AEM 4321 / EE 4231: HW#2

1. **First Order Systems: Step Response**

   Consider the following first order system:

$$10\dot{x} + 3x = -4u \tag{1}$$
$$x(0) = 1 \tag{2}$$

   (a) Is this system stable? What is the time constant for the system?

   (b) Sketch the response for this sysem with the following input:

$$u(t) = \begin{cases} 0 & t < 0 \text{ sec} \\ 6 & t \geq 0 \text{ sec} \end{cases} \tag{3}$$

   Label on your plot the initial value, final (steady-state) value, and the approximate time to reach the steady state.

2. **Cruise Control: Open Loop**

   In class we discussed a simple model for a car:

$$m\dot{v} = -bv + F \tag{4}$$

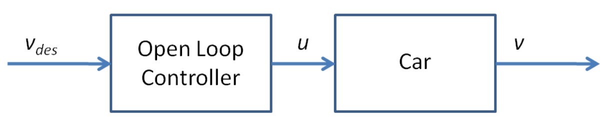
   where $v :=$ velocity (m/s), $m :=$mass (kg), and $b :=$ wind drag coefficient (N s/m). F is the force generated due to the engine (N). Assume that F is proportional to the engine throttle angle: $F = ku$ where $u :=$ engine throttle (deg) and $k :=$ force constant (N/deg). Then the vehicle model can be written as:

$$m\dot{v} = -bv + ku \tag{5}$$
$$v(0) = v_0 \tag{6}$$

   (a) The course webpage contains a `Simulink` diagram of this vehicle model. The webpage also has a file `OpenLoopPlotsA.m` that simulates the system and generates a plot of $v(t)$ vs. t. The files use the following parameters: $m = 2000$ kg, $b = 20$ N s/m, and $k = 40$ N/deg. Modify the files (or create your own) to simulate the system with $v(0) = 15$m/sec and $u = 15$ deg. Hand in a plot of $v(t)$ vs t. Make sure all axes are labeled with the signal names and units. Label on your plot the initial value, final (steady-state) value, and the approximate time to reach the steady state.

   (b) In steady state, what value of the throttle is required to make the car reach a desired velocity v_{des}? This gives the throttle u needed to achieve a v_{des}, i.e. $u = f(v_{des})$.

   (c) Use the formula from part (b) to implement an open-loop strategy in `Simulink`. Specifically, modify your diagram from part (a) to include the open-loop controller:



   Simulate this model with $v(0) = 15$ m/sec and the desired velocity profile:

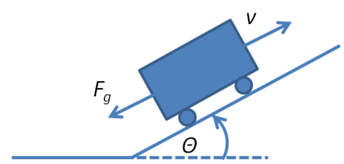
$$v_{des}(t) = \begin{cases} 15 \text{ m/sec} & t \leq 5 \text{ sec} \\ 25 \text{ m/sec} & t > 5 \text{ sec} \end{cases} \tag{7}$$

   Hand in a plot of the response of $v(t)$ vs. t. At $t = 5$ sec, v_{des} changes to 25 m/sec. How long does it take for the car to reach 24 m/sec? Does this seem like a reasonable speed of response for a cruise control?

   (d) When the car is on a slope, there will be an additional gravitational force $F_g(t) = mg \sin\theta(t)$, where $\theta(t)$ is the road slope at time t.

   Including this disturbance force in our car model gives:

$$m\dot{v} = -bv + ku - F_g \tag{8}$$
$$v(0) = v_0 \tag{9}$$



Add this disturbance force to your `Simulink` model assuming the force profile is:

$$F_g(t) = \begin{cases} 0 \text{ N} & t \leq 10 \text{ sec} \\ 100 \text{ N} & t > 10 \text{ sec} \end{cases} \tag{10}$$

Simulate the model with $v(0) = 25$ m/sec and $v_{des}(t) = 25$ m/sec for $t \geq 0$. Hand in a plot of the response of $v(t)$ vs. t. How much does the vehicle speed change due to the road slope disturbance? How much would the vehicle speed change if $F_g(t) = 350$ N for $t > 10$ sec? Note that 350 N corresponds to a very small road slope (≈ 1 deg). Does this seem like acceptable performance?

(e) Hand in a print-out of your final `Simulink` diagram.

3. **Cruise Control: Proportional Feedback Control**

In this problem we investigate a simple closed-loop strategy known as Proportional Control. Here, the throttle is chosen to be proportional to the tracking error:

$$u(t) = K_p(v_{des} - v(t)) \tag{11}$$

where $v_{des} - v(t)$ is the tracking error, and K_p is the proportional gain (deg sec/m).

(a) Take your `Simulink` diagram from Problem 2 and replace the open loop controller with the proportional controller given above, making the appropriate signal connections. Simulate the model with $v(0) = 15$ m/s, $F_g = 0$ N, and

$$v_{des}(t) = \begin{cases} 15 \text{ m/sec} & t \leq 5 \text{ sec} \\ 25 \text{ m/sec} & t > 5 \text{ sec} \end{cases} \tag{12}$$

Perform simulations with 3 different gains $K_p = 5, 10, 20$ (deg sec/m). On a single plot, show $v(t)$ vs. t for each of the gains (hint: use the `hold on` command in MATLAB). Label each curve based on its corresponding value of K_p. On another plot, show the throttle $u(t)$ vs. t for each gain (again, label the plots). How does the performance change based on K_p? How does the performance compare to the open loop performance in Problem 1(c)? In particular, comment on the speed of the response and steady-state value of $v(t)$.

(b) Next, simulate the system with $v(0) = 25$ m/sec and $v_{des}(t) = 25$ m/sec for $t \geq 0$, and

$$F_g(t) = \begin{cases} 0 \text{ N} & t \leq 10 \text{ sec} \\ 100 \text{ N} & t > 10 \text{ sec} \end{cases} \tag{13}$$

Again, simulate with the three gains $K_p = 5, 10, 20$ (deg sec/m) and generate plots of $v(t)$ and $u(t)$ vs. t. How does the ability of the proportional controller to maintain v_{des} compare to the open-loop results in Problem 1(d)? How does the performance vary with K_p? Hand in a print out of your final `Simulink` diagram.

(c) You may have noticed that the responses in part (a) don't converge to v_{des}. In other words, there is a steady state error. Let's use our understanding of first order systems to understand the steady-state error that appears in the simulation results. The closed-loop system is described by (ignoring F_g):

$$m\dot{v} = -bv + ku, \quad v(0) = v_o \tag{14}$$

$$u = K_p(v_{des} - v) \tag{15}$$

$$v_{des} = \begin{cases} v_o \text{ m/sec} & t \leq T \text{ sec} \\ v_1 \text{ m/sec} & t > T \text{ sec} \end{cases} \tag{16}$$

Put this closed-loop system in the form of our standard first order system with v_{des} as the input and v as the output variable. Do not plug in the numeric values of the variables. In other words, all your work should be done symbolically. What is the time constant of the closed loop system? What is the steady-state value of the speed? How do the speed of response and steady state value of $v(t)$ depend on K_p?

(d) One practical consideration is that the throttle cannot be opened by more than 90 deg. Thus, the throttle input on the real system must satisfy $0 \leq u(t) \leq 90$ deg for all time. This is known as actuator saturation. Based on the simulation results in parts (a) and (b), what limitations does this place on K_p? What impact will this have on the speed of response?