28,39], which they term "spectral POD" (SPOD). They present illustrative examples of applying SPOD on flowfields obtained from a large-scale turbulent jet simulation and a vertical wind turbine experiment, extracting the frequency spectra and modal profiles. Chavarin and Luhar extend the resolvent formalism to analyze and predict the effect of riblets on the drag within a turbulent channel flow [24]. The resolvent gains are found to be relevant predictors of drag increase and decrease, creating opportunities to exploit these low-order resolvent models in riblet shape optimization. The resolvent framework is also used to analyze cavity flows with stable and unstable base states by Sun et al. [23]. In their work, guiding steps for resolvent analysis are provided with examples of laminar and turbulent compressible open cavity flows to elucidate physics insights. Ansell and Mulleners tailor the empirical mode decomposition (EMD) [40] to analyze the multiscale vortex characteristics of dynamic stall [26]. The EMD approach is used to distill PIV data from wind tunnel experiments into a set of intrinsic modes that reflect the complex physics associated with vortex formation, vortex pairing, and shear-layer roll-up.

The utility of modal decomposition methods for predictive and control-oriented modeling is also demonstrated in a number of papers in this collection. Xu et al. use projection-based reduced-order models to expedite numerical simulations of a quasi-one-dimensional rocket combustor without compromising predictive performance [27]. POD–Galerkin models are devised to model the combustion instabilities in an upstream domain, which are then integrated with full-order models of the downstream domain. Bai et al. show that the DMD framework can be combined with ideas from compressed sensing to solve challenging problems in system identification [21]. The approach is demonstrated for the identification of input–output models from heavily subsampled data of flow past a pitching airfoil. Not only are these models ideal for subsequent control analysis and design tasks, but they are also shown to be physically interpretable. Kalur and Hemati show that modal decomposition techniques can be used for control-oriented model reduction, allowing feedback control design using computationally intensive convex-optimization-based techniques [22]. By combining balanced truncation with output projection onto POD modes, optimal feedback controllers are designed to minimize the maximum transient energy growth of flow perturbations within a channel flow. Symon et al. use variational data assimilation within the resolvent framework to

[§]Presentations from the invited session at Aviation 2016 have been made available online at https://www.youtube.com/playlist?list=PLOl5RDXeYOMCNXko4fMx2ciVmTbbIGcB5.

reconstruct the mean and unsteady components of flow over a cylinder from a single point sensor [25]. Resolvent analysis is further used to develop strategies for placing multiple sensors.

The publication of this Virtual Collection in the *AIAA Journal* completes the activities of the *Modal Analysis of Aerodynamic Flows Discussion Group*. The efforts of this group have evolved and continue through a new discussion group on *Reduced-Complexity Modeling and Analysis of Fluid Flows* chaired by the second author and Karthik Duraisamy. We greatly look forward to the future developments of modal analysis techniques and their applications on complex fluid flows. Before closing, we gratefully acknowledge the support and encouragement from Douglas Smith, Steven Brunton, Scott Dawson, Tim Colonius, Mitul Luhar, Karthik Duraisamy, Alexander Smits, Peyman Givi, and the contributors to our discussion group, who have made this endeavor possible and enjoyable.

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