AEM 4321 / EE 4231: HW#7

1. **Robustness Margins**

   Consider the feedback diagram in Figure 1 below and let $L(s) := G(s)K(s)$ denote the loop transfer function for the feedback system shown below. Consider two pairs of plants and controllers:

   i) $K(s) = \frac{10(s+3)}{s}$ and $G(s) = \frac{-0.5(s^2-2500)}{(s-3)(s^2+50s+1000)}$

   ii) $K(s) = \frac{0.4s+1}{s}$ and $G(s) = \frac{1}{s+1}$

   Perform the following calculations for each plant/controller pair:

   (a) Verify that the feedback system is stable.

   (b) Use the Bode plot of $L(s)$ to find all phase-crossover frequencies, i.e. frequencies ω_0 such that $\angle L(j\omega_0) = \pm 180^o$. Use this information to compute the gain margin(s) of the feedback system.

   (c) For each gain margin c_0 calculated in part b), construct the closed-loop using the perturbed loop transfer function $c_0 L(s)$. Verify that the closed-loop has poles at $\pm j\omega_0$ and hence the gain variation c_0 causes instability.

   (d) Use the Bode plot of $L(s)$ to find all gain-crossover frequencies, i.e. frequencies ω_0 such that $|L(j\omega_0)| = 1$. Use this information to compute the phase and time delay margins of the feedback system.

   You can use the `allmargin` command to check your answers.

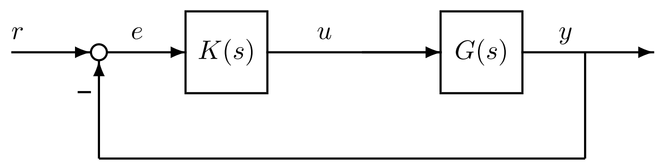

Figure 1: Feedback Loop

2. **Nyquist Plots**

   Consider again the feedback system in Figure 1 with $L(s) := G(s)K(s)$. Define two loop transfer functions:

   (a) $L(s) = \frac{-8}{s+4}$

   (b) $L(s) = \frac{2s+9}{s-4}$

   Sketch, by hand, the Nyquist plot for each loop transfer function. Then apply the Nyquist stability theorem to predict the number of closed-loop poles of the feedback system.

3. **Supercavitating Vehicle Control**

   Supercavitating vehicles are an attractive technology because they can travel underwater at extremely high speeds. Current research (see References 1 and 2 below) suggests that supercavitating vehicles could one day travel up to 3 times faster than conventional torpedoes. The increase in speed is accomplished through a dramatic reduction in drag on the vehicle body. The drag is reduced by enveloping the vehicle in an air pocket, or supercavity. The supercavity is induced by a cavitator at the nose of the vehicle. As a result, only the cavitator is in contact with the water, which reduces drag on the entire vehicle.

   - Reference 1: J.E. Dzielski, Longitudinal Stability of a Supercavitating Vehicle, IEE Journal of Oceanic Engineering, Vol. 26, No. 4, 2011.
   - Reference 2: B. Vanek, Control Methods for High-Speed Supercavitating Vehicles, PhD Thesis, University of Minnesota, 2008.

The longitudinal motion of a supercavitating vehicle can be controlled by rotating the cavitator. You will design a controller to meet a set of performance and robustness specifications using the loopshaping approach. Figure 2 shows the feedback diagram. The dynamics of the supercavitating vehicle from the cavitator input δ_c (rad) to the pitch rate q (rad/s) measurement are given by the following transfer function (Reference 2):

$$G(s) = \frac{-519.8s - 92650}{s^2 + 98.35s + 27840} \tag{1}$$

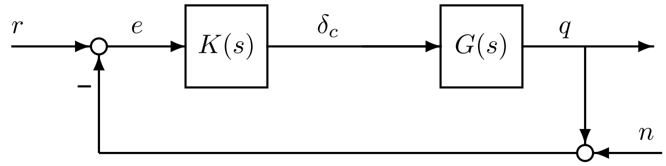

Figure 2: Supercavitating Vehicle Pitch Rate Control

Design specifications for the closed-loop supercavitating vehicle are:

i. The closed-loop system is stable.

ii. The loop transfer function has a crossover frequency near 300 rad/s.

iii. The loop transfer function has positive gain at zero frequency, i.e. $L(0) > 0$.

iv. The system has zero steady-state tracking error for a unit step reference command r.

v. The system can track a 5 rad/s sinusoid reference command r with less than 10% error.

vi. The loop transfer function must attenuate noise by -30 dB at frequencies above 3000 rad/s.

vii. The system has at least ± 6 dB of gain margin.

viii. The system has at least 45 deg phase margin.

Perform the following design steps:

(a) Use loopshaping to satisfy requirements i. through vi. As a recommendation, first choose the proportional gain to satisfy requirements ii. and iii. Then use the other loop-shaping design stages to satisfy the low and high frequency performance specifications. Turn in a single Bode plot comparing the plant dynamics $G(s)$ to the loop transfer function. Use the `legend` command to add a legend which distinguishes the various plots.

(b) For the loop transfer function designed in part (a), compute the gain and phase margins from the Bode plot. Turn in a Bode plot indicating the gain and phase margins. You can use the Matlab command `margin` to check your answers, but you should also turn in a Bode plot marked up by hand. If your design does not satisfy requirements vii. and viii., then go back to part (a) and modify your controller.

(c) Use the Simulink model provided with the homework and fill in the transfer function block with your control design so far. Simulate with the input $r(t) = \sin(5t)$. On the same plot, show the input r, the output q, and the tracking error e. Include a legend to distinguish the various plots.

(d) A more realistic model would include the cavitator actuator dynamics. In the Simulink model, opening the plant subsystem will reveal an actuator block. Currently, the actuator is model is neglected with a simple unity transfer function. Modify the actuator transfer function block to represent the following second order low-pass model:

$$G_{act}(s) = \frac{562500}{s^2 + 1061s + 562500} \tag{2}$$

What is the new loop transfer function when these actuator dynamics are included? Turn in a Bode plot of the new loop transfer function with the gain and phase margins marked up. Do you still satisfy all the specifications? Comment on any specifications that are violated by the inclusion of the actuator dynamics.

(e) Most likely your design has insufficient phase margin after including the actuator model. A lead control can be used to increase the phase margins. Add a lead stage to the controller from part part (a). The lead stage should be designed to increase the phase margin until requirement viii. is satisfied. This may cause you violate some of the other performance requirements. Iterate on your previous design if necessary until all requirements are satisfied. Turn in a Bode plot of the new loop transfer function with the gain and phase margins marked up.

(f) Use the Simulink model to simulate a doublet input. The input signal is already provided. On the same plot, show the input r and the output q. Include a legend to distinguish the various plots.