Disturbance Propagation in Large Interconnected Systems

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Abstract

This paper will focus on control of vehicle strings and the theoretical issues arising from this problem. The key result is that a control structure where each vehicle uses only information about its predecessor is fundamentally sensitive to disturbances. Specifically, small disturbances acting on one vehicle can propagate and have a large effect on another vehicle. A similar, though less general, result is derived for a control structure where each vehicle looks at both neighbors.

1 Introduction

The problem, in its most basic form, is to move a collection of vehicles from one point to another point. One application of this work is an Automated Highway System (AHS) [5] where the goal is to reduce traffic congestion by using closed loop control. To maximize the traffic throughput, the vehicles travel in closely spaced platoons (Figure 1). Centralized control is impractical for medium to large sized platoons. Thus a decentralized controller should be used. Furthermore, treating the vehicles independently is an unsafe approach because the inter-vehicle spacings are required to be small. A reasonable decentralized control strategy is for each vehicle to use a radar to keep a fixed distance behind the preceding vehicle (Figure 2). The reference trajectory for the $(i+1)^{th}$ vehicle is a fixed distance, δ_i, behind the preceding vehicle: $r_{i+1} = x_i - \delta_i$. The feedback loops are coupled and it is possible for disturbances acting on one vehicle to propagate and affect other vehicles in the string. In fact, we show that for any linear control law, $K(s)$, it is possible for a small disturbance acting on one vehicle to have an arbitrarily large effect on another vehicle.

The possibility of disturbance propagation in vehicle strings has been known for some time. Chu showed that an infinite string of vehicles could not be stabilized using the strategy depicted in Figure 2 with a proportional control law [2]. A similar result was shown via a transfer function analysis [9]. In the early 90s, renewed interest in AHS spurred further research on the control of vehicle strings [3, 4, 10, 11, 12, 13, 14]. Swaroop developed rigorous definitions of string stability and relations to error propagation transfer functions [12]. The research on vehicle strings can be generalized and studied as a spatially invariant system [1].

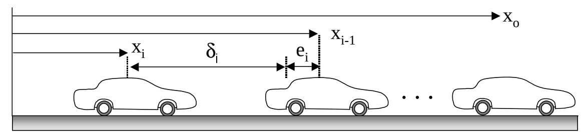

Figure 1: An AHS Platoon

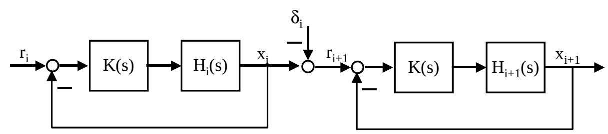

Figure 2: Coupled Feedback Loops

To summarize, we note that many researchers have shown that "string stability" cannot be obtained when vehicles use only relative spacing information to maintain a constant distance behind their predecessor. All of these results have been for specific control laws. In this paper we show that if vehicles use only relative spacing information, then we have "string instability" for *any* linear controller. This result motivates the need for communication in a vehicle string and highlights that 'local' decentralized controllers may be sensitive to disturbances.

2 Problem Formulation

The problem is motivated by the control of an AHS platoon (Figure 1). The platoon is a string of $N+1$ vehicles. Let $x_0(t)$ denote the position of the first car and $x_i(t)$ $(1 \le i \le N)$ denote the position of the i^{th} follower in the string. Define the vehicle spacing errors as: $e_i(t) = x_{i-1}(t) - x_i(t) - \delta_i(t)$ $(1 \le i \le N)$ where $\delta_i(t)$ is the desired vehicle spacing. The goal is to force these spacing errors to zero and ensure that small disturbances acting on one vehicle cannot have a large effect on another vehicle. Before proceeding, we call attention to some of our assumptions.

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Assumption 1: All the vehicles have the same model.
Assumption 2: The vehicle model is linear and SISO.
Assumption 3: All vehicles use the same control law.
Assumption 4: The desired spacing is a constant.

We are more interested in performance at the platoon level rather than individual vehicle control. Thus Assumptions 1 and 2 are reasonable abstractions of the problem at this scale. Assumption 3 is a simplification for ease of implementation. Finally, there are a variety of other spacing laws (see [12]) and the constant spacing policy is chosen for this analysis.

Given any time-domain signal, $x(t)$, we denote its Laplace Transform by $X(s)$. Applying the assumptions, we can model each vehicle in the Laplace domain as (assuming the vehicles start from rest):

$$X_i(s) = H(s)U_i(s) + \frac{x_i(0)}{s} \qquad \text{for } 1 \leq i \leq N \tag{1}$$

where $H(s)$ has two poles at the origin and $x_i(0)$ is the initial position of the i^{th} vehicle. A simple point mass model for a car is $H(s) = \frac{1}{s^2}$ with the vehicle acceleration as the control input. In general, $H(s)$ can include actuator dynamics. The spacing error is given by $e_i(t) = x_{i-1}(t) - x_i(t) - \delta$. We assume the platoon starts with zero spacing errors and the leader starts at $x_0(0) = 0$. Hence, $x_i(0) = -i\delta$ for $0 \leq i \leq N$.

3 Error Propagation

In this section we give a simple analysis of three decentralized control laws. We will make use of the following norm: $\|X(s)\|_\infty := \sup_{\omega \in \mathbb{R}} \bar{\sigma}\left(X(j\omega)\right)$.

3.1 Predecessor Following

A linear control law based only on relative spacing error with respect to the predecessor is given by:

$$U_i(s) = K(s)E_i(s) \tag{2}$$

We can obtain the spacing error dynamics from Equations 1, 2 and the platoon initial conditions:

$$E_1(s) = \frac{1}{1 + H(s)K(s)} X_0(s) := S(s)X_0(s) \tag{3}$$

$$E_i(s) = \frac{H(s)K(s)}{1 + H(s)K(s)} E_{i-1}(s) := T(s)E_{i-1}(s) \tag{4}$$

$$\text{for } i = 2, \ldots, N$$

These equations show that the transfer function from $X_0(s)$ to $E_1(s)$ is the sensitivity function, $S(s)$. The transfer function from $E_{i-1}(s)$ to $E_i(s)$ is the complementary sensitivity function, $T(s)$. There is a classical trade-off between making $|S(j\omega)|$ and $|T(j\omega)|$ small. In the context of Equations 3 and 4, the $S(s)$ vs. $T(s)$ trade-off has the interpretation of limiting the first spacing error (making $|S(j\omega)|$ small) and limiting the propagation of errors (making $|T(j\omega)|$ small). We would like $|T(j\omega)| < 1$ at all frequencies so that propagating errors are attenuated. In fact, it is not possible to attenuate propagating errors at all frequencies. Note that if $K(s)$ stabilizes the closed loop, then $H(s)K(s)$ has two poles at $s = 0$. Thus $T(0) = 1$ and hence $\|T(s)\|_\infty \geq 1$. The next theorem implies that the inequality is strict: $\|T(s)\|_\infty > 1$. This is a simplified version of a theorem by Middleton and Goodwin [8, 7].

Theorem 1 *Suppose that $H(s)$ is a rational transfer function with at least two poles at the origin. If the associated feedback system is stable, then the complementary sensitivity function must satisfy:*

$$\int_0^\infty \ln|T(j\omega)| \frac{d\omega}{\omega^2} \geq 0 \tag{5}$$

This integral relation is similar to the more common Bode Sensitivity integral. The integral implies that the area of error amplification is greater than or equal to the area of error attenuation. A simple consequence of this theorem is that for *any* stabilizing controller, there exists a frequency, ω, such that $|T(j\omega)| > 1$. Figure 3 shows an example of this result. The vehicle model is a double integrator with first order actuator dynamics and a lead controller is used to follow the preceding vehicle:

$$H(s) = \frac{1}{s^2(0.1s+1)} \qquad K(s) = \frac{2s+1}{0.05s+1} \tag{6}$$

Figure 3 is a plot of $|T(j\omega)|$ and $|S(j\omega)|$. As predicted by Theorem 1, there is a frequency such that $|T(j\omega)| > 1$. Specifically, $\|T(s)\|_\infty = 1.21$ and is achieved at $\omega_0 = 0.93\ rads/sec$. Errors acting at this frequency will be amplified as they propagate.

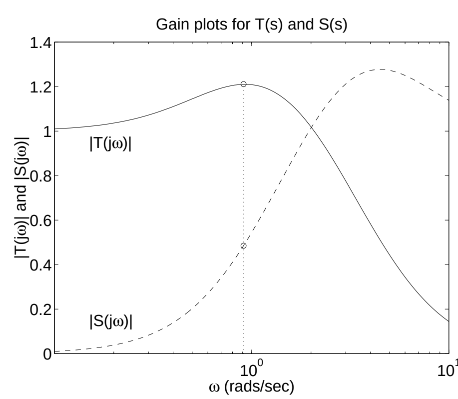

Figure 3: Plots of $|T(j\omega)|$ and $|S(j\omega)|$

We elaborate on this last statement. Consider a 6 car platoon ($N = 5$) starting from rest with initial conditions $x_i(0) = -i\delta$ for $i = 0, \ldots 5$. The desired spacing is $\delta = 5m$. The lead vehicle accelerates from rest to 20 m/s over 12 seconds using the following input:

$$U_0(s) = \frac{1}{s^2}\left[e^{-s} - e^{-3s} - e^{-11s} + e^{-13s}\right] \qquad (7)$$

In the time domain, this corresponds to a trapezoidal input with peak acceleration of $2m/s^2$. The lead vehicle motion, $X_0(s) = H(s)U_0(s)$, causes an initial spacing error, $E_1(s) = S(s)X_0(s)$. Figure 4 shows that $|E_1(j\omega)|$ has substantial low-frequency content. Figure 3 shows that $|T(j\omega)| > 1$ at low frequencies, so we expect low-frequency content to be amplified. Figure 4 confirms that low frequency content is amplified as it propagates from $E_1(s)$ to $E_5(s)$.

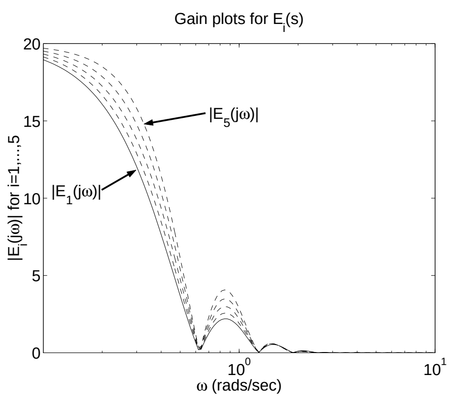

Figure 4: Frequency domain plots of spacing errors with the predecssor following strategy.

This error amplification can also be interpreted in the time domain (Figure 5). In this example, the vehicles farthest from the leader experience the largest peak spacing error. Results on peak error amplification can be found in [12]. It also possible to show that the control effort also propagates via $T(s)$: $U_i(s) = T(s)U_{i-1}(s)$. The same statements regarding amplification of control effort apply here. If more cars are added to the platoon, then either the actuators on the trailing cars will saturate or a collision may occur.

3.2 Predecessor and Leader Following

In this section, we add lead vehicle information to the predecessor-following control law.

$$U_i(s) = K_p(s)E_i(s) + K_l(s)\left(X_0(s) - X_i(s) - \frac{i\delta}{s}\right) \qquad (8)$$

This controller tries to keep the errors with respect to the preceding vehicle and with respect to the lead vehicle small. The leader motion is essentially the reference for the string. Intuitively, this control law gives each vehicle some preview information of this reference. As before, we can obtain the error dynamics:

$$E_1(s) = \frac{1}{1 + H(s)\left(K_p(s) + K_l(s)\right)}X_0(s) := S_{lp}(s)X_0(s)$$

$$E_i(s) = \frac{H(s)K_p(s)}{1 + H(s)\left(K_p(s) + K_l(s)\right)}E_{i-1}(s) := T_{lp}(s)E_{i-1}$$

$$2 \leq i \leq N \qquad (9)$$

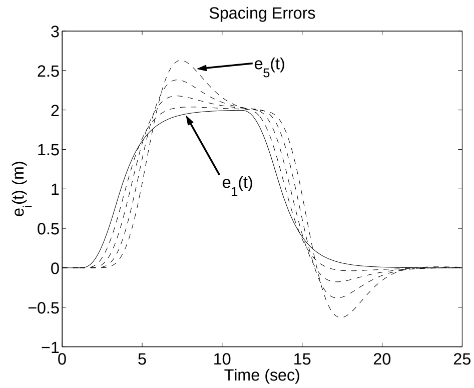

Figure 5: Time domain plots of spacing errors with the predecssor following strategy.

If $K_l(s) \equiv 0$ then these equations reduce to the corresponding equations in the previous section. Note that we are free from the constraint $T(0) = 1$. For example, if we choose $K_l(s) = K_p(s)$ then $T_{lp}(0) = 0.5$. More importantly, we can easily design $K_l(s)$ and $K_p(s)$ so that $\|T_{lp}(s)\|_\infty < 1$.

We compare this strategy to the predecessor following strategy described in Section 3.1. We use the same vehicle model given in Equation 6 and the control law in Equation 8 with $K_l(s) = K_p(s) = \frac{1}{2}K(s)$. $T_{lp}(s) = \frac{1}{2}T(s)$, so the peak magnitude is dropped to $\|T_{lp}(s)\|_\infty = 0.605$. Thus all frequency content of propagating errors is attenuated. Figure 6 shows the time responses for comparison. Figure 6 shows that the spacing errors are attenuated as they propagate down the chain. Furthermore, it is possible to show that the control effort will not grow unbounded down the chain.

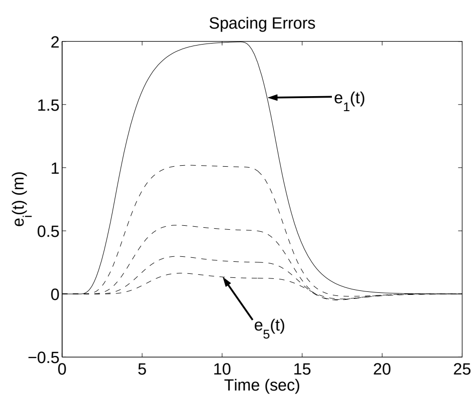

Figure 6: Time domain plots of spacing errors with the predecssor and leader following strategy.

4 Sensitivity to Disturbances

In this section we further the analysis and determine the effect of disturbances acting on each vehicle.

4.1 Look-ahead

Let us first consider an $N+1$ car platoon where each vehicle has an input reflected disturbance:

$$X_i(s) = H(s)\left(U_i(s) + D_i(s)\right) + \frac{-i\delta}{s} \qquad 1 \le i \le N \quad (10)$$

We will now derive the closed loop transfer function matrix from disturbances to errors when each vehicle uses preceding and lead vehicle information. The i^{th} spacing error is given by $E_i(s) = X_{i-1}(s) - X_i(s) - \frac{\delta}{s}$. Using the vehicle model (Equation 10), we can write the spacing error dynamics for the platoon as:

$$\overline{E}(s) = P_{11}(s)\begin{bmatrix} X_0(s) \\ \overline{D}(s) \end{bmatrix} + P_{12}(s)\overline{U}(s) \qquad (11)$$

where we have defined:

$$\overline{E}(s) := [E_1(s)\dots E_N(s)]^T, \quad \overline{D}(s) := [D_1(s)\dots D_N(s)]^T, \quad \overline{U}(s) := [U_1(s)\dots U_N(s)]^T$$

$$P_{11}(s) := \left[\begin{array}{c|ccccc} 1 & -H(s) & & & & \\ 0 & H(s) & -H(s) & & & \\ \vdots & & \ddots & \ddots & & \\ 0 & & & & H(s) & -H(s) \end{array}\right]$$

$$P_{12}(s) := \begin{bmatrix} -H(s) & & & \\ H(s) & -H(s) & & \\ & \ddots & \ddots & \\ & & H(s) & -H(s) \end{bmatrix}$$

We assume each vehicle uses the control law given in Equation 8. This control law can be rewritten in terms of the platoon spacing errors:

$$U_i(s) = K_p(s)E_i(s) + K_l(s)\left(\sum_{k=1}^{i} E_i(s)\right) \qquad (12)$$

This form of the control law is strictly for convenience in the derivation that follows. The vector of platoon inputs is given by:

$$\overline{U}(s) = \overline{K}(s)\overline{E}(s) \qquad (13)$$

$$\overline{K}(s) = \begin{bmatrix} K_l(s)+K_p(s) & & & \\ K_l(s) & K_l(s)+K_p(s) & & \\ \vdots & \ddots & \ddots & \\ K_l(s) & \cdots & K_l(s) & K_l(s)+K_p(s) \end{bmatrix}$$

We can eliminate $\overline{U}(s)$ from Equations 11 and 13 to obtain the closed loop equation:

$$\overline{E}(s) = \left[\left(I - P_{12}(s)\overline{K}(s)\right)^{-1} P_{11}(s)\right]\begin{bmatrix} X_0(s) \\ \overline{D}(s) \end{bmatrix} \quad (14)$$

Substitute for the matrices ($P_{11}(s)$, $P_{12}(s)$, $\overline{K}(s)$):

$$\overline{E}(s) = \begin{bmatrix} 1 \\ T_{lp}(s) \\ \vdots \\ T_{lp}(s)^{N-1} \end{bmatrix} S_{lp}(s)X_0(s) - S_{lp}(s)H(s)\times$$
$$\begin{bmatrix} 1 & & & \\ (T_{lp}(s)-1) & 1 & & \\ \vdots & \ddots & \ddots & \\ (T_{lp}(s)-1)T_{lp}(s)^{N-2} & \cdots & (T_{lp}(s)-1) & 1 \end{bmatrix}\overline{D}(s) \qquad (15)$$

$T_{lp}(s)$ and $S_{lp}(s)$ are as defined in Equation 9. The transfer function vector from $X_0(s)$ to $\overline{E}(s)$ agrees with the analysis Section 3.2. Two cases show the trade-off between disturbance propagation and safety. If we use only leader information then $T_{lp}(s) = 0$. There is no disturbance propagation in this case, but $D_i(s)$ affects $E_{i+1}(s)$ through $S_{lp}(s)H(s)$. On the other hand, if we use only preceding vehicle information, then the effect of $D_i(s)$ on $E_{i+1}(s)$ is through $-(T_{lp}(s)-1)S_{lp}(s)H(s)$. Typically, $|T_{lp}(s)-1| << 1$ near $\omega = 0$, so the use of preceding vehicle information reduces the effect of a disturbance on the spacing error. The price for this safety is that there always exists a frequency such that $|T_{lp}(j\omega)| > 1$. Hence disturbances may amplify as they propagate through the chain. For example, the effect of $D_1(s)$ on $E_k(s)$ is given by $-(T_{lp}(s)-1)T_{lp}^k(s)S_{lp}(s)H(s)$. If $|T_{lp}(\omega)| > 1$, then this effect is amplified geometrically for increasing k. The control law proposed in Section 3.2 provides a compromise to this trade-off. This discussion of disturbance propagation is made rigorous in the ensuing theorem. The proof makes use of the following lemma:

Lemma 1 *Given any complex numbers, $a, b \in \mathbb{C}$, define the following sequence of matrices:*

$$X_N := \begin{bmatrix} 1 & & & & \\ b & 1 & & & \\ ab & b & 1 & & \\ \vdots & \ddots & \ddots & \ddots & \\ a^{N-2}b & \cdots & ab & b & 1 \end{bmatrix} \in \mathbb{C}^{N\times N} \qquad (16)$$

If $|a| < 1$ then $\bar{\sigma}(X_N) \le 1 + \frac{|b|}{1-|a|}$ for all N.

Proof. For any matrix, $\rho(A) \le \|A\|_1$ where $\rho(A)$ is the spectral radius and $\|A\|_1$ is the induced 1-norm [6]. Applying this fact to $A = X_N^* X_N$ gives inequality (a) in the following upper bound of $\bar{\sigma}(X_N)$:

$$\bar{\sigma}(X_N)^2 \overset{(a)}{\le} \|X_N^* X_N\|_1 \le \|X_N^*\|_1 \|X_N\|_1 = \|X_N\|_\infty \|X_N\|_1$$

For all N, $\|X_N\|_1 = \|X_N\|_\infty = 1 + \frac{|b|(1-|a|^{N-1})}{1-|a|}$. If $|a| < 1$ then $\bar{\sigma}(X_N) \le 1 + \frac{|b|}{1-|a|}$ $\forall N$. ■

Theorem 2 *Assume $H(s)$ has 2 poles at the origin and the closed loop is stable. Let $\overline{T}_{de}(s) \in C^{N\times N}$ be the transfer function matrix from $\overline{D}(s)$ to $\overline{E}(s)$ in Equation 15. If $\|T_{lp}(s)\|_\infty > 1$, then given any $M > 0$ $\exists N$ such that $\|\overline{T}_{de}(s)\|_\infty \ge M$. If $\|T_{lp}(s)\|_\infty < 1$, then $\exists M > 0$ such that $\|\overline{T}_{de}(s)\|_\infty \le M$ $\forall N$.*

Proof. For the first part of the theorem, there exists a frequency ω_0 such that $|T_{lp}(j\omega_0)| > 1$. Given any $M > 0$, choose N to satisfy the following inequality:

$$|T_{lp}(j\omega_0)|^{N-2} > \frac{M}{|S_{lp}(j\omega_0)H(j\omega_0)(T_{lp}(j\omega_0)-1)|}$$

There is one technical subtlety in choosing N. The right side is infinite if $H(s)$ has a zero at $s = j\omega_0$ or $K(s)$ has a pole at $s = j\omega_0$. Since $H(s)$ and $K(s)$ have a finite number of poles and zeros, we can choose ω_0 such that $|T_{lp}(j\omega_0)| > 1$ and $|S_{lp}(j\omega_0)H(j\omega_0)| \neq 0$. Hence the right hand side of the inequality is finite and it is possible to choose N to satisfy the inequality. Let $e_1 \in \mathbb{R}^N$ be the first basis vector. By choice of N, $\bar{\sigma}\left(\overline{T}_{de}(j\omega_0)\right) \geq \|\overline{T}_{de}(j\omega_0)e_1\|_2 > M$. Hence $\|\overline{T}_{de}(s)\|_\infty \geq M$.

For the second part of the theorem, fix ω and define two complex numbers: $a := T_{lp}(j\omega)$ and $b := T_{lp}(j\omega) - 1$. Given these complex numbers, define the sequence of matrices, X_N, as in Equation 16. We can apply Lemma 1 to conclude that if $|a| < 1$ then $\bar{\sigma}(X_N) \leq 1 + \frac{|b|}{1-|a|}$ for all N. Therefore, if $|T(j\omega)| < 1$, then the gain from disturbance to error at the frequency ω can be upper bounded for all N:

$$\bar{\sigma}\left(\overline{T}_{de}(j\omega)\right) \leq |S_{lp}(j\omega)H(j\omega)| \cdot \left(1 + \frac{1 + |T_{lp}(j\omega)|}{1 - |T_{lp}(j\omega)|}\right) \quad (17)$$

By assumption, $\|T_{lp}(s)\|_\infty < 1$ and the closed loop is stable. Closed loop stability implies $\|S_{lp}(s)H(s)\|_\infty < \infty$. Equation 17 can be applied to upper bound the peak gain from disturbances to errors uniformly in N:

$$\|\overline{T}_{de}(s)\|_\infty \leq \|S_{lp}(s)H(s)\|_\infty \cdot \left(1 + \frac{1 + \|T_{lp}(s)\|_\infty}{1 - \|T_{lp}(s)\|_\infty}\right) < \infty$$

■

As noted in Section 3.1, if we use the predecessor following strategy, then for any stabilizing, linear controller we have $\|T_{lp}(s)\|_\infty > 1$. From Theorem 2 we conclude that this strategy will always lack scalability because the gain from disturbances to errors grows without bound as the platoon length grows. However, if we use leader information, then it is possible to make $\|T_{lp}(s)\|_\infty < 1$. In this case, the theorem states that the algorithm is scalable because the gain from disturbances to errors is uniformly bounded as the platoon length grows.

The consequence of this theorem is displayed in Figure 7. The plot shows the disturbance to error gain as a function of frequency for strategies with (Right subplot) and without (Left subplot) leader information. N is the number of followers in the platoon. $H(s)$, $K(s)$, $K_l(s)$, and $K_p(s)$ are the same as those used in the previous sections. The right subplot shows that the disturbance to error gain is relatively independent of vehicle size if leader information is used. The left subplot, on the other hand, shows that if the predecessor following strategy is used, then the platoon becomes sensitive to disturbances as N grows.

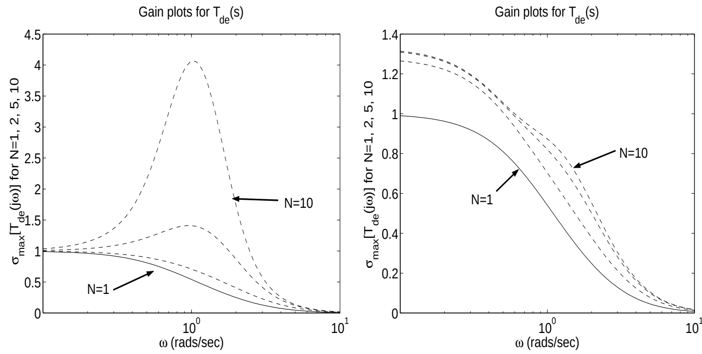

Figure 7: $\bar{\sigma}\left(\overline{T}_{de}(j\omega)\right)$ for $N = 1, 2, 5, 10$. *Left:* Predecessor following strategy . *Right:* Leader and predecessor following strategy.

4.2 Bidirectional

In the previous section, we showed that a vehicle following control law based only on relative spacing error is not scalable. The algorithm can be made scalable if all vehicles have knowledge of the lead vehicle motion. However, the latter algorithm requires a network to communicate this information to all vehicles while the former algorithm can be implemented with only on-board sensors. In this section, we try to construct a scalable control law that relies only on 'local' measurements, i.e. no communication is necessary.

We consider platoon controllers which use relative spacing error with respect to adjacent vehicles. In this section, vehicles use a bidirectional controller:

$$U_i(s) = K_p(s)E_i(s) - K_f(s)E_{i+1}(s) \quad (18)$$

Since the last vehicle in the chain does not have a follower, it uses the control law: $U_N(s) = K_p(s)E_N(s)$. P_{11}, P_{12} are as defined previously, but the controller matrix for the entire platoon, $\overline{K}(s)$, is given by:

$$\overline{K}(s) = \begin{bmatrix} K_p(s) & -K_f(s) & & \\ & \ddots & \ddots & \\ & & \ddots & -K_f(s) \\ & & & K_p(s) \end{bmatrix}$$

For this control structure, the closed loop equation from disturbances to errors is again given by Equation 14. We will focus on the effect of disturbances which is given by:

$$\begin{aligned} \overline{E}(s) &= \left[\left(I - P_{12}(s)\overline{K}(s)\right)^{-1} P_{12}(s)\right]\overline{D}(s) \\ &= \left(P_{12}^{-1} - \overline{K}(s)\right)^{-1}\overline{D}(s) \end{aligned} \quad (19)$$

where:

$$P_{12}^{-1}(s) = -\frac{1}{H(s)} \begin{bmatrix} 1 & & \\ \vdots & \ddots & \\ 1 & \cdots & 1 \end{bmatrix}$$

The next theorem shows that this strategy also fails to be scalable for a class of these bidirectional controllers.

Theorem 3 *Assume $H(s)$ has 2 poles at the origin and the closed loop is stable. Assume the bidirectional controller is symmetric: $K_p(s) = K_f(s)$ and $K_f(s)$ has no poles at $s = 0$. Let $\overline{T}_{de}(s) \in C^{N \times N}$ be the transfer function matrix from $\overline{D}(s)$ to $\overline{E}(s)$ in Equation 19. Given any $M > 0$ $\exists N$ such that $\|\overline{T}_{de}(s)\|_\infty \geq M$.*

Proof. Given the assumptions in the theorem, the disturbance to error transfer function at $s = 0$ simplifies to:

$$\overline{E}(0) = -\frac{1}{K_f(0)} \begin{bmatrix} 1 & \cdots & 1 \\ & \ddots & \vdots \\ & & 1 \end{bmatrix} \overline{D}(0)$$

Let U_N be the $N \times N$ matrix with ones on the upper triangle and let e_N be the N^{th} basis vector. Then $\bar{\sigma}(U_N) \geq \|U_N e_N\| = \sqrt{N}$. Given any M, choose N such that $\frac{\sqrt{N}}{|K_f(0)|} > M$. For this N, $\|\overline{T}_{de}(s)\|_\infty \geq |\overline{T}_{de}(0)| > M$. ■

The left subplot of Figure 8 shows an example of the effect stated in Theorem 3. This plot shows the disturbance to error gain as a function of frequency when $K_f(s)$ has no poles at $s = 0$. The controller is given by $K_p(s) = K_f(s) = \frac{2s+1}{0.05s+1}$. As predicted by Theorem 3, the steady state gain grows as N increases. The right subplot shows the effect of adding an integrator to the control law: $K_p(s) = K_f(s) = \frac{2s^2+s+0.1}{s(0.05s+1)}$. The integrator causes the steady state gain to be 0 for all N. However, the peak gain from disturbances to errors changes greatly as vehicles are added to the platoon. In this example, the peak gain is actually greater for $N = 5$ than for $N = 10$. This behavior is in contrast to the predecessor/leader following strategy. Using that strategy, we can be assured that the disturbance to error gain is relatively independent of platoon size (Right subplot of Figure 7).

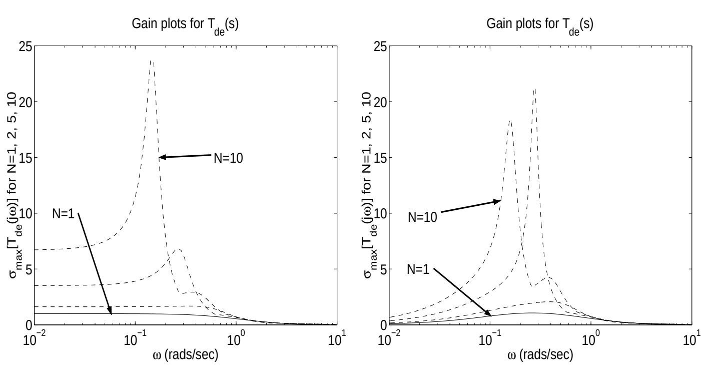

Figure 8: $\bar{\sigma}\left(\overline{T}_{de}(j\omega)\right)$ for $N = 1, 2, 5, 10$. *Left:* Bidirectional strategy: $K_f(s)$ with no poles at $s = 0$. *Right:* Bidirectional strategy: $K_f(s)$ with a pole at $s = 0$.

5 Conclusions

The key point of this paper is that extending single vehicle designs to large platoons can lead to unintended problems. We demonstrated that some control structures with only local information fail to be scalable. It was also shown that the problem can be solved if we include reference information in the control structure.

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