Robust Analysis and Synthesis for Linear Parameter Varying Systems

Pete Seiler
Department of Aerospace Engineering and Mechanics
University of Minnesota
Work with: G. Balas, A. Packard, H. Pfifer, S. Wang, R. Venkataraman, B. Taylor, C. Regan, & A. Hjartarson

Lockheed Martin BFF

Gary Balas (1960-2014)

Outline

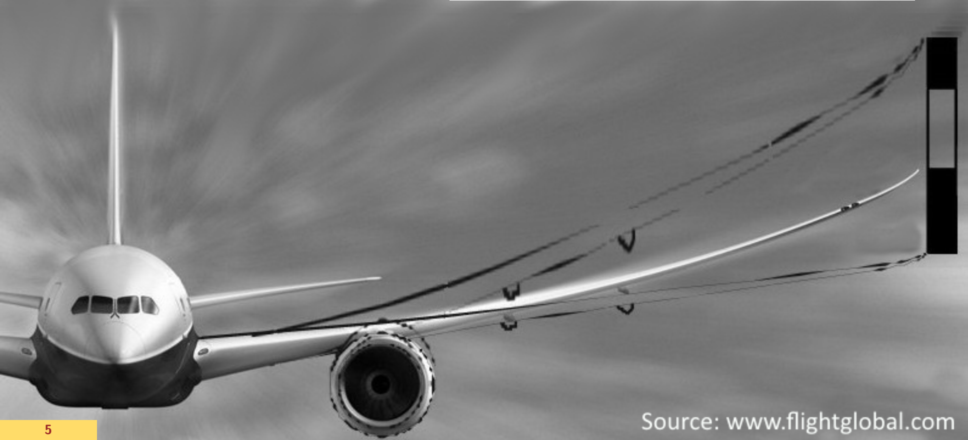
1. Flexible Aircraft
2. Linear Parameter Varying Systems
3. Robustness Analysis
4. Summary

Outline

Aeroelasticity

Efficient aircraft design

- lightweight structures
- high aspect ratios



Why Flexible Wings?

Breguet Range Equation

$$\text{Range} = V \times \underbrace{I_{sp}}_{\text{propulsion efficiency}} \times \underbrace{\frac{\text{Lift}}{\text{Drag}}}_{\text{glide number}} \times \underbrace{\ln\left(\frac{m_{\text{takeoff}}}{m_{\text{landing}}}\right)}_{\text{structural mass}}$$

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$$\text{Induced Drag} = \frac{\text{Lift}^2}{\pi\,\Lambda}$$

$\rightsquigarrow$ Maximize wing aspect ratio Λ

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Main contributions to total mass:

$$m_{\text{takeoff}} = m_{\text{structure}} + m_{\text{payload}} + m_{\text{fuel}}$$
$$m_{\text{landing}} = m_{\text{structure}} + m_{\text{payload}}$$

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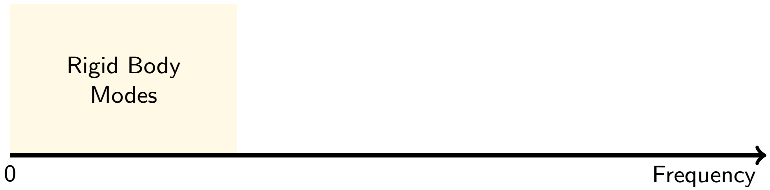
Light weight, high aspect ratio, flexible wings

Classical Approach



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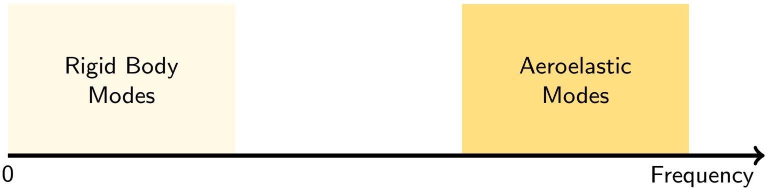

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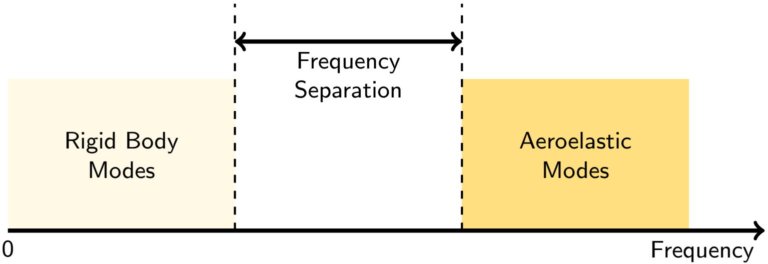

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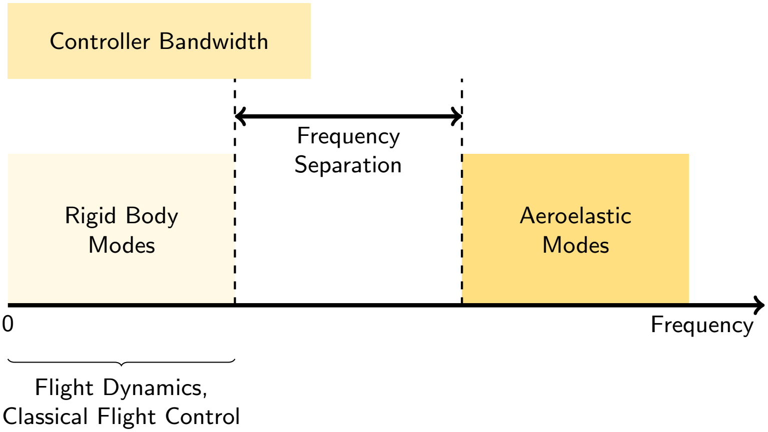

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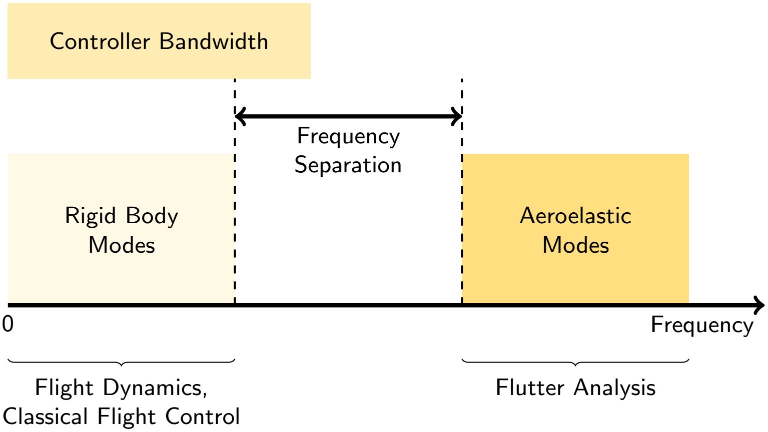

Flutter

NASA Dryden Flight Research

Flexible Aircraft Challenges

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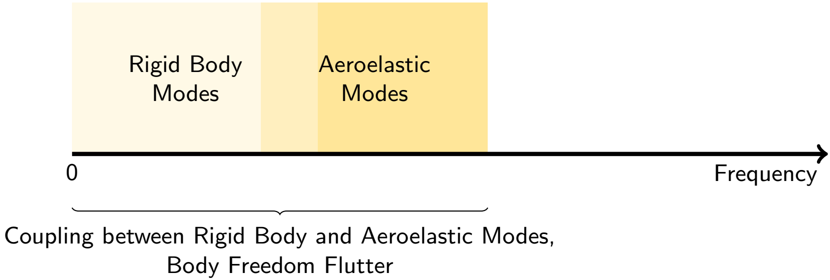
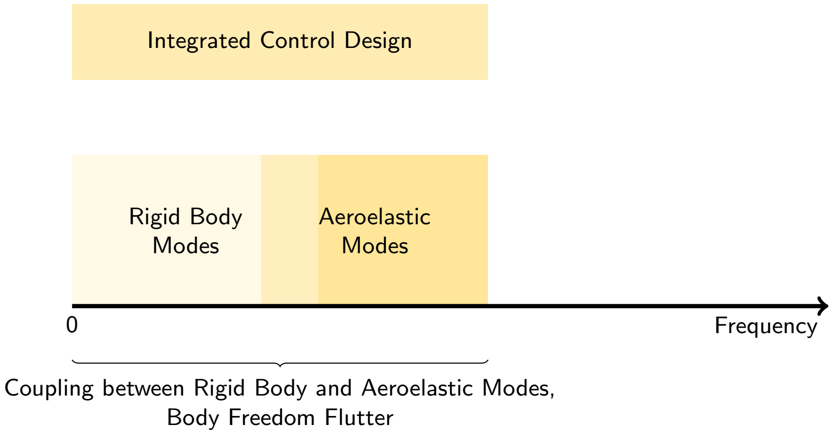


Flexible Aircraft Challenges

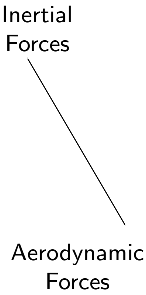


Body Freedom Flutter

Lockheed Martin BFF

Aeroservoelastic Model

Flight Dynamics

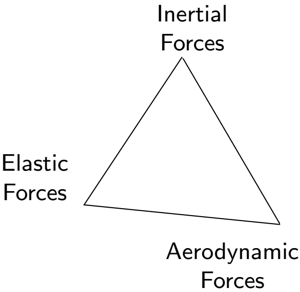


Rigid Body Dynamics

- Classical 6 degree of freedom equations of motion
- Steady aerodynamics

Aeroservoelastic Model

Aeroelasticity

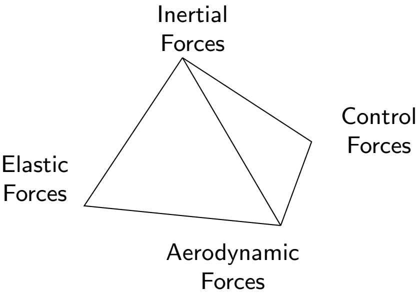


Flexible Aircraft

- Rigid body dynamics (6 DoF)
- Structural dynamics (typically 6-8 modes)
- Unsteady aerodynamics (typically 2 lag states per mode)

Aeroservoelastic Model

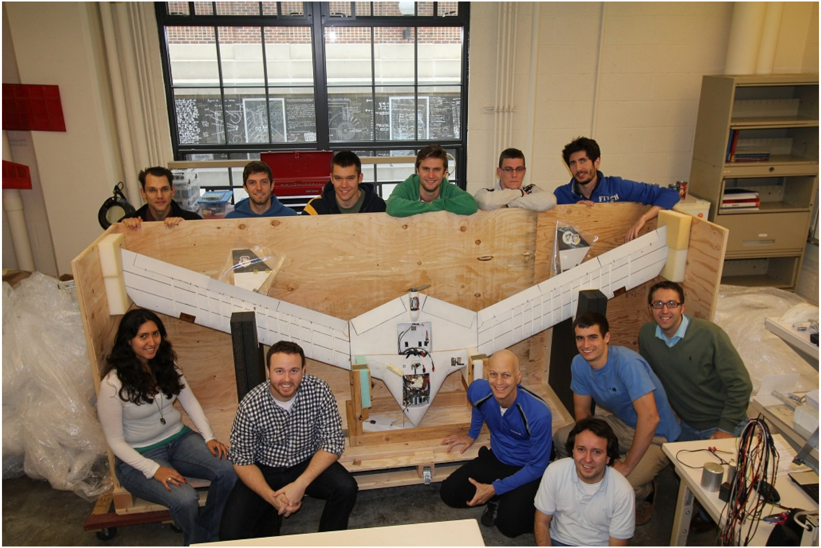
Aeroservoelasticity



High dimensional, strongly coupled models

- Rigid body dynamics (from flight dynamics)
- Structural dynamics (from finite element method)
- Unsteady aerodynamics (from potential theory)

Body Freedom Flutter Vehicle



mini-MUTT Aircraft at UMN


UMN mini-MUTT

Key Features:

- Low-cost, modular flight research infrastructure
- Design based on the Lockheed Martin BFF vehicle
- Parallels X-56 Flight test program at NASA
- Fabricated completely in-house
- Detachable wings of various flexibility

Flight Test of Rigid Wing mini-MUTT

Current Status of the Flexible Wing

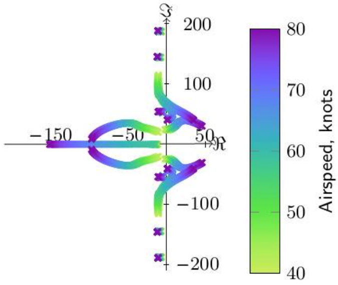
Next Steps:

- Finish building flexible wings
- Flight test campaign this summer

Limitation of Classical Approaches

Classical approaches are not suitable for control of flexible aircraft

Parameter Dependent Dynamics



Model Uncertainty

Aerodynamics:

- Simple potential theory based model
- Rational approximation of unsteady effects

Structural Dynamics:

- Simple beam model
- Estimates of mass and inertia properties

Outline

Aeroservoelastic Models

BFF Vehicle

Nonlinear equation of motion:

$$\dot{x}(t) = f(x(t), u(t), \rho(t))$$
$$y(t) = h(x(t), u(t), \rho(t)),$$

where ρ is a vector of measurable, exogenous signals, in this case airspeed.

Parameterized Trim Points: Assume there are trim points $(\bar{x}(\rho), \bar{u}(\rho), \bar{y}(\rho))$ parameterized by ρ:

$$0 = f(\bar{x}(\rho), \bar{u}(\rho), \rho)$$
$$\bar{y}(\rho) = h(\bar{x}(\rho), \bar{u}(\rho), \rho)$$

Aeroservoelastic Models


BFF Vehicle

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Time-Varying Linearization: Linearize around $(\bar{x}(\rho(t)), \bar{u}(\rho(t)), \bar{y}(\rho(t)), \rho(t))$

$$\dot{\delta}_x = A(\rho)\delta_x + B(\rho)\delta_u + \Delta_f(\delta_x, \delta_u, \rho) - \dot{\bar{x}}(\rho)$$
$$\dot{\delta}_y = C(\rho)\delta_x + D(\rho)\delta_u + \Delta_h(\delta_x, \delta_u, \rho)$$

where $A(\rho) := \frac{\partial f}{\partial x}(\bar{x}(\rho), \bar{u}(\rho), \rho)$, etc.

LPV Systems

$$\dot{x}(t) = A(\rho(t))x(t) + B(\rho(t))u(t)$$
$$y(t) = C(\rho(t))x(t) + D(\rho(t))u(t)$$

Parameter vector ρ lies within a set of admissible trajectories

$$\mathcal{A} := \{\rho : \mathbb{R}^+ \to \mathbb{R}^{n_\rho} \;:\; \rho(t) \in \mathcal{P},\ \dot{\rho}(t) \in \dot{\mathcal{P}}\ \forall t \geq 0\}$$

Comments:

- LPV theory is an extension of classical gain-scheduling used in industry, e.g. flight controls.
- Large body of literature in 90s: Shamma, Packard, Gahinet, Scherer, and many others.
- `LPVTools`: Toolbox developed by Balas, Packard, Seiler, and Hjartarson.

LPV Systems

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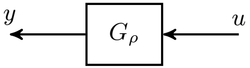
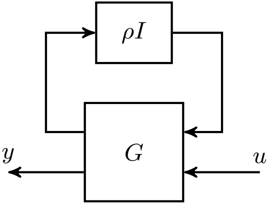
Grid based LPV systems



LFT based LPV systems

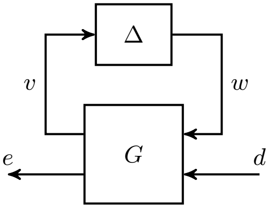


Outline

1. Flexible Aircraft
2. Linear Parameter Varying Systems
3. **Robustness Analysis**
4. Summary

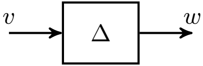
Integral Quadratic Constraints (IQCs)

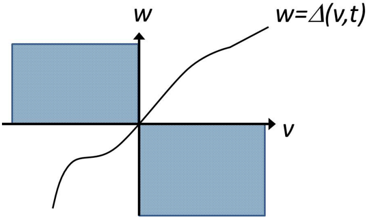
IQCs provide a general framework for analysis of a known **LTI** system G under perturbations Δ (Megretski & Rantzer, '97 TAC).



Goal: Extend framework to cases where known system is **LPV**, e.g. robustness margins for flexible aircraft.

Example: Passive System





$w = \Delta(v,t)$ is a passive system (pointwise in time).

$$\Downarrow$$

$$2v(t)^T w(t) \geq 0 \ \forall t$$

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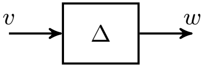

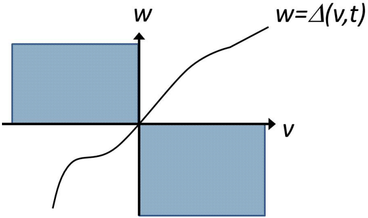

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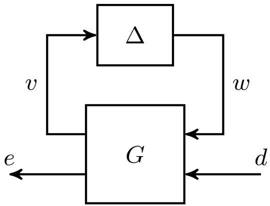
$$\Downarrow$$

$$\begin{bmatrix} v(t) \\ w(t) \end{bmatrix}^T \begin{bmatrix} 0 & I \\ I & 0 \end{bmatrix} \begin{bmatrix} v(t) \\ w(t) \end{bmatrix} \geq 0 \ \forall t$$

Pointwise quadratic constraint

Analysis: Circle Criterion



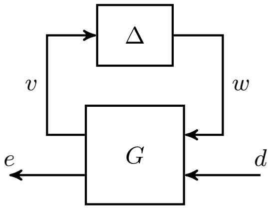
Theorem: Assume:

1. Interconnection is well-posed.
2. Δ is (pointwise) passive.
3. $\exists\, V \geq 0$ such that

$$\dot{V} + \begin{bmatrix} v(t) \\ w(t) \end{bmatrix}^T \begin{bmatrix} 0 & I \\ I & 0 \end{bmatrix} \begin{bmatrix} v(t) \\ w(t) \end{bmatrix} < d^T d - e^T e$$

Then gain from d to e is ≤ 1.

Analysis: Circle Criterion



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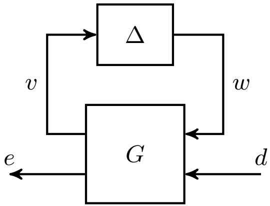
Proof: Let $d \in L[0, \infty)$ be any input signal and $x(0) = 0$. Integrate:

$$V(x(T)) + \int_0^T \begin{bmatrix} v(t) \\ w(t) \end{bmatrix}^T \begin{bmatrix} 0 & I \\ I & 0 \end{bmatrix} \begin{bmatrix} v(t) \\ w(t) \end{bmatrix} dt < \int_0^T d(t)^T d(t) dt - e(t)^T e(t) dt$$

Left side is ≥ 0 by $V \geq 0$ **and passivity.**

Analysis: Circle Criterion



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Then gain from d to e is ≤ 1.

Comments:

1. The proof relied on $V \geq 0$ and the passivity constraint. More general integral quadratic constraints (IQCs) can be incorporated, e.g. Zames-Falb.

2. Eq (1) is a matrix inequality when G is LTI and V is quadratic. Convex optimization can be used to efficiently search over combinations of IQCs.

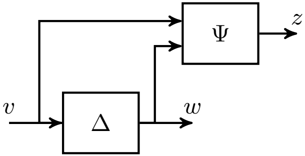
General IQCs (Megretski/Rantzer, '97 TAC)

Time Domain:

Let Ψ be a stable, LTI system and M a constant matrix.
Δ satisfies IQC defined by Ψ and M if

$$\int_0^T z(t)^T M z(t) dt \geq 0$$

$\forall v \in L_2[0, \infty)$, $w = \Delta(v)$, and $T \geq 0$.



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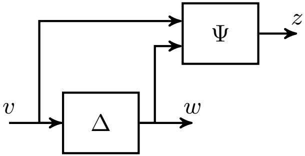
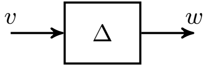


Frequency Domain:

Let $\Pi : j\mathbb{R} \to \mathbb{C}^{m\times m}$ be Hermitian-valued.
Δ satisfies IQC defined by Π if

$$\int_{-\infty}^{\infty} \begin{bmatrix} \hat{v}(j\omega) \\ \hat{w}(j\omega) \end{bmatrix}^* \Pi(j\omega) \begin{bmatrix} \hat{v}(j\omega) \\ \hat{w}(j\omega) \end{bmatrix} d\omega \geq 0$$

$\forall v \in L_2[0,\infty)$ and $w = \Delta(v)$.



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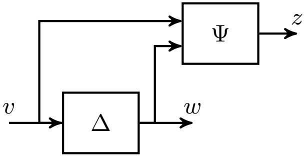

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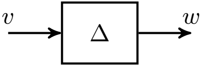
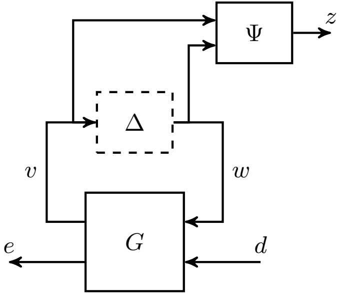


A non-unique factorization $\Pi = \Psi^{\sim} M \Psi$ connects the two definitions.

IQC Analysis (Megretski/Rantzer, '97 TAC)



Summary:

1. Analysis involves frequency domain conditions on G and IQC multiplier(s) Π.
2. Proof uses a homotopy method.
3. **Any** stable factorization $\Pi = \Psi^{\sim} M \Psi$ and KYP lemma leads to an LMI.
4. LMI condition can be written as:

$$\dot{V} + z^T M z < d^T d - e^T e$$

Neither $V \geq 0$ nor $\int_0^T z(t)^T M z(t)\, dt \geq 0$ holds, in general.

Question:

Is there an equivalent dissipation inequality proof?

Equivalence of Approaches (Seiler, '15 TAC)

Summary:
Under some technical conditions, the frequency-domain conditions in (M/R, '97 TAC) are equivalent to the time-domain dissipation inequality conditions.

Equivalence of Approaches (Seiler, '15 TAC)

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Under some technical conditions, the frequency-domain conditions in (M/R, '97 TAC) are equivalent to the time-domain dissipation inequality conditions.

Def.: $\Pi = \Psi^\sim M \Psi$ is a **J-Spectral factorization** if $M = \begin{bmatrix} I & 0 \\ 0 & -I \end{bmatrix}$ and Ψ, Ψ^{-1} are stable.

Thm.: If $\Pi = \Psi^\sim M \Psi$ is a J-spectral factorization then:

1. Δ satisfies the freq. domain IQC (Π) iff it satisfies the time domain IQC (Ψ, M).
2. All solutions of KYP LMI satisfy $P \geq 0$.

Proof: Uses LQ dynamic games, (Willems. '72 TAC) and (Engwerda, '05).

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Thm.: Partition $\Pi = \begin{bmatrix} \Pi_{11} & \Pi_{21}^* \\ \Pi_{21} & \Pi_{22} \end{bmatrix}$. Π has a J-spectral factorization if $\Pi_{11}(j\omega) > 0$ and $\Pi_{22}(j\omega) < 0$ $\forall \omega \in \mathbb{R} \cup \{+\infty\}$.
Proof: Use equalizing vectors thm. of Meinsma (SCL, 1995) ■.

Utility of Time-Domain Approach

Summary:
Under some technical conditions, the frequency-domain conditions in (M/R, '97 TAC) are equivalent to the time-domain dissipation inequality conditions.

Consequences:
The time-domain dissipation inequality conditions can be extended for:

1. LPV robustness analysis (Pfifer & Seiler, '14 IJRNC); (Pfifer & Seiler, in prep.)
2. LPV robust synthesis for general case (Wang, Pfifer, & Seiler, submitted to Aut) and robust filter/feedforward synthesis (Venkataraman & Seiler, in prep.)
3. Optimization analysis with ρ-hard IQCs (Lessard, Recht, & Packard)
4. Nonlinear analysis using SOS techniques

Item 1 has been implemented in `LPVTools`. Item 2 parallels results by (Scherer, Kose, and Veenman) for LFT-type LPV systems.

Outline

Upcoming Flight Test Plans

NASA X-56a:

- A. Hjartarson (Musyn) used `LPVTools` to synthesis (nominal) LPV controllers and assess robustness.
- NASA designed their own gain-scheduled control law.

UMN mini-MUTT:

- Finish flex wing and begin flight tests.
- Validate control-oriented aeroelastic models incorporating data from flight tests and high fidelity CFD/CSD models.
- New approaches for model order reduction required to obtain LPV models suitable for control design.
- Other team members (D. Schmidt, STI, Va. Tech, CMSoft, Aurora) will play key roles in modeling, design and analysis.

Lockheed Martin X-56a

UMN mini-MUTT

Acknowledgements

1. NASA NRA NNX14AL36A: "Lightweight Adaptive Aeroelastic Wing for Enhanced Performance Across the Flight Envelope," , Technical Monitor(s): Dr. John Bosworth and Daniel Moerder (Interim).
2. National Science Foundation under Grant No. NSF-CMMI-1254129 entitled "CAREER: Probabilistic Tools for High Reliability Monitoring and Control of Wind Farms," Program Manager: Dr. George Chiu.
3. NASA Langley NRA NNX12AM55A: "Analytical Validation Tools for Safety Critical Systems Under Loss-of-Control Conditions," Technical Monitor: Dr. Christine Belcastro.
4. Air Force Office of Scientific Research: Grant No. FA9550-12-0339, "A Merged IQC/SOS Theory for Analysis of Nonlinear Control Systems," Technical Monitor: Dr. Fariba Fahroo.

Summary

Conclusions:

- More efficient, flexible aircraft require integrated flight control systems.
- IQCs can be used in time-domain dissipation-inequalities without loss of conservatism.

Additional Details:

1. `http://www.aem.umn.edu/~SeilerControl/`
2. `http://paaw.net/`

Brief Summary of LPV Lower Bound Algorithm

There are many exact results and computational algorithms for LTV and periodic systems (Colaneri, Varga, Cantoni/Sandberg, many others)

The basic idea for computing a lower bound on $\|G_\rho\|$ is to search over periodic parameter trajectories and apply known results for periodic systems.

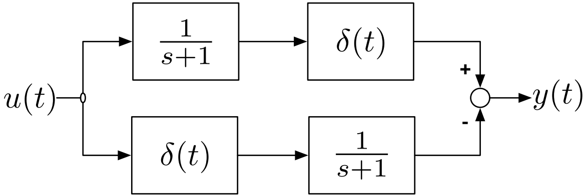
$$\|G_\rho\| := \sup_{\rho \in \mathcal{A}} \sup_{u \neq 0, u \in \mathcal{L}_2} \frac{\|G_\rho u\|}{\|u\|} \geq \sup_{\rho \in \mathcal{A}_h} \sup_{u \neq 0, u \in \mathcal{L}_2} \frac{\|G_\rho u\|}{\|u\|}$$

where $\mathcal{A}_h \subset \mathcal{A}$ denotes the set of admissible *periodic* trajectories.

Ref: T. Peni and P. Seiler, Computation of lower bounds for the induced $\mathcal{L}_2$ norm of LPV systems, submitted to the 2015 CDC.

Numerical example

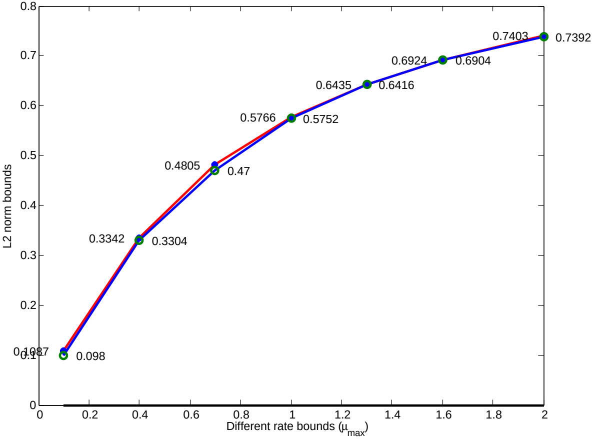
Simple, 1-parameter LPV system:



with $-1 \leq \delta(t) \leq 1$, and $-\overline{\mu} \leq \dot{\delta}(t) \leq \overline{\mu}$

The upper bound was computed by searching for a polynomial storage function.

Upper and Lower Bounds



Question: Can this approach be extended to compute lower bounds for uncertain LPV systems?