Frequency Domain System Identification

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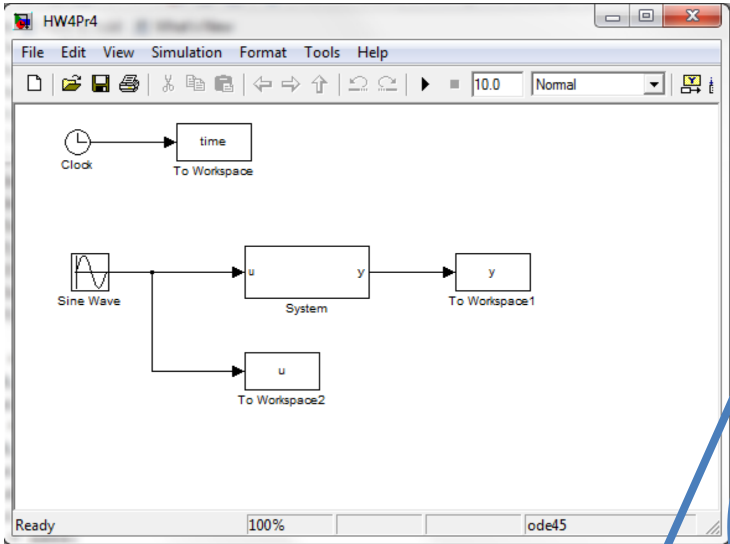
Homework 5

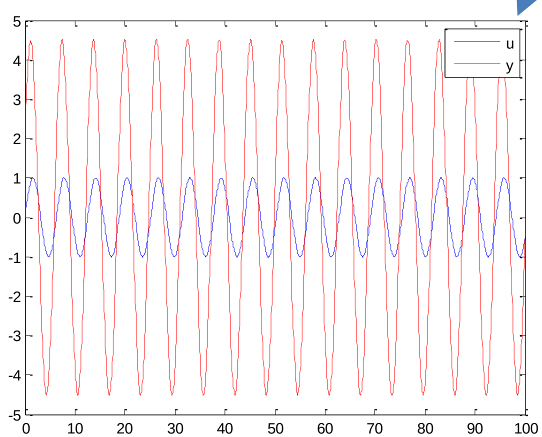


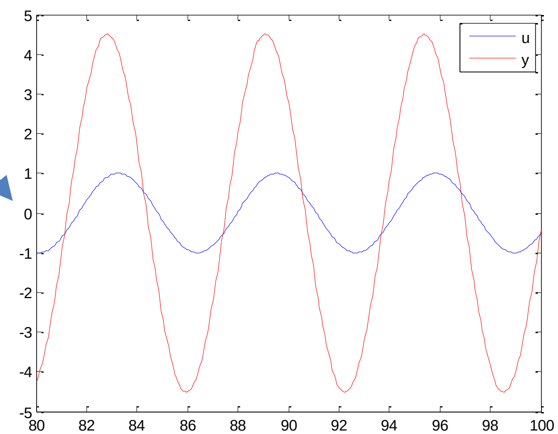
Given: Unknown “black box” system

Question: How can you construct an ODE model?

Approach:

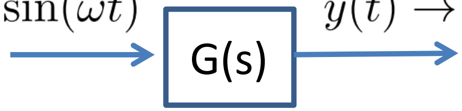
1. Simulate system for a long time with a sinusoid of chosen frequency
2. Measure the steady-state phase and gain shift of output relative to the input.
3. Repeat 1. and 2. for different frequencies
4. Draw Bode plot from 1-3 and try to construct the system model.

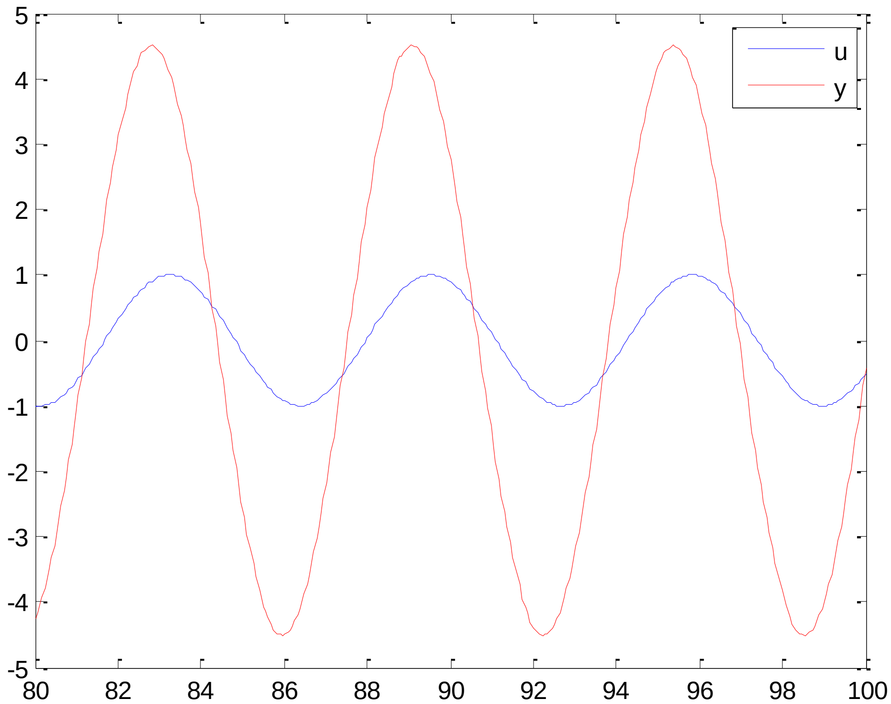




Homework 5

$u(t) = \sin(\omega t)$ $\quad$ $y(t) \to |G(j\omega)| \sin(\omega t + \angle G(j\omega))$





Magnitude: If *u(t)* has amplitude equal to 1 then *|G(jω)|* is equal to the amplitude of *y(t)* in steady state.

Phase: Note that the output *y(t)* can be re-written as follows:

$$y(t) \to |G(j\omega)| \sin\left(\omega\left(t + \frac{\angle G(j\omega)}{\omega}\right)\right)$$

$$= |G(j\omega)| u(t + \Delta t)$$

Thus the time offset between the input and output is related to the phase by:

$$\Delta t = \frac{\angle G(j\omega)}{\omega}$$

The phase can be determined from this relation by measuring the time difference Δt between peaks on u(t) and y(t). The phase is >0 if the output "leads" the input and <0 if the output "lags" the input.

University of Minnesota UAV Group

Ultra Stick 25e

Mass: 1.9 kg
Wing Span: 1.27 m
Mean Chord: 0.25 m

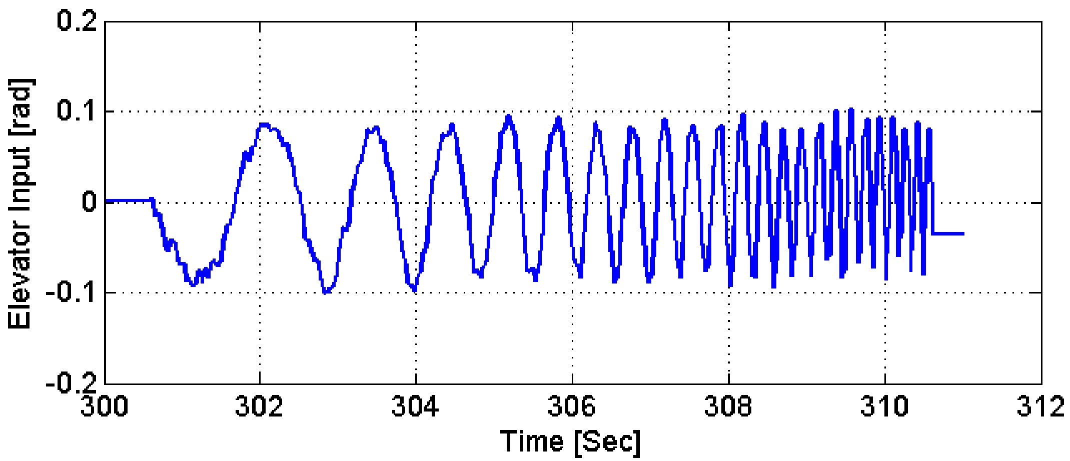
Airspeed: 15-25 m/s

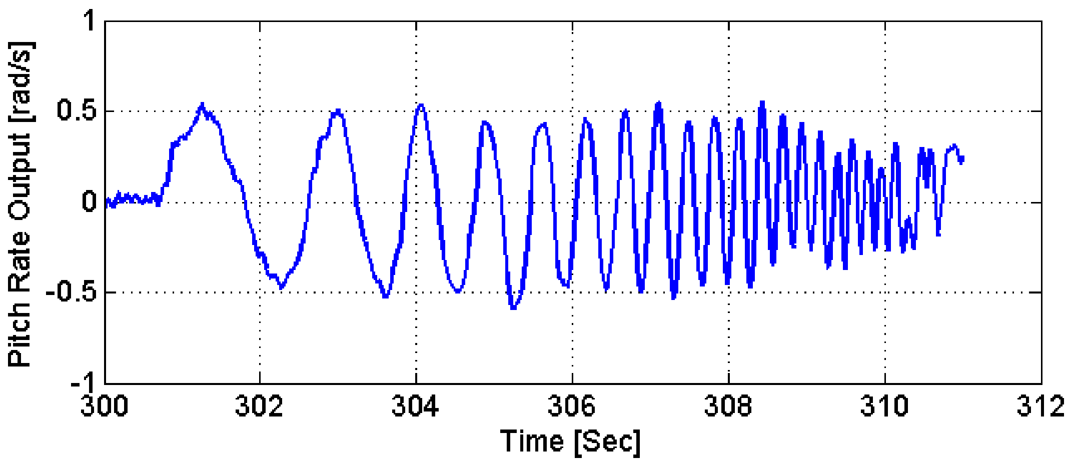
Key Properties:
- Battery Powered
- RC Controlled
- Autonomous Flight
- IMU
- Flight Computer

Experimental Flight Data: Input/Output

Input: Elevator Angle δ_e

Output: Pitch Rate q

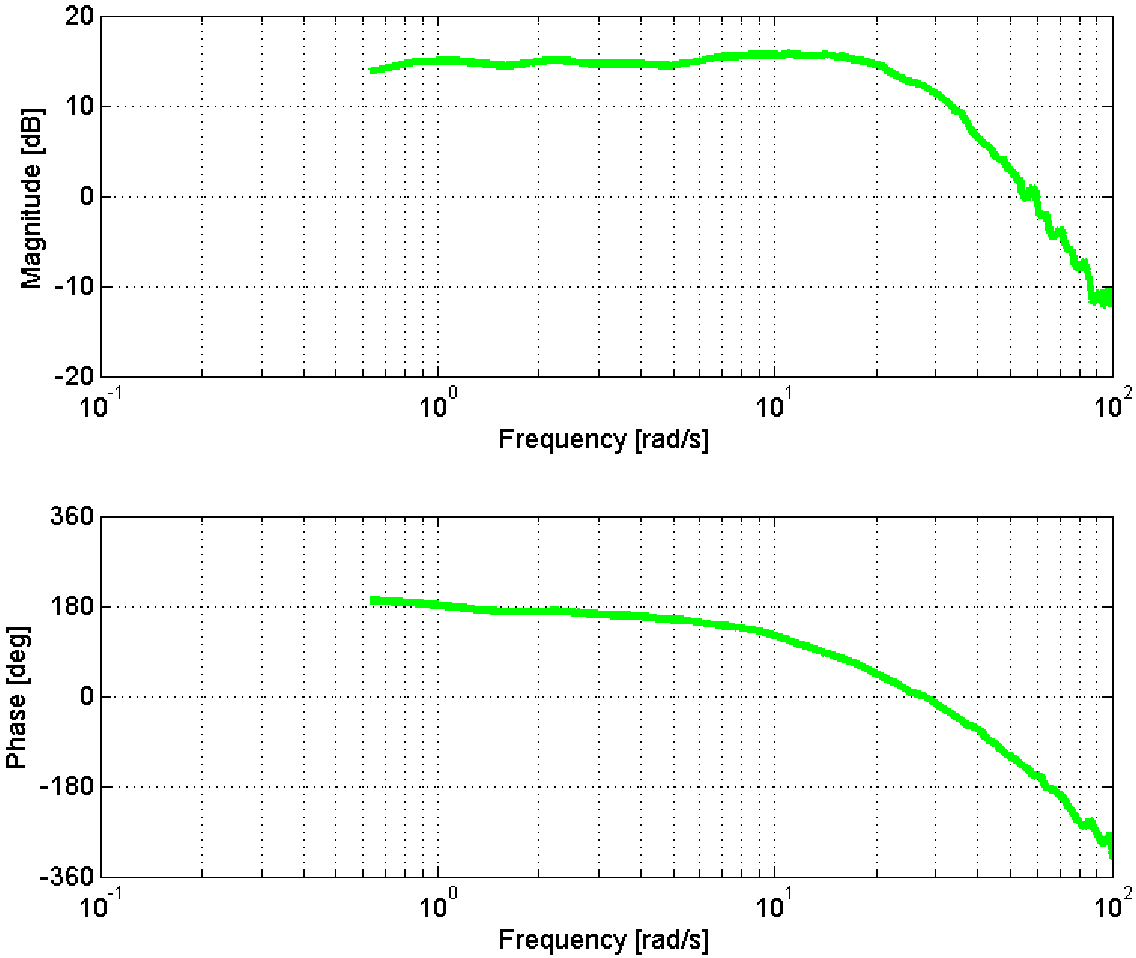




Experimental Frequency Response

Input: Elevator Angle δ_e

Output: Pitch Rate q



Short Period Mode

Short Period Mode:
2nd Order System with 1 zero

$$\delta_e \rightarrow \boxed{\frac{K\,(s+a_1)}{s^2 + 2\zeta\omega_n s + \omega_n^2}} \rightarrow q$$

Input: Elevator Angle δ_e

Output: Pitch Rate q

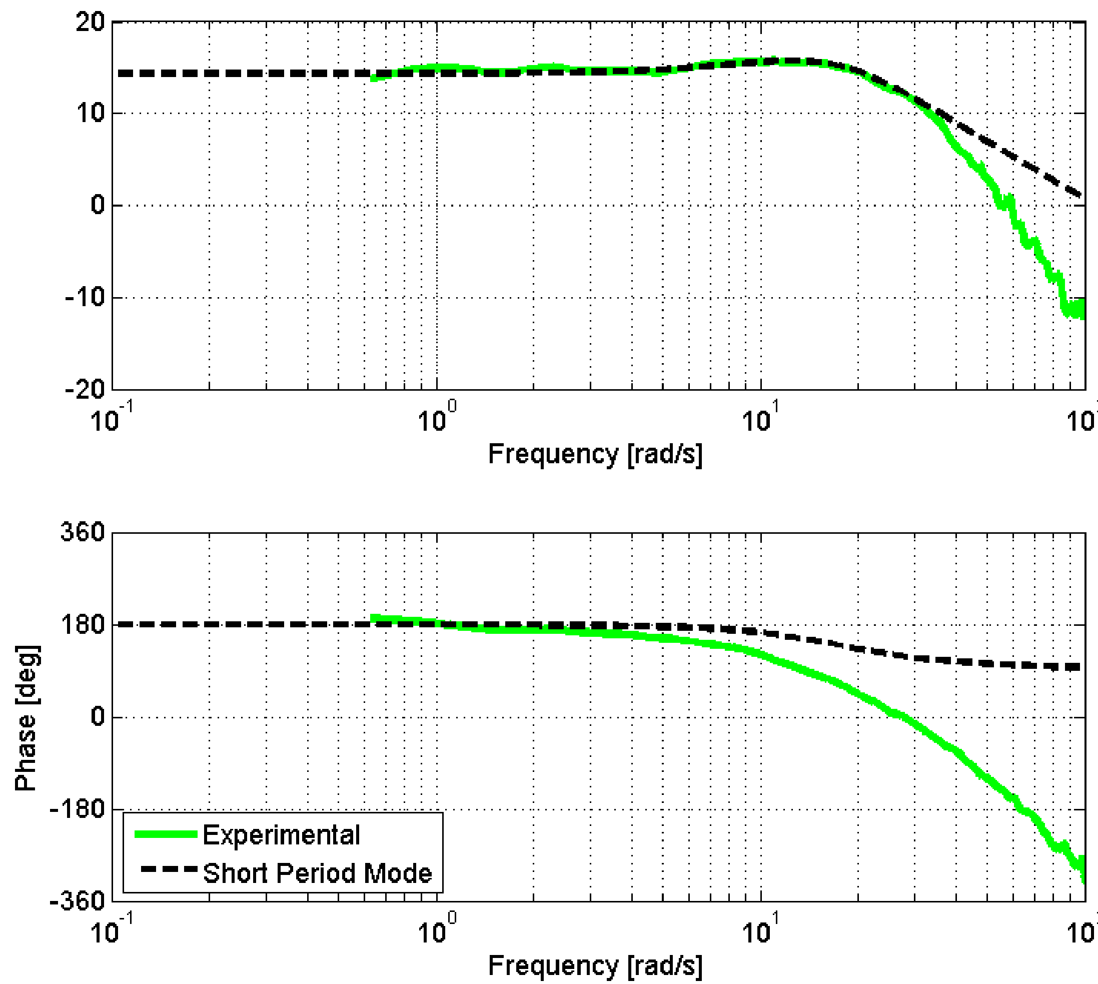
SP Params:

$K = -107.42$

$a_1 = 14.45$

$\zeta = 0.7$

$\omega_n = 17.28$



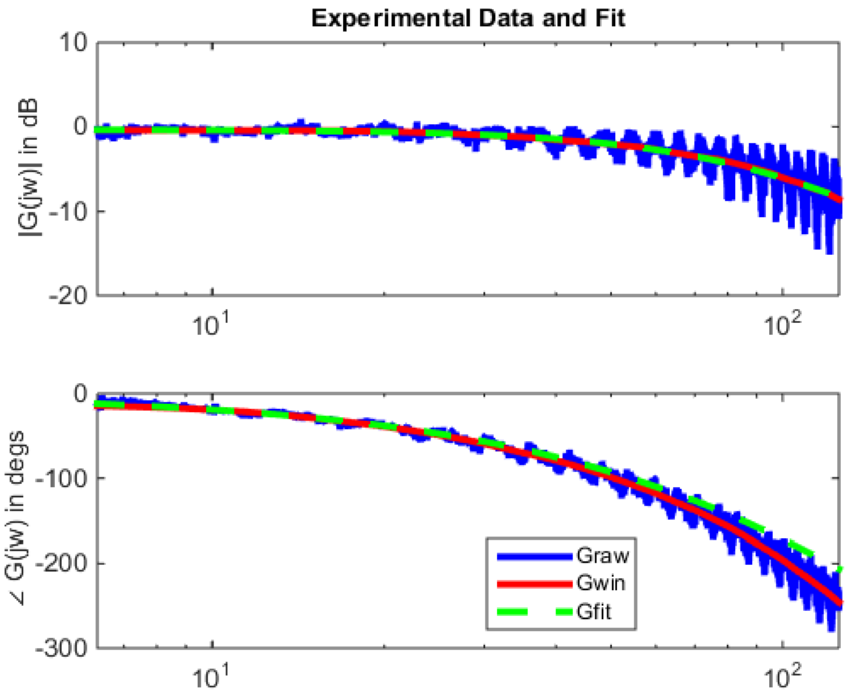
Actuator Dynamics

Low-cost servos drive the surfaces

Experimental test to model servos.

https://www.flickr.com/photos/100936386@N02/15419670923/

Actuator Model




```
ans =

                         9067
  exp(-0.014*s) * --------------------
                  s^2 + 183.4 s + 9441

Continuous-time transfer function.
```

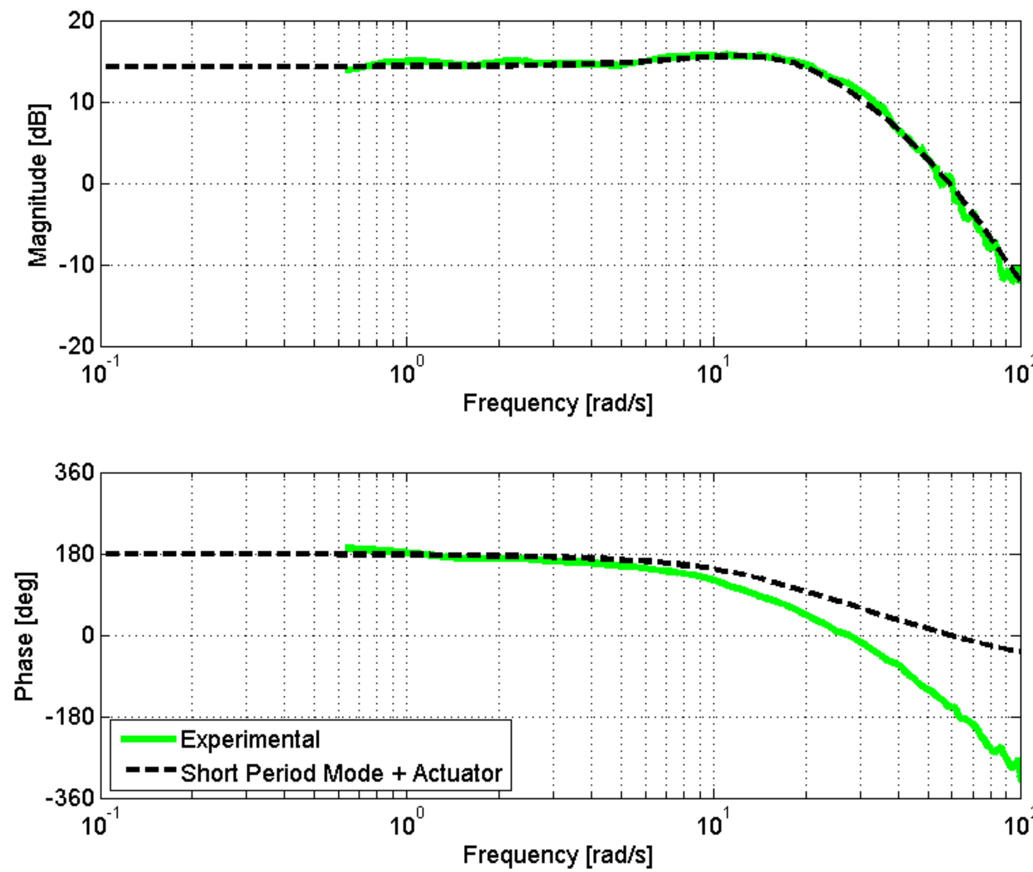
Actuator Dynamics

Include
Actuator Dynamics

$\delta_e \rightarrow \boxed{\dfrac{\omega_a^2}{s^2 + 2\zeta_a\omega_a s + \omega_a^2}} \rightarrow \boxed{\dfrac{K\,(s+a_1)}{s^2 + 2\zeta\omega_n s + \omega_n^2}} \rightarrow q$

Input: Elevator Angle δ_e

Output: Pitch Rate q



SP Params:	Act Params:
$K = -107.42$	$\zeta_a = 0.8$
$a_1 = 14.45$	$\omega_a = 50.27$
$\zeta = 0.7$	
$\omega_n = 17.28$	

Time Delay: 25 Hz Signal » 45 msec Delay

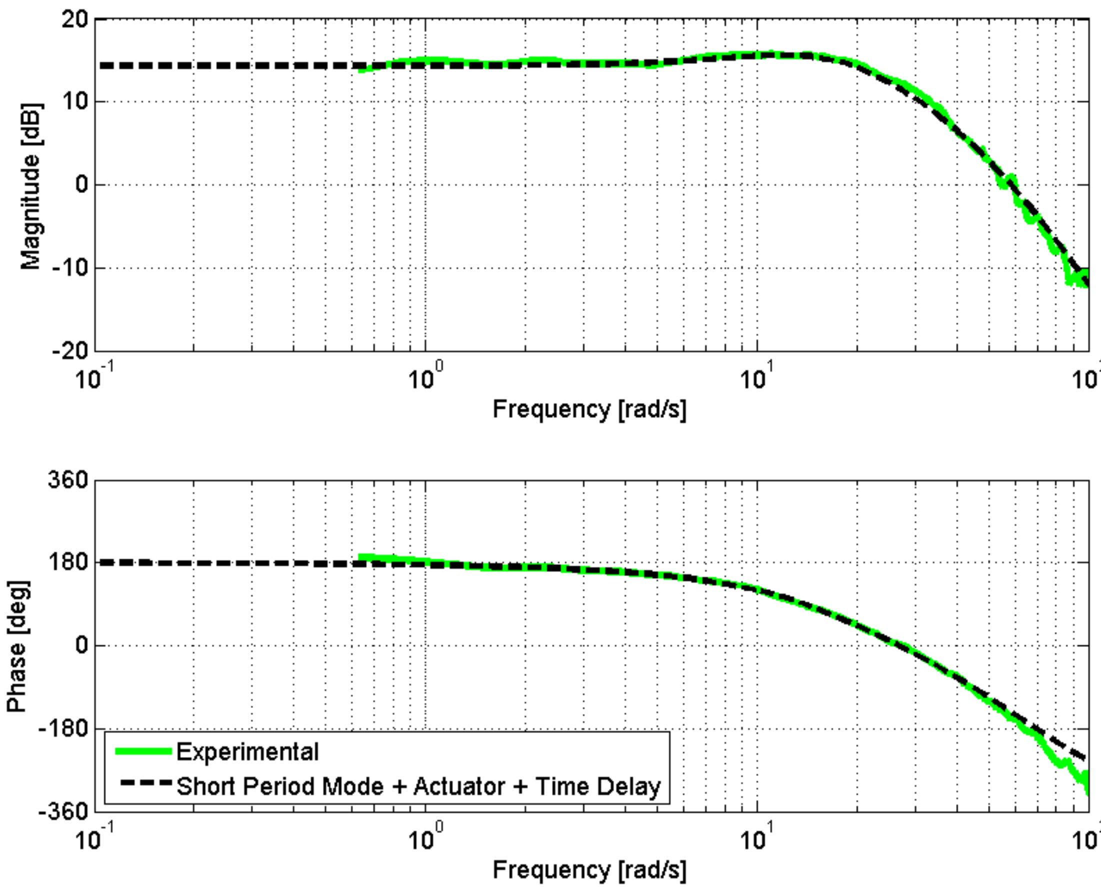
Include Time Delay

δ_e → [45 msec Delay] → $\left[\dfrac{\omega_a^2}{s^2 + 2\zeta_a\omega_a s + \omega_a^2}\right]$ → $\left[\dfrac{K(s+a_1)}{s^2 + 2\zeta\omega_n s + \omega_n^2}\right]$ → q

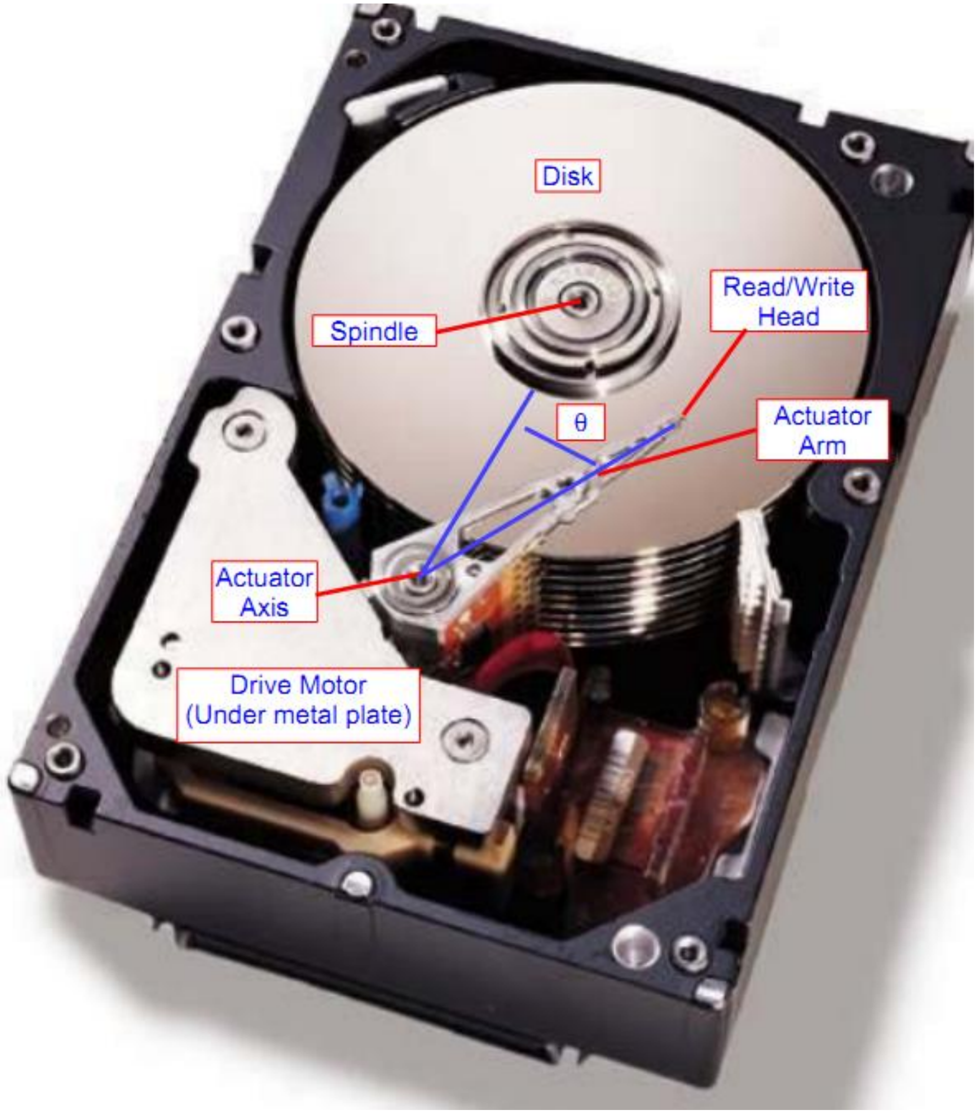
Input: Elevator Angle δ_e

Output: Pitch Rate q

SP Params:	Act Params:
$K = -107.42$	$\zeta_a = 0.8$
$a_1 = 14.45$	$\omega_a = 50.27$
$\zeta = 0.7$	
$\omega_n = 17.28$	Delay: 45 msec



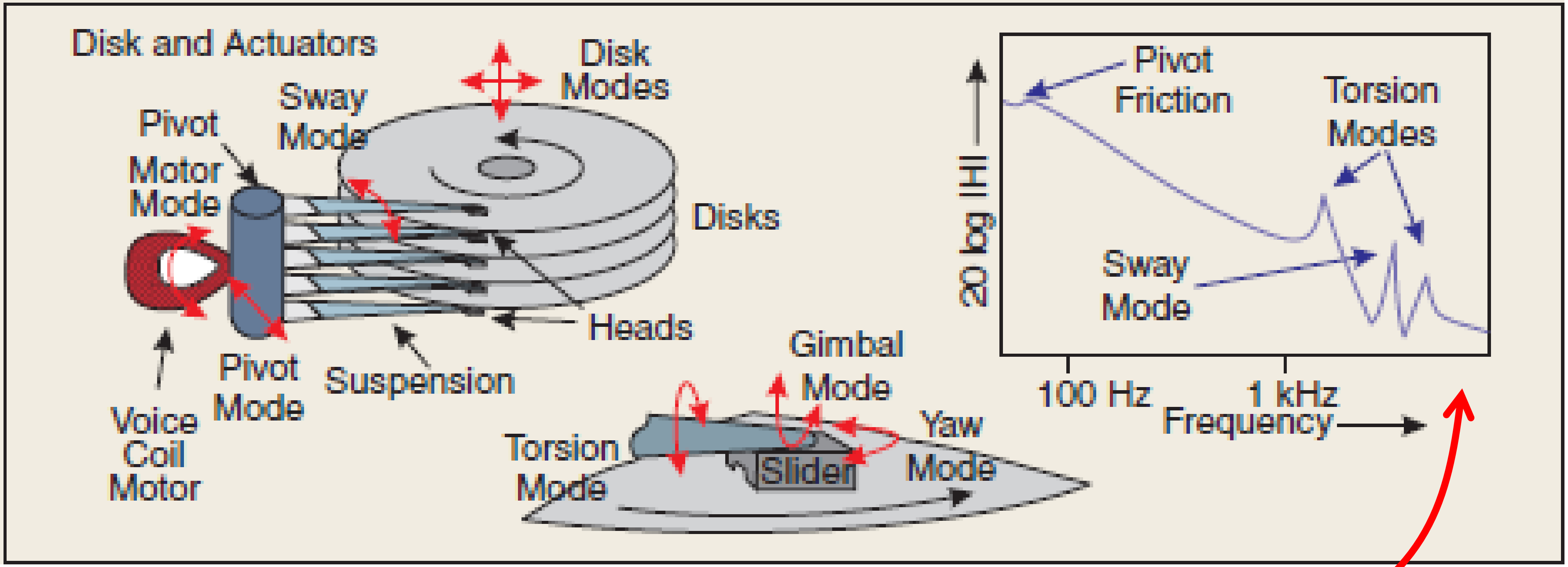
Hard Drive System Identification



Input: Motor Voltage δ

Output: Actuator Angle Θ

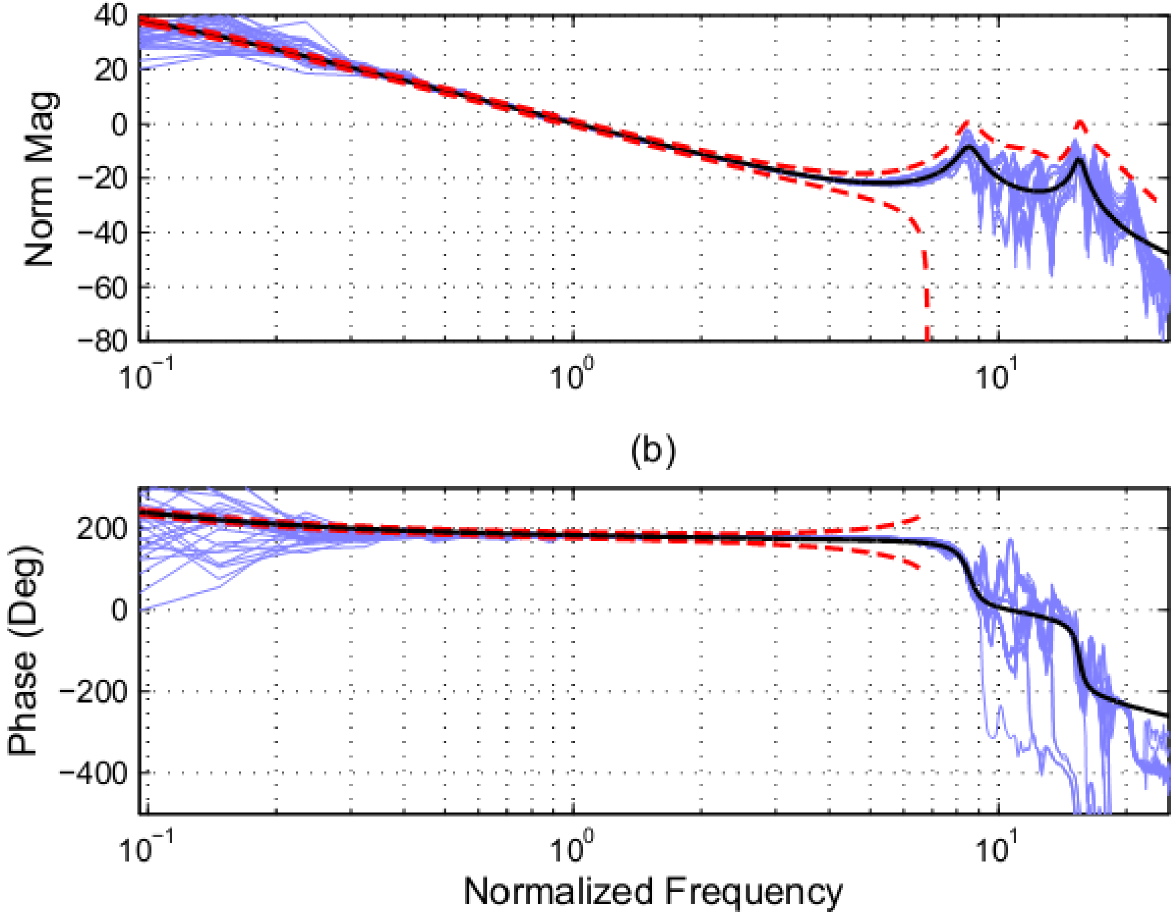
Hard Drive System Identification



Identify this transfer function using
frequency domain system identification

Taken From: D. Abramovitch and G. Franklin, “A brief history of disk drive control,"
IEEE Control Systems Magazine, p.28-42, June 2002.

Experimental Results: Voice Coil Motor



Taken From: M. Honda and P. Seiler, “Uncertainty Modeling for Hard Disk Drives,” American Control Conference, 2014.