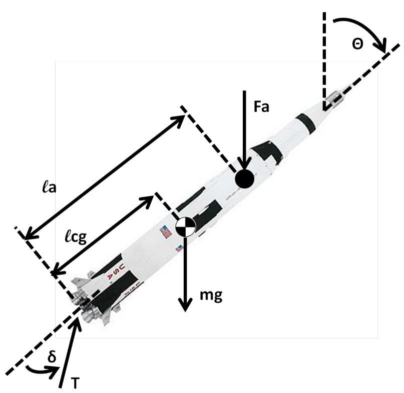
**AEM 4321 / EE 4231: HW#4**

1. **Rocket Attitude Control**

Rockets require precise control of their heading direction to ensure that they reach their desired final destination. Even small deviations in their heading can lead to large errors in the final position once the rocket reaches outer space. Modern rockets control their heading, i.e. their attitude, by using thrust vectoring. Specifically, the rocket nozzle can be rotated to change the direction of the thrust and hence control the heading direction of the rocket. In this problem, you'll design an algorithm to control the rocket attitude. Additional details on rocket attitude control can be found at: `http://microgravity.grc.nasa.gov/education/rocket/gimbaled.html`

The free-body diagram below shows the key forces involved in the attitude control problem. $T$ is the thrust force produced by the engine rocket and $F_a$ is the aerodynamic force. $\theta$ is the rocket attitude, i.e. the heading angle. $\delta$ is the angle between the rocket thrust and the centerline of the rocket. The objective is to use the control input $\delta$ to ensure that the rocket attitude $\theta$ tracks some desired reference heading $r$. The angles $\theta$, $\delta$, and $r$ are all in units of radians. It is important to note that the rocket nozzle can only be rotated by small angles. For this problem we'll assume the rotation is limited to $|\delta| \leq 0.2$ rad.




Rocket Free-body Diagram

| | | |
|---|---|---|
| $I$ | Moment of inertia about CG | 2.49e8 kg m$^2$ |
| $l_{cg}$ | Length from nozzle to CG | 30.48 m |
| $l_a$ | Length from nozzle to aero. center | 33.53 m |
| $T$ | Engine thrust | 5.16e7 N |
| $m$ | Rocket mass | 1.456e6 kg |
| $F_a$ | Aerodynamic force | 1e7 N |

Rocket Parameters

The parameters for our model are taken from the following paper:

- "Inversion based multibody control: Launch vehicle with fuel slosh," by K. Krishnaswamy and D. Bugajski, 2005 AIAA Guidance, Navigation, and Control Conference, paper number AIAA 2005-6149.

Our model makes several simplifying assumptions, e.g. we are ignoring the moments due to the fuel sloshing in the fuel tank. In addition we'll assume $F_a$ acts vertically downward as drawn in the diagram. However, the model will be sufficient to highlight the key issues in attitude control. The rocket attitude dynamics are given by Newton's second law for rotational systems. Summing the moments about the center of gravity gives:

$$I\ddot{\theta} = T\ l_{cg} \sin\delta + F_a\,(l_a - l_{cg}) \sin\theta \tag{1}$$

This is a nonlinear ODE due to the $\sin\delta$ and $\sin\theta$ terms. The system has an equilbrium point at $(\dot{\theta}, \theta, \delta) = (0, 0, 0)$. Linearizing around this trim condition gives the following linear ODE:

$$I\ddot{\theta} = T\ l_{cg}\delta + F_a\,(l_a - l_{cg})\,\theta \tag{2}$$

After plugging in the parameter values from the table and re-arranging terms we obtain:

$$\ddot{\theta} - 0.1225\ \theta = 6.3163\ \delta \tag{3}$$

We'll use the model in Equation 3 for our design and analysis. The control law should be designed so that the closed-loop system with input $r(t)$ and output $\theta(t)$ is stable. In addition, the closed loop should satisfy the following transient response requirements when $r(t)$ is any step reference command of magnitude less than 0.1 rad:

i. has a settling time $< 2$ sec,

ii. has an overshoot $< 5$ %,

iii. has steady state error $|e_{ss}| < 0.005$ rad,

iv. has $|\delta(t)| \leq 0.2$ rad for all $t \geq 0$

Questions:

(a) Is the rocket (with no controller) stable or unstable? Sketch the approximate response of the rocket, $\theta(t)$ vs. $t$, if the initial conditions are $(\theta(0), \dot{\theta}(0)) = (0.01$ rad, $0$ rad/sec$)$ and $\delta(t) = 0$ for all $t \geq 0$. Your sketch only needs to roughly capture the long-term characteristics of $\theta(t)$.

(b) Consider a proportional control law: $\delta(t) = K_p(r(t) - \theta(t))$. Derive an ODE that models the closed-loop dynamics with input $r$ and output $\theta$. Sketch a graph by hand showing the locations of the closed-loop poles as a function of the gain $K_p$. On this graph label the locations of the closed-loop poles for $K_p < 0$, $K_p = 0$, and $K_p > 0$. Can you satisfy the design constraints using a proportional control law?

(c) Consider a control law of the form: $\delta(t) = K_p(r(t) - \theta(t)) - K_d\dot{\theta}(t)$. As discussed in class, this is a version of proportional-derivative control. This form avoids the differentiation of the reference command $r(t)$ and is sometimes called proportional control with rate feedback.

Derive an ODE that models the closed-loop dynamics with this control law. Choose the gains $K_p$ and $K_d$ to make the closed-loop stable and to satisfy requirements i), ii), and iii).

(d) The course website contains a `Simulink` model with the rocket dynamics and the control law. It also contains a block for a sensor model that will be used in the next part of the problem. Use the `Simulink` model to simulate your control law with $r(t) = 0.1$ rad for $t \geq 0$ and see if your design satisfies requirement iv). If iv) is not satisfied then modify your gains until you are able to satisfy all design requirements. This may require some iteration. Hand in plots of $\theta(t)$ vs. $t$ and $\delta(t)$ vs. $t$ for your final choices of $K_p$ and $K_d$. Also, hand in work to justify the choices of your gains.

(e) One issue with the implementation of the PD control law is that differentiation amplifies sensor noise. A very simple model of the attitude sensor is:

$$\theta_{meas}(t) = \theta(t) + n(t) \tag{4}$$

where $n(t)$ is sensor noise. The key point of this model is that the measured value of the rocket attitude is not exactly equal to the true value of the rocket attitude. Typically sensor noise would have small amplitude but high frequency. Modify the `Attitude Sensor` block in the `Simulink` diagram to implement this sensor model with the following value for noise:

$$n(t) = 0.002\sin(500t) \tag{5}$$

You can use the `Sine Wave` block in the `Sources` folder. Simulate your control law with $r(t) = 0.1$ rad for $t \geq 0$. Hand in a plots of $\theta(t)$, $\theta_{meas}(t)$, and $\delta(t)$ vs. time. Comment on the impact of the sensor noise. Does your controller still satisfy all the design requirements?

(f) A typical solution for this noise issue is to use another sensor to independently measure $\dot{\theta}$, e.g. a rate gyroscope. Assume that a rate gyroscope is available to independently measure $\dot{\theta}$. We'll again use the following simple model for this sensor:

$$\dot{\theta}_{meas}(t) = \dot{\theta}(t) + n(t) \tag{6}$$

where $n(t)$ is given by the small amplitude, high frequency sinusoid in Equation 5.

Modify your `Simulink` diagram to simulate an implementation of the PD control law that uses this rate gyroscope measurement. First, add another subsystem to model the rate gyroscope sensor. You can make a copy of the `Attitude Sensor` subsystem and simply change the signal names. Second, modify the `Controller` block to include another input for $\dot{\theta}_{meas}$. Third use the $\dot{\theta}_{meas}$ measurement for the derivative term rather than differentiating $\theta_{meas}$. This avoids the need to numerically differentiate the measurement of $\theta(t)$.

Simulate your control law with $r(t) = 0.1$ rad for $t \geq 0$. Hand in a plot of $\delta(t)$ vs. time. Comment on the impact of the sensor noise with this implementation. Does this implementation of your controller satisfy all the design requirements?