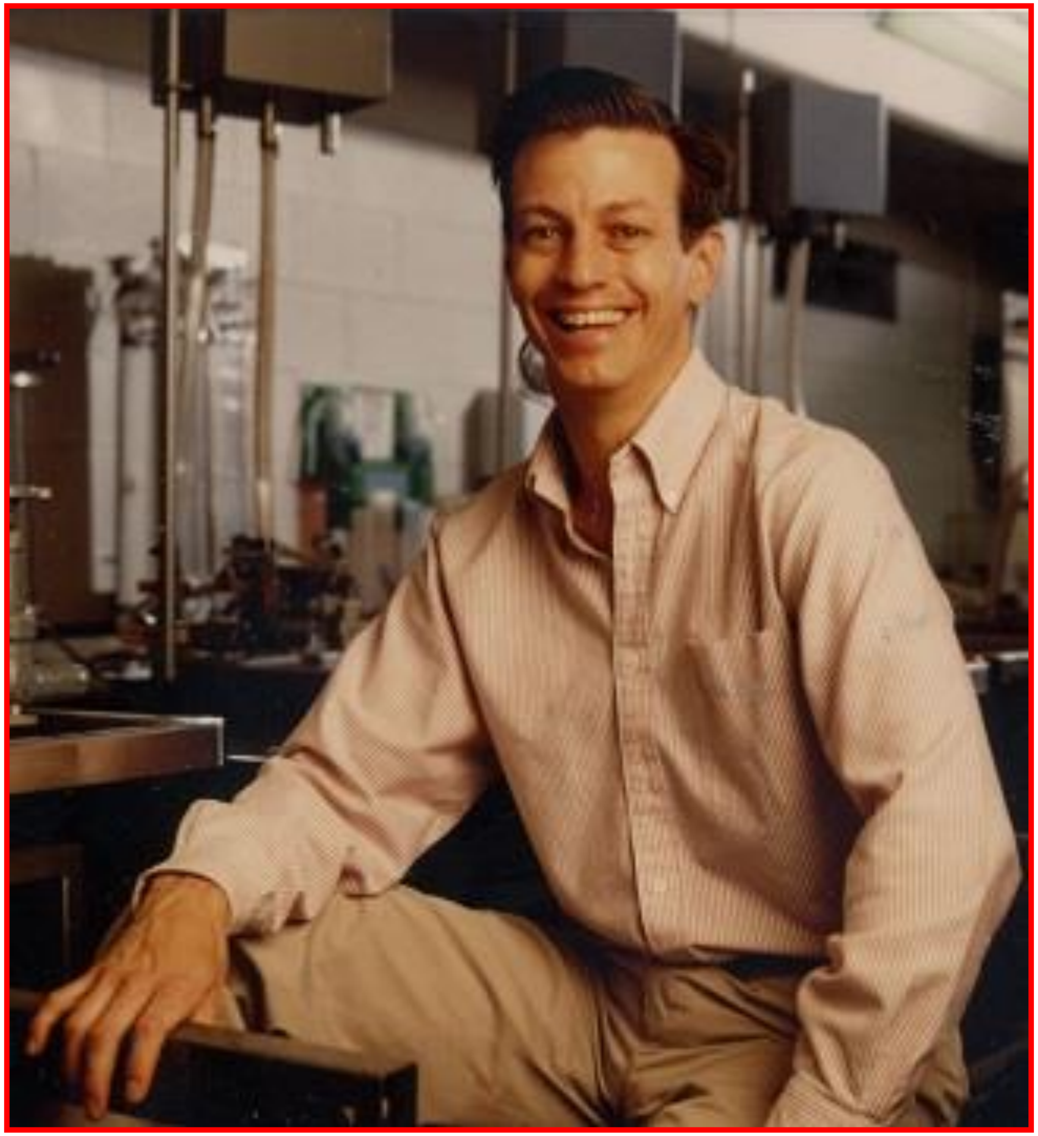
Robust Analysis and Synthesis for Linear Parameter Varying Systems

Peter Seiler
University of Minnesota

1st IFAC Workshop on Linear Parameter Varying Systems
In Memory of Gary J. Balas



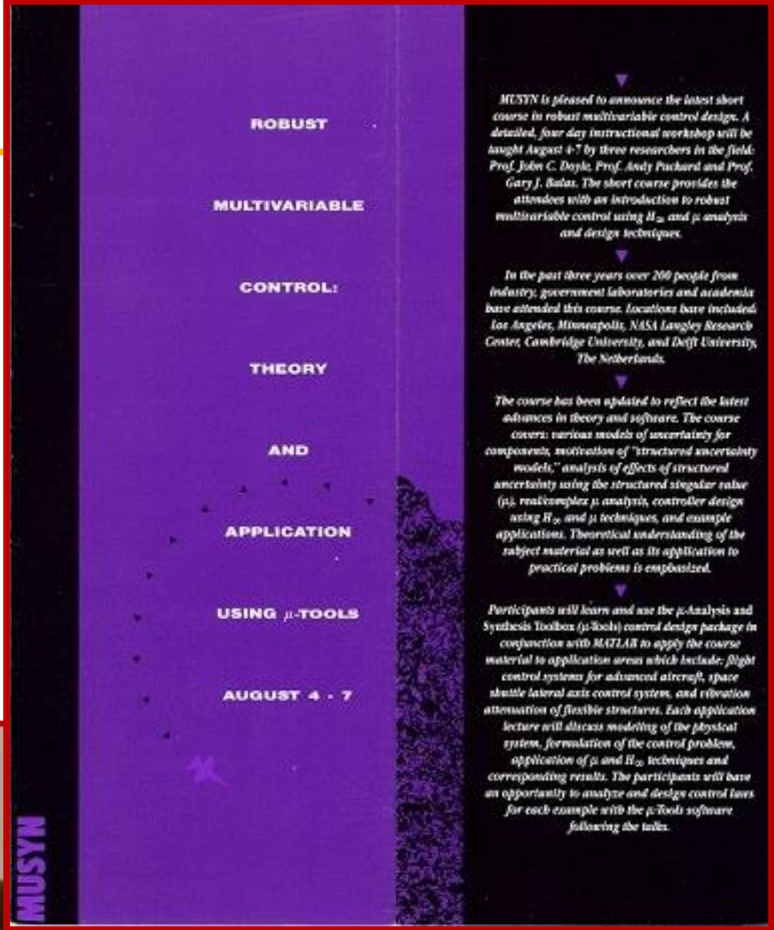
Gary J. Balas (Sept. 27. 1960 – Nov. 12, 2014)



Gary and Andy Packard

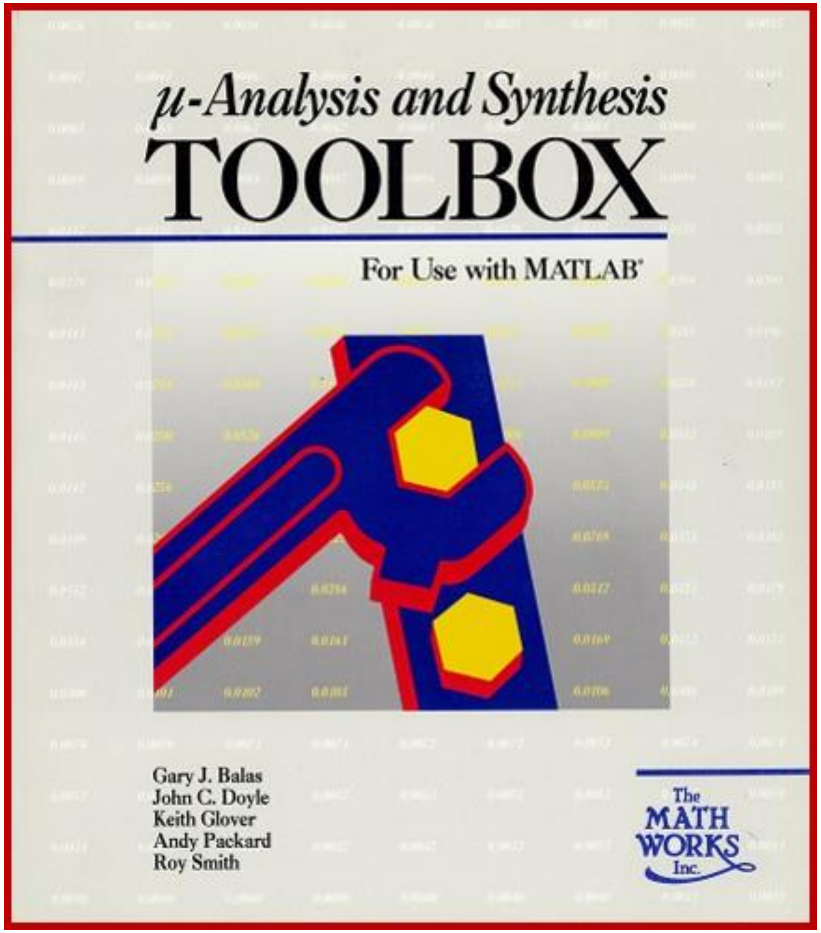
Spreading the Word

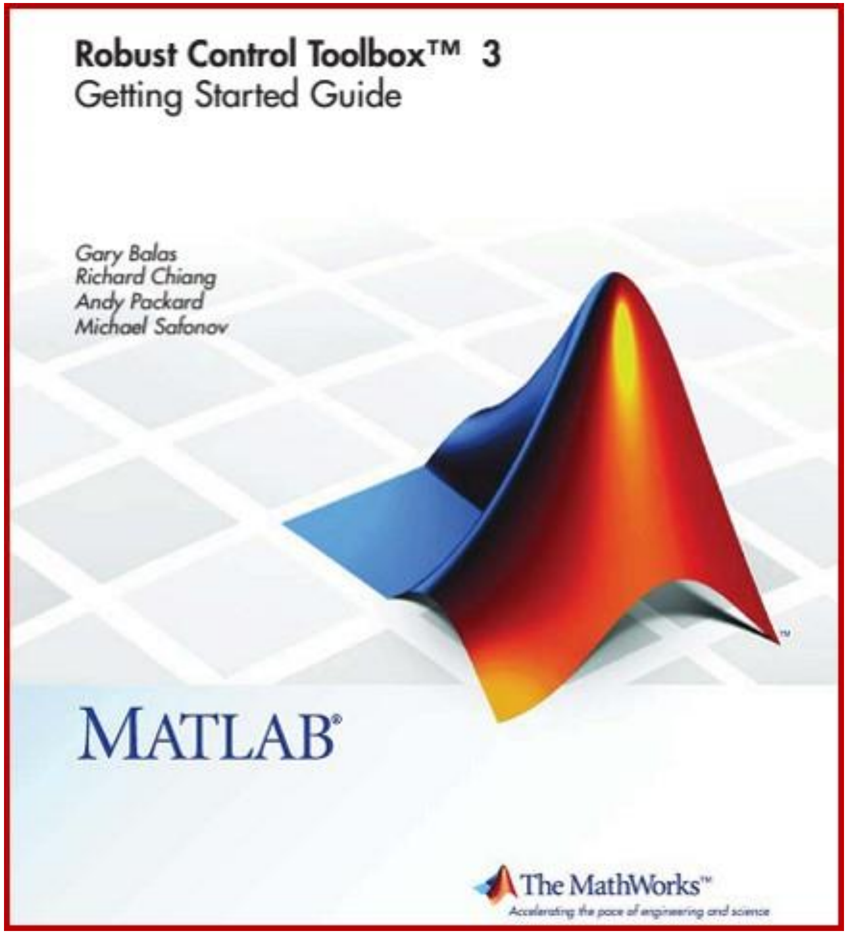
MUSYN Robust Control Theory Short Course (Start: 1989)



Software Development

μ-Analysis and Synthesis (μ-Tools) Matlab Toolbox (1990)





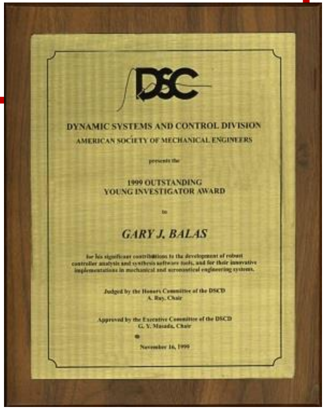
μ-Tools merged with the Matlab Robust Control Toolbox (2004)

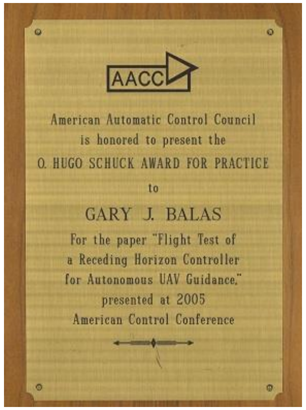
Awards

Honors and Awards

2012	Honorary Member, Hungarian Academy of Engineering
2012	Plenary Speaker, 7th IFAC Symposium on Robust Control Design, Aalborg, Denmark
2010	Prize for the Development of the Hungarian Aeronautical Science, Hungarian Scientific Association for Transport
2010	Plenary Speaker, 2nd Workshop on Clearance of Flight Control Laws, Stockholm, Sweden
2009	Plenary Speaker, 49th Israel Annual Conference on Aerospace Sciences, Tel Aviv and Haifa, Israel
2007	Distinguished McKnight University Professor, University of Minnesota
2006	O. Hugo Schuck Best Paper Award, American Automatic Control Council (with T. Keviczky)
2005	Control Systems Technology Award, IEEE Control System Society (with Prof. A.K. Packard)
2005-2006	Fellow, Committee on Institutional Cooperation Academic Leadership Program
2005	Semi-Plenary Speaker, 16th International Federation of Automatic Control (IFAC) World Congress, Prague, Czech Republic (with Prof. J. Bokor)
2004	Fellow, IEEE
2004	Plenary Speaker, Technical University of Delft Center for Systems and Control, "Challenges for the 21st Century"
2003	Institute of Technology George Taylor Distinguished Research Award, University of Minnesota
2003	Semi-Plenary Speaker, European Control Conference, Cambridge, England
2002	Associate Fellow, AIAA
2002-04	Senior Member, IEEE
2002	Session Plenary Speaker, International Council of Aeronautical Sciences Conference, Toronto
1999	Outstanding Young Investigator Award, ASME Dynamic Systems and Control
1993-1995	McKnight-Land Grant Professorship, University of Minnesota
1989-90, 2002	American Control Conference Best Paper Presentation in Session
1986-89	NASA Graduate Student Fellowship
1986	Donald Wills Douglas Fellowship in Aeronautics
1982-84	Hughes Aircraft Graduate Student Fellowship
1980-82	Hughes Aircraft Undergraduate Student Fellowship

Schuck Award w/ T. Keviczky ('05 ACC)





Head of Aerospace Eng. & Mechanics (7/06-1/14)

ON BEHALF OF
THE MINNEAPOLIS CHAPTER OF THE AMERICAN INSTITUTE OF ARCHITECTS,
PRESERVE MINNEAPOLIS,
AND
THE MINNEAPOLIS HERITAGE PRESERVATION COMMISSION,

THIS AWARD IS PRESENTED IN RECOGNITION OF

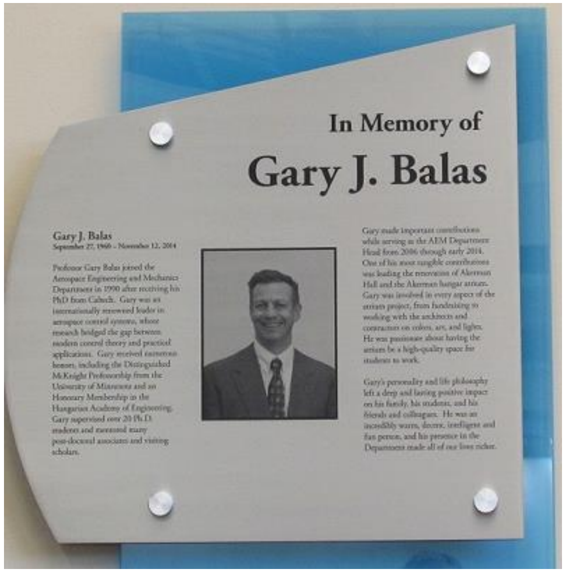
AKERMAN HALL HANGAR RENOVATION

THIS PROJECT IS A MERITORIOUS EXAMPLE OF AN ADAPTIVELY REUSED HISTORIC BUILDING, TRANSFORMED AND REINVENTED TO INCORPORATE BOTH NEW AND OLD ELEMENTS IN A WAY THAT THAT IS RESPECTFUL OF, BUT DIFFERENT FROM, THE FORM OF THE PAST.

2011 MINNEAPOLIS HERITAGE PRESERVATION AWARDS

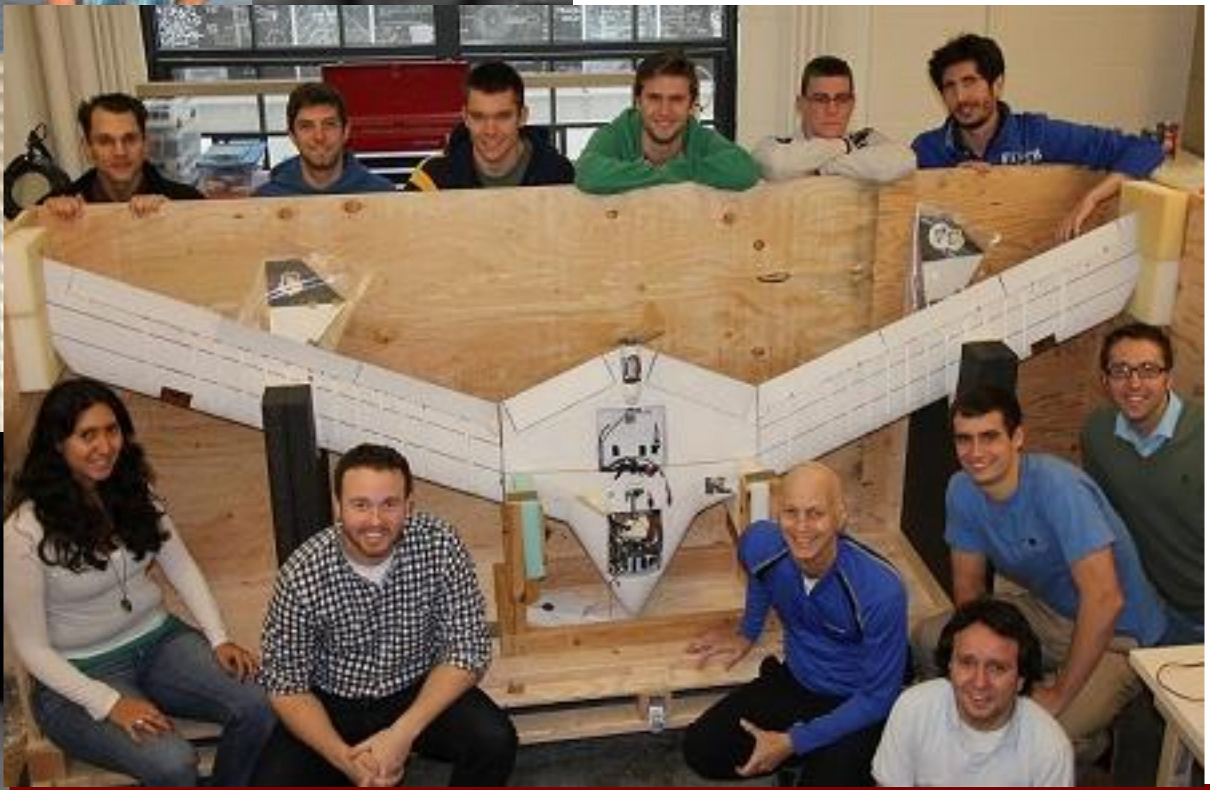
CHAIR | PRESIDENT | BOARD PRESIDENT

MINNEAPOLIS HERITAGE PRESERVATION COMMISSION | AIA Minneapolis | preserve minneapolis



Students & Visitors

and many others…



Collaborations


UNIVERSITY OF MINNESOTA

On behalf of the faculties of the University of Minnesota
and colleagues in Aerospace Engineering and Mechanics,
we congratulate the

Budapest University of
Technology and Economics

on the initiation of the reciprocal exchange and cooperation
memorandum of understanding between the two universities.
ay the spirit of collaboration continue between our two universities,







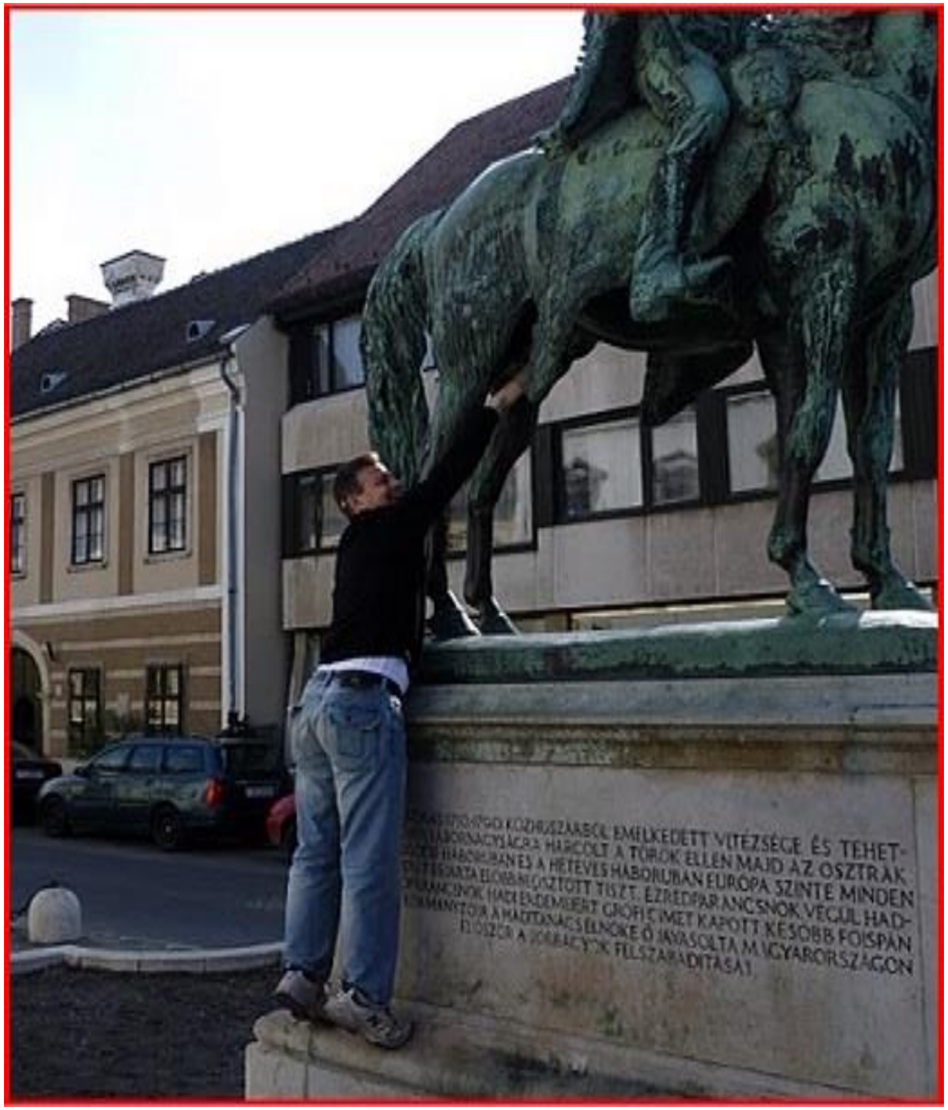

Enjoying Conferences

Biking….All year round in Minnesota!

Outline

- Applications
  - Flexible Aircraft
  - Wind Farms
- Numerical Tools
  - LPVTools
- LPV Theory
  - Lower Bounds
  - Analysis with IQCs
  - Model Reduction
  - Jacobian Linearization

Outline



- **Applications**
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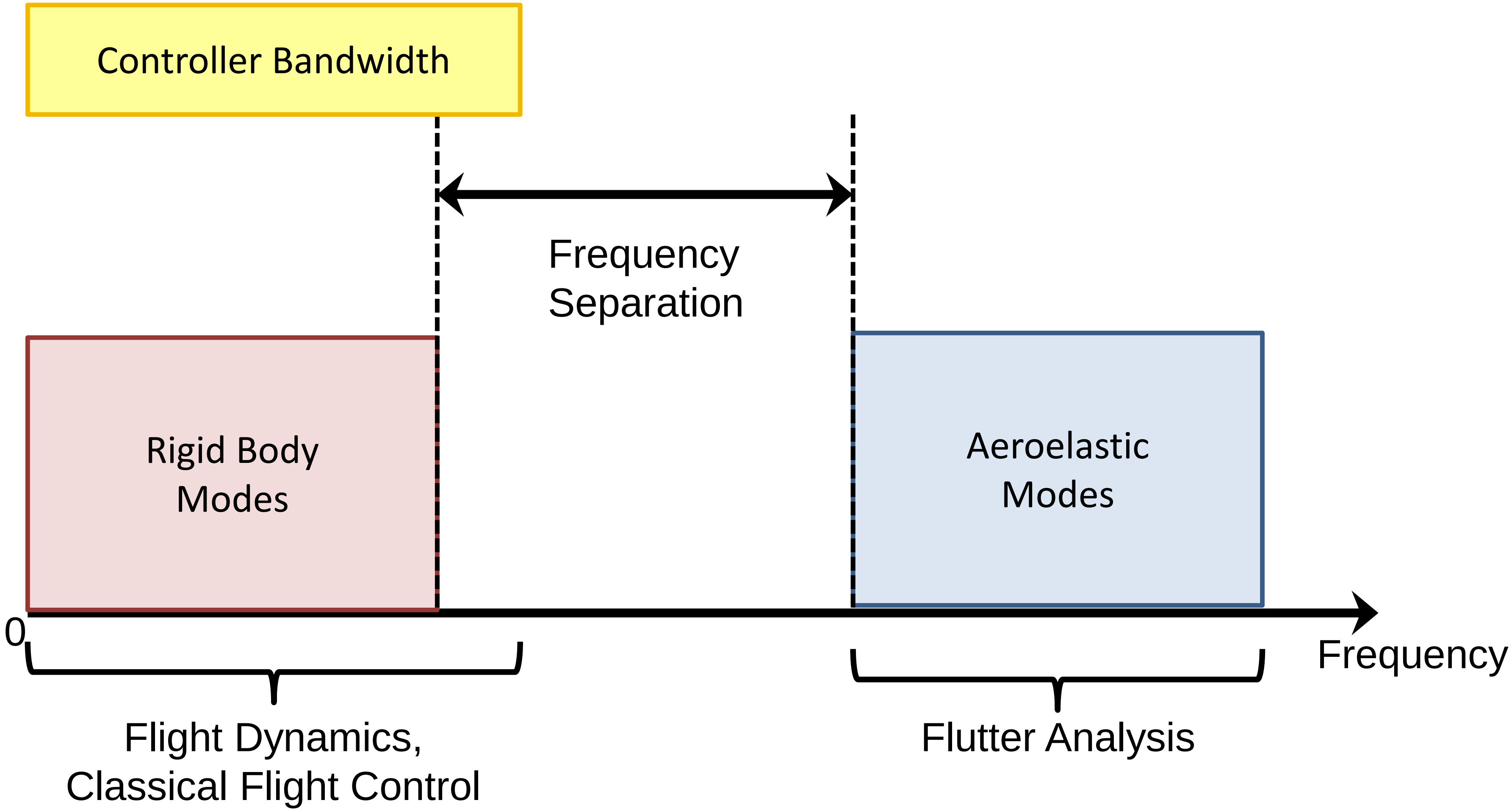
Aeroservoelasticity (ASE)

Efficient aircraft design

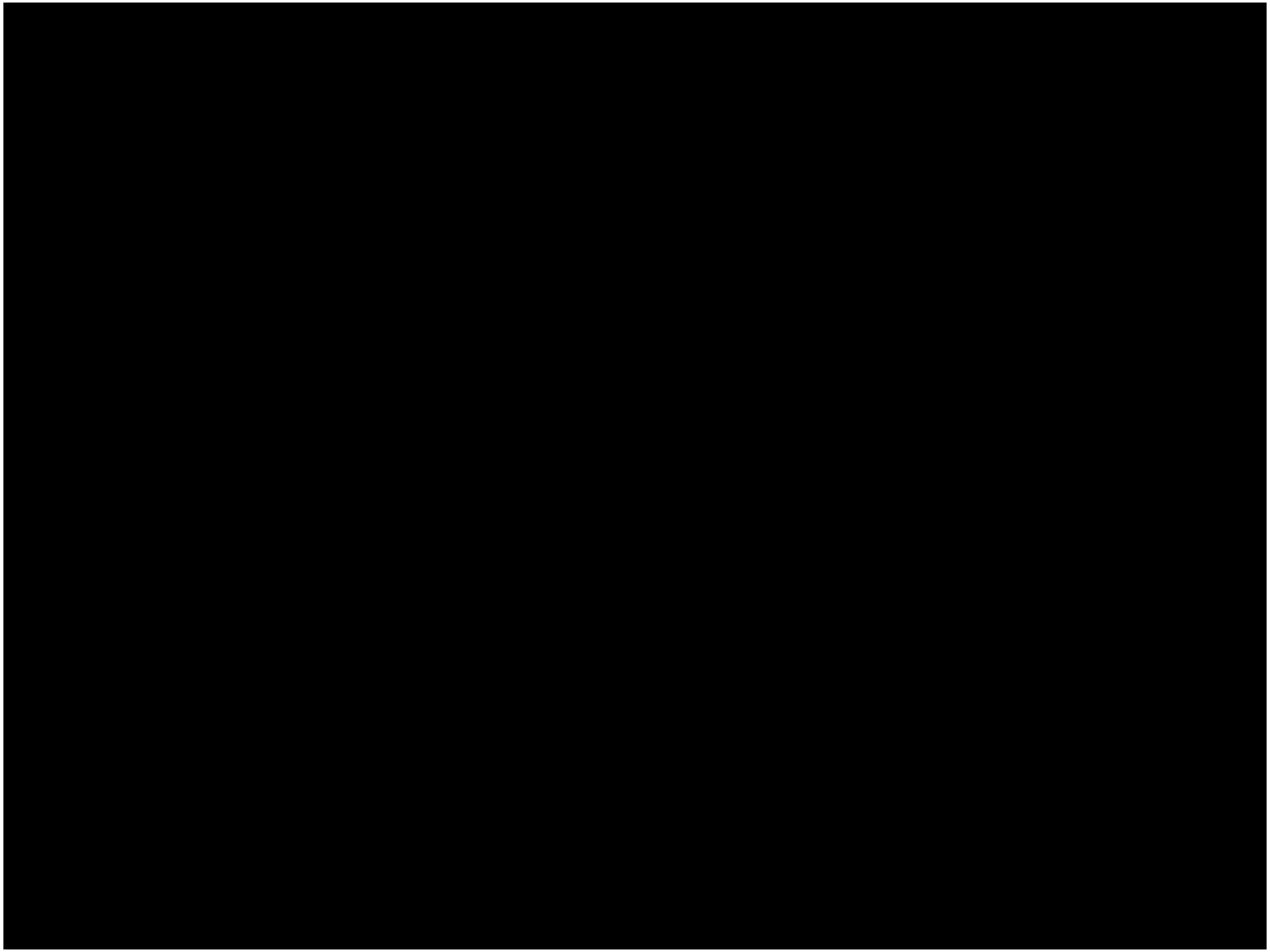
- Lightweight structures
- High aspect ratios

Source: www.flightglobal.com

Classical Approach

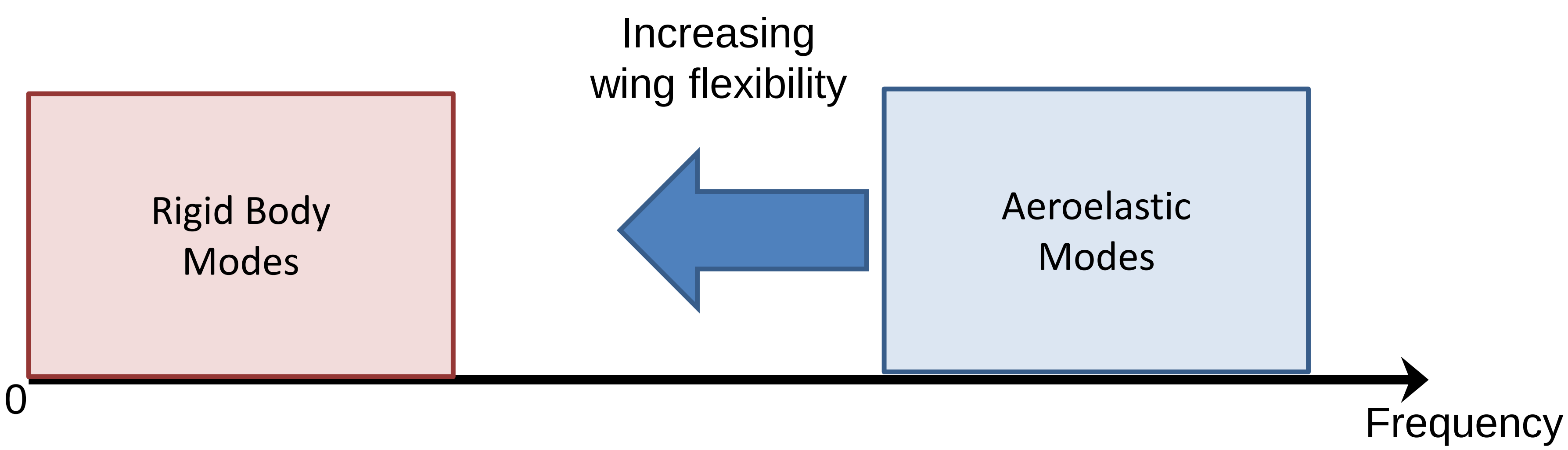


Flutter

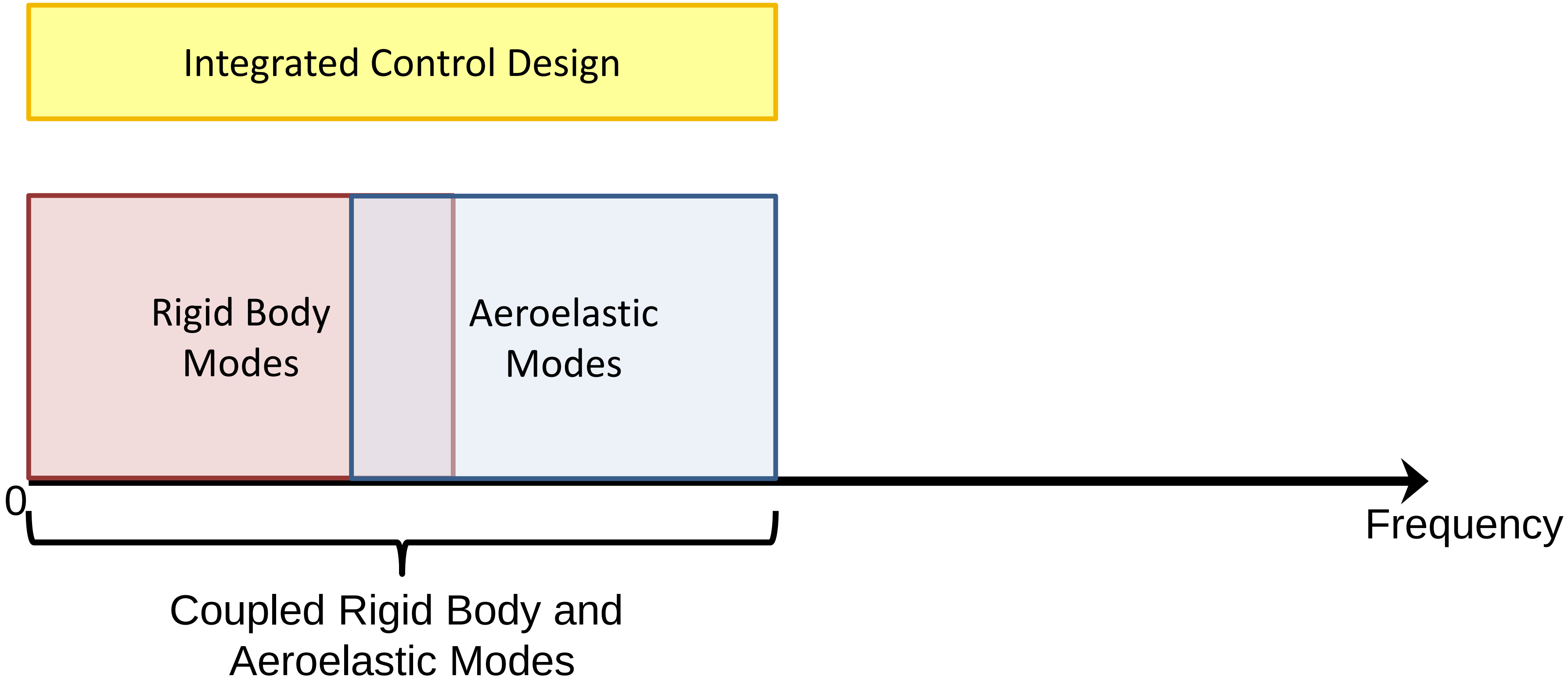


Source: NASA Dryden Flight Research

Flexible Aircraft Challenges



Flexible Aircraft Challenges



Body Freedom Flutter

Performance Adaptive Aeroelastic Wing (PAAW)

- Goal: Suppress flutter, control wing shape and alter shape to optimize performance
  - Funding: NASA NRA NNX14AL36A
  - Technical Monitor: Dr. John Bosworth
  - Two years of testing at UMN followed by two years of testing on NASA's X-56 Aircraft

Aurora FLIGHT SCIENCES

CMSoft

LM BFF

UMN Mini-Mutt

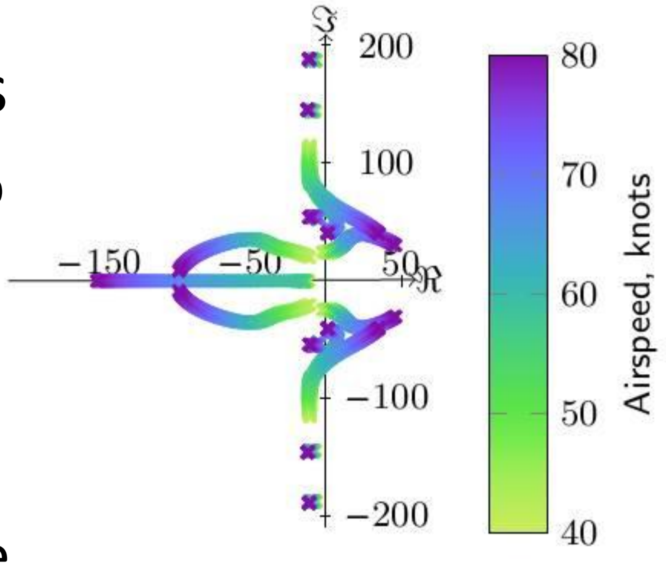
LM/NASA X-56

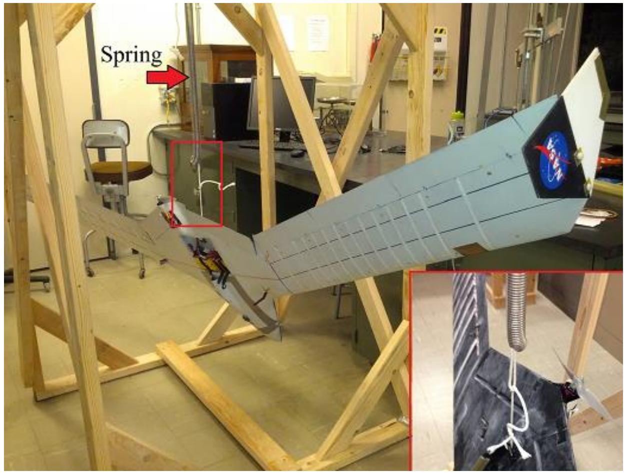
Schmidt & Associates

VirginiaTech Invent the Future

Modeling and Control for Flex Aircraft

1. Parameter Dependent Dynamics
   - Modes depend on airspeed due to structural/aero interactions
   - LPV is a natural framework.
2. Model Reduction
   - High fidelity CFD/CSD models have many (millions) of states.
3. Model Uncertainty
   - Use of simplified low order models OR reduced high fidelity models
   - Unsteady aero, mass/inertia & structural parameters





Additional Details

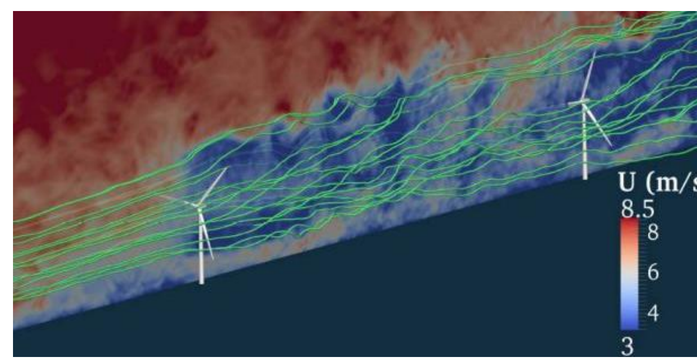
- Webpages: All models, flight data, etc
  - http://paaw.net/
  - http://www.uav.aem.umn.edu/
- References
  - Burnett, et al., Ndof simulation model forflight control development with flight test correlation, AIAA 2010.
  - Pfifer, et al., LPV Techniques Applied to Aeroservoelastic Aircraft: In Memory of Gary Balas, Thursday 14:00-14:20, ThP1T1.1.
  - Dowell (Ed), A Modern Course in Aeroelasticity, 2004
  - Schmidt, Modern Flight Dynamics, 2011.
- EU H2020 Project: "Flutter Free FLight Envelope eXpansion for ecOnomical Performance improvement" (FlexOp)
  - B. Vanek, PI (Sztaki) with control design supported by T. Peni (Sztaki), A. Marcos (Bristol), and A. Wildschek (Airbus).
  - Inspired by the work of Gary, aiming at developing active flutter mitigation control laws for industrial consideration.

Modeling and Control for Wind Farms

1. Parameter Dependent Dynamics
   - Modes depend on windspeed due to structural/aero interactions
   - LPV is a natural framework.
2. Model Reduction
   - High fidelity CFD/CSD models have many (millions) of states.
3. Model Uncertainty
   - Use of simplified low order models OR reduced high fidelity models

Eolos: http://www.eolos.umn.edu/

Saint Anthony Falls: http://www.safl.umn.edu/



Simulator for Wind Farm Applications, Churchfield & Lee http://wind.nrel.gov/designcodes/simulators/SOWFA

Refs:

J. Annoni and P. Seiler, Parameter varying dynamic mode decomposition, Submitted to Int. Journal of Robust and Nonlinear control, 2015.

J. Annoni, P.M.O. Gebraad, and P. Seiler, Wind farm modeling using input-output dynamic mode decomposition, Submitted to ACC, 2015.

J. Annoni, Modeling for Wind Farm Control, MS Thesis, 2014.

Outline

- Applications
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Classes of LPV Models

LPV systems depend on a time varying parameter *ρ(t)*

$$\begin{bmatrix} \dot{x}(t) \\ y(t) \end{bmatrix} = \begin{bmatrix} A(\rho(t)) & B(\rho(t)) \\ C(\rho(t)) & D(\rho(t)) \end{bmatrix} \begin{bmatrix} x(t) \\ u(t) \end{bmatrix}$$

Three main classes of LPV systems

- **Grid-based** (Jacobian Linearization) Models
  - $A(\rho)$, $B(\rho)$, $C(\rho)$, and $D(\rho)$ are arbitrary functions of ρ.
  - State matrices defined on a grid of parameter values ρ_k
- **Linear Fractional Transformation** (LFT) Models
  - $A(\rho)$, $B(\rho)$, $C(\rho)$, and $D(\rho)$ are rational functions of ρ.
- **Polytopic** Models
  - $A(\rho)$, $B(\rho)$, $C(\rho)$, and $D(\rho)$ are polytopic functions of ρ.
  - Affine models as a special case.

LPVTools: Matlab Toolbox for LPV Systems

- Developed by MuSyn: Balas, Packard, Seiler, Hjartarson
  - Funded by NASA SBIR contract #NNX12CA14C
  - Contract Monitor: Dr. Martin J. Brenner, NASA Armstrong.

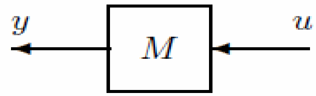
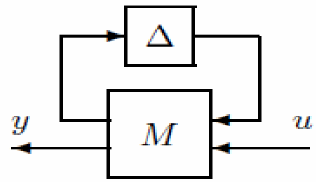
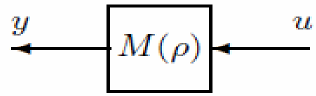
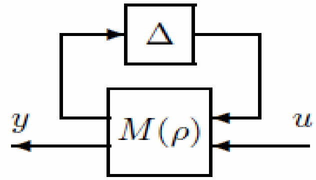
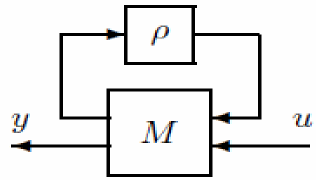
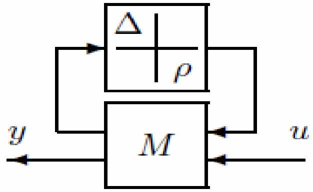
- Goal: Unified framework for grid/LFT based LPV
  - Modeling
  - Synthesis
  - Analysis
  - Simulation
- MATLAB/Simulink integration
  - Compatible with Control Toolbox, Robust Control Toolbox, Simulink.
  - Uses MATLAB object-oriented class programming

(A Subset of) LPV Software Tools

- LFT
  - SMAC, LFR, LFRT-SLK, and Robust Feedforward Design Toolboxes (ONERA: Magni, Biannic, Roos, Ferreres, Demourant,…)
  - Enhanced LFR-toolbox (DLR: Hecker, Varga, Pfifer,…)
  - LPV Robust Control Toolbox (Milan: De Vito, Lovera; NGC Aerospace: Kron, de Lafontaine)
  - LFR-RAI (Siena: Garulli, Masi , Paoletti, Türkoğlu)
  - LPV Analysis & Synthesis (Stuttgart: Scherer, Veenman, Köse, Köroğlu,…)
- Grid-based
  - LMI Control Toolbox, HINFSTRUCT, Simulink LPV Blocks (Matlab)
- Polytopic
  - TP Toolbox (Sztaki: Baranyi, Takarics,…)

Data Structures

LPVTools

Object Type	Block	Matrix	System	Frequency Response
Nominal		double	ss	frd
Uncertain		umat	uss	ufrd
Nominal Gridded LPV		pmat	pss	pfrd
Uncertain Gridded LPV		upmat	upss	upfrd
Nominal LFT LPV		pmatlft	psslft	
Uncertain LFT LPV		pmatlft	psslft	

LPVTools: Open Source Release

- Release 1.0:
  - http://www.aem.umn.edu/~SeilerControl/software.shtml
  - Google Search: SeilerControl
  - Static release under GNU Affero GPL License
  - Full documentation (manual, command line, Matlab "doc")
  - Ref: A. Hjartarson, A. Packard, and P. Seiler, LPVTools: A Toolbox for Modeling, Analysis, and Synthesis of Parameter Varying Control Systems. Thursday 16:30-16:50, ThP2T1.1.
- Basic objects and results implemented
  - LFT Analysis and Synthesis (Packard, Scherer, Gahinet, Apkarian, …)
  - Gridded Analysis and Synthesis (Wu, Packard, Becker, …)
  - Model Reduction with Generalized Gramians (Wood, Glover, Widowati, …)
  - Simulink interface

LPVTools: Open Source Release

Many gaps remain including

- LPV System identification (Toth, Verdult, Verhaegen, Lovera, van Wingerden, Gebraad, Lee, Poolla, Bamieh, …)
- Robust Synthesis / Full block S procedure (Scherer, Veenman, Köse, Köroğlu, …)
- Polytopic systems (Baranyi, Takarics,…)
- Large scale model reduction (Amsallem, Farhat, Carlberg, Poussot-Vassal,…)
- Reduced complexity controllers (Scorletti, Fromion,…)
- LPV with Delays, Saturation (Wu, Briat, …)
- Large Scale Systems (Werner, Kulcsár, Mohammadpour, Grigoridis, …)
- FDI (Bokor, Vanek, Szabo, Peni, Sename, …)

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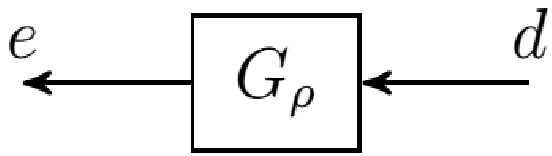
LPV Analysis



Gridded LPV System

$$\dot{x}(t) = A(\rho(t))\; x(t) + B(\rho(t))\; d(t)$$
$$e(t) = C(\rho(t))\; x(t) + D(\rho(t))\; d(t)$$

$$\rho \in \mathcal{A} := \text{Set of allowable trajectories}$$

Induced L_2 Gain

$$\sup_{\rho \in \mathcal{A}} \|G_\rho\|_{2\to 2} = \sup_{\rho \in \mathcal{A}} \sup_{0 \neq d \in L_2} \frac{\|e\|_2}{\|d\|_2}$$

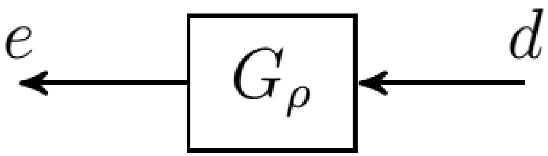
(Standard) Dissipation Inequality Condition

Theorem

If there exists $V(x,\rho) \geq 0$ such that

$$\dot{V} + e^T e \leq \gamma^2 d^T d$$

then $\sup_{\rho \in \mathcal{A}} \|G_\rho\|_{2\to 2} \leq \gamma$.



Proof: Integrate the dissipation inequality

$$\underbrace{V(x(T))}_{\geq 0} + \underbrace{V(x(0))}_{=0} + \int_0^T e(t)^T e(t) dt \leq \gamma^2 \int_0^T d(t)^T d(t) dt \qquad \blacksquare$$

Comments

- Dissipation inequality can be expressed/solved as LMIs.
  - Finite dimensional LMIs for LFT/Polytopic LPV systems
  - Parameterized LMIs for Gridded LPV (requires basis functions, gridding, etc)
- **Condition is IFF for LTI systems but only sufficient for LPV**

LPV Lower Bounds

- **Questions:** The dissipation inequality gives an upper bound on the induced L2 gain.
  - Can we compute lower bounds?
  - Can we compute “bad” parameter trajectories?
- **Simple Approach:** Frozen LTI Analysis
  - Let $\mathcal{A}_{const}$ denote a set of constant parameter trajectories.
  - The system G_ρ is LTI for each frozen $\rho \in \mathcal{A}_{const}$
  - Evaluate H_∞ norm on a grid of parameter values:

$$\sup_{\rho\in\mathcal{A}} \|G_\rho\|_{2\to 2} \geq \sup_{\rho\in\mathcal{A}_{const}} \|G_\rho\|_\infty$$

LPV Lower Bounds

- **Questions:** The dissipation inequality gives an upper bound on the induced L2 gain.
  - Can we compute lower bounds?
  - Can we compute "bad" parameter trajectories?
- **Enhanced Approach:** Periodic LTV Analysis
  - Let $\mathcal{A}_{per}$ denote a set of PLTV parameter trajectories.
  - Apply results to exactly compute the induced L_2 gain for LTV and periodic systems (Colaneri, Varga, Cantoni/Sandberg, etc).
  - Optimize lower bound over set of PLTV trajectories

$$\sup_{\rho \in \mathcal{A}} \|G_\rho\|_{2\to 2} \geq \sup_{\rho \in \mathcal{A}_{per}} \|G_\rho\|_{2\to 2}$$

Refs:

T. Peni & P. Seiler, Computation of lower bounds for the induced L_2 norm of LPV systems, IJRNC, 2015.

M. Cantoni & H. Sandberg, Computing the L_2 gain for linear periodic continuous-time systems. Aut. 2009.

LPV Lower Bounds

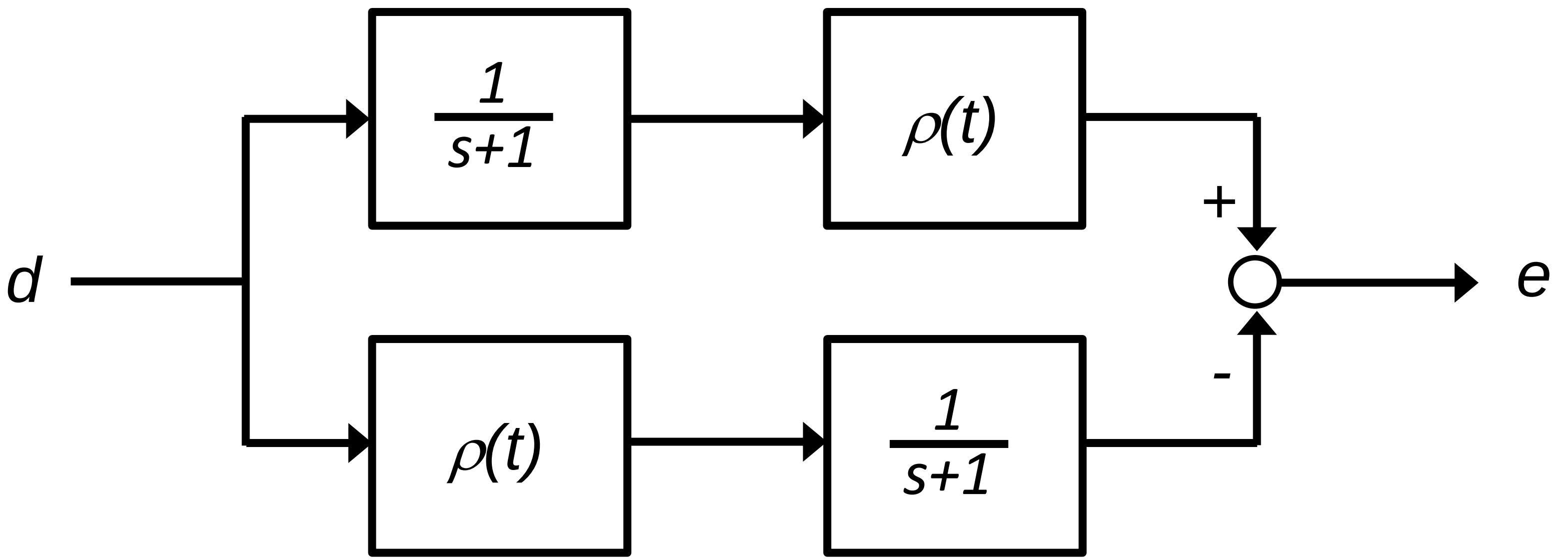
- **Questions:** The dissipation inequality gives an upper bound on the induced L2 gain.
  - Can we compute lower bounds?
  - Can we compute “bad” parameter trajectories?
- **Enhanced Approach:** Periodic LTV Analysis
- **Possible Extensions**
  - Improved algorithm (choice of bases functions, etc)
  - Finite Horizon LTV analysis
  - Uncertain LPV lower bounds

Refs:

T. Peni & P. Seiler, Computation of lower bounds for the induced L_2 norm of LPV systems, IJRNC, 2015.

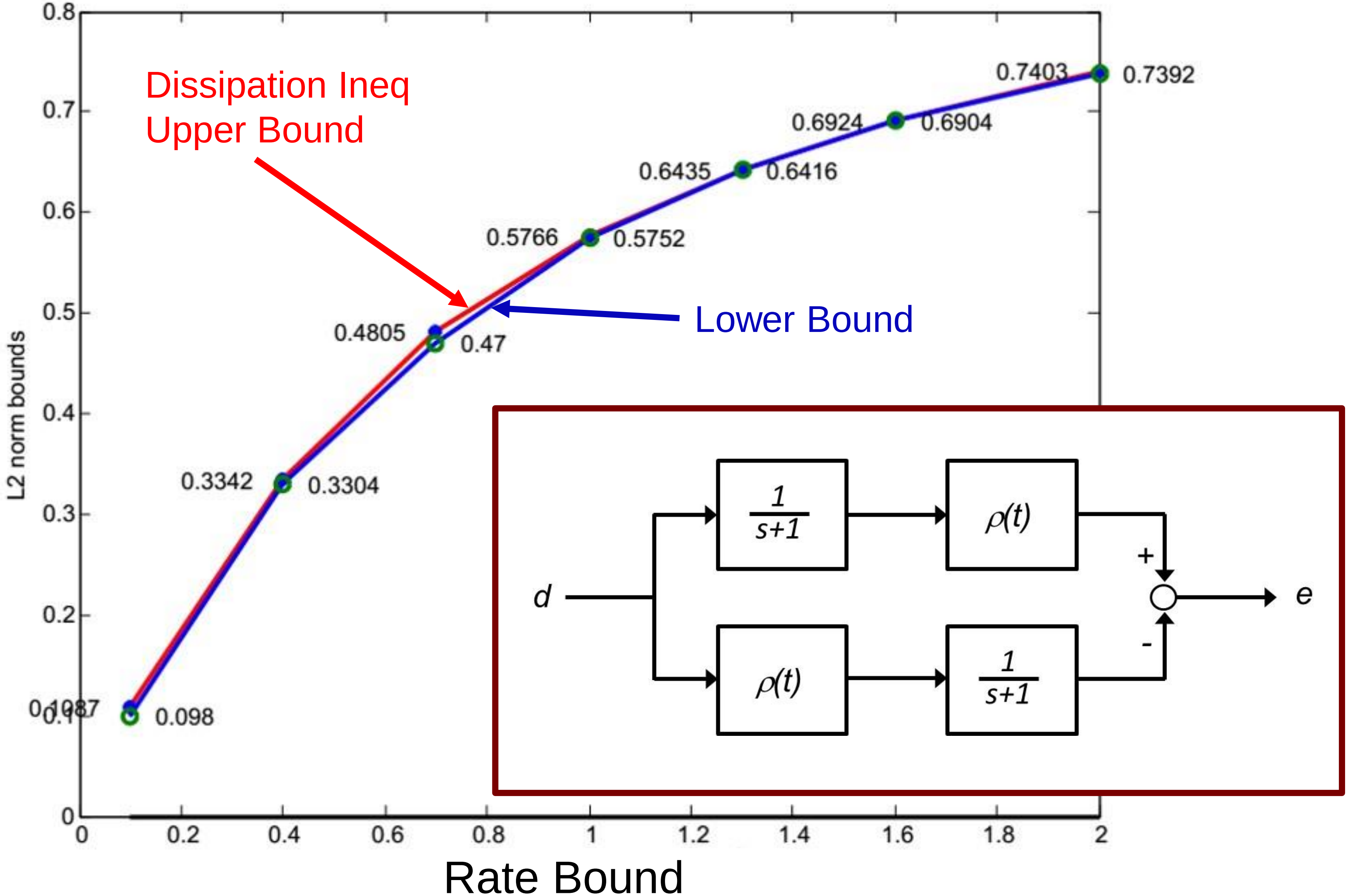
M. Cantoni & H. Sandberg, Computing the L_2 gain for linear periodic continuous-time systems. Aut. 2009.

Example: LPV Induced L_2 Gain



Note: Gain from *d* to *e* is *0* if $\rho(t)$ is constant.

Example: LPV Induced L_2 Gain



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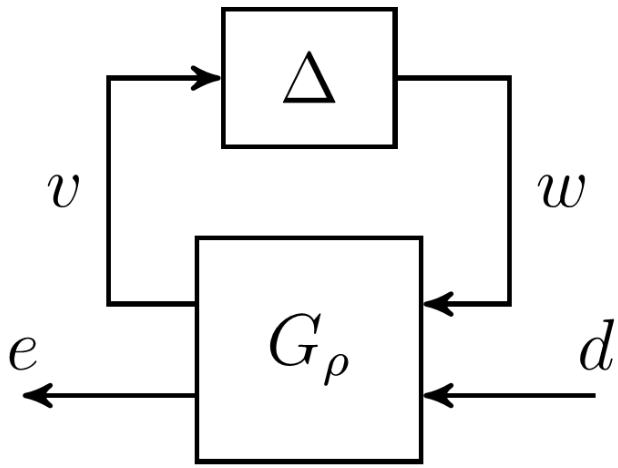
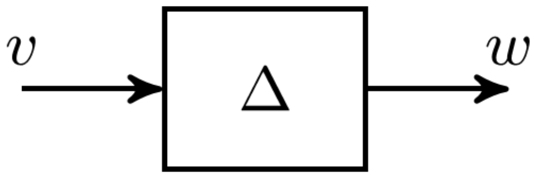
Robustness Analysis for LPV Systems

- **Goal:** Assess the impact of model uncertainty for an LPV system.



- **Approach:**
  - LFT Model: Separate uncertainty Δ from nominal system G_ρ .
  - "Uncertainty" Δ can be parametric, LTI dynamic, and/or nonlinearities (saturation, etc).
  - Use Integral Quadratic Constraints to model input/output behavior (Megretski & Rantzer, TAC 1997).
  - Extend dissipation inequality approach for robustness analysis
- **Results for Gridded Nominal system**
  - Parallels earlier results for LFT nominal system by Scherer, Veenman, Köse, Köroğlu.

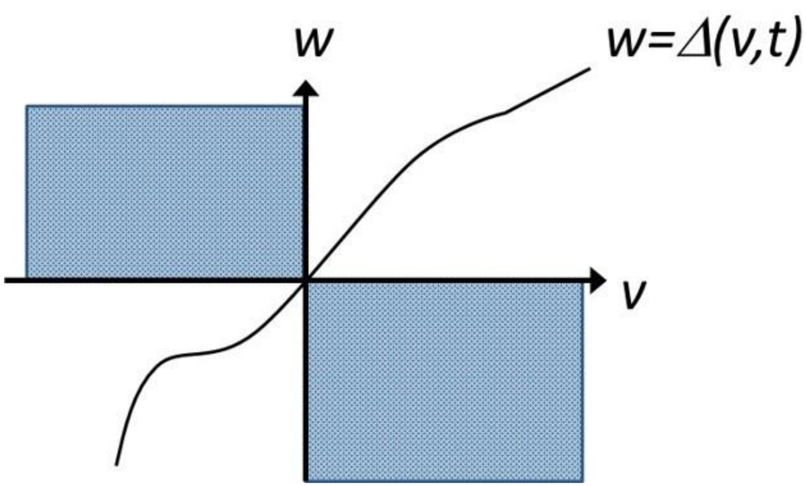
IQC Example: Passive System



$w = \Delta(v,t)$ is a passive system (pointwise in time).

$$2v(t)^T w(t) \geq 0 \; \forall t$$

$$\begin{bmatrix} v(t) \\ w(t) \end{bmatrix}^T \begin{bmatrix} 0 & I \\ I & 0 \end{bmatrix} \begin{bmatrix} v(t) \\ w(t) \end{bmatrix} \geq 0 \, \forall t$$



Pointwise Quadratic Constraint

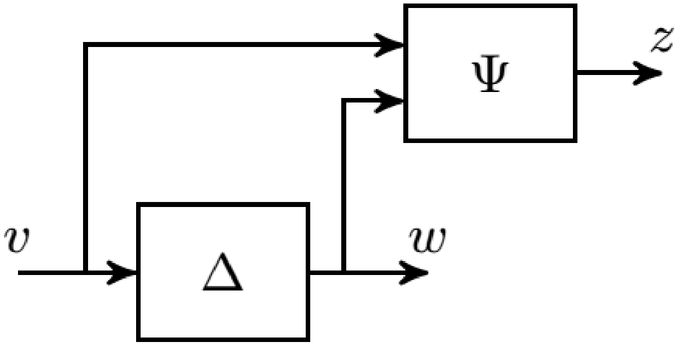
General (Time Domain) IQCs

General IQC Definition:

Let Ψ be a stable, LTI system and M a constant matrix.
Δ satisfies IQC defined by Ψ and M if

$$\int_0^T z(t)^T M z(t) dt \geq 0$$

$\forall v \in L_2[0,\infty)$, $w = \Delta(v)$, and $T \geq 0$.



Comments:

- Megretski & Rantzer ('97 TAC) has a library of IQCs for various components.
- IQCs can be equivalently specified in the freq. domain with a multiplier Π
- A non-unique factorization connects $\Pi=\Psi^*M\Psi$.
- Multiple IQCs can be used to specify behavior of Δ.

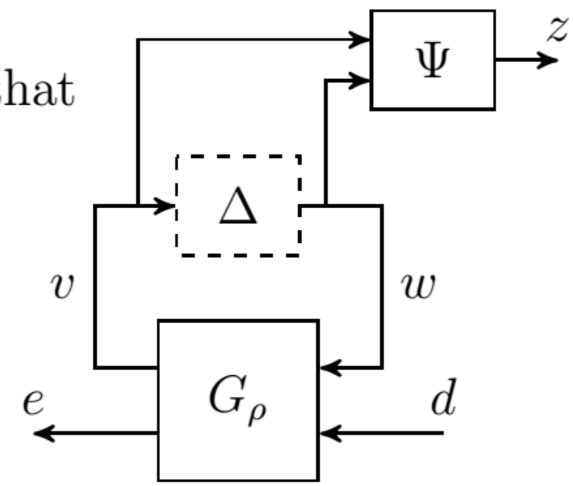
IQC Dissipation Inequality Condition

Theorem

If $\Delta \in IQC(\Psi, M)$ and there exists $V(x,\rho) \geq 0$ such that

$$\dot{V} + z^T M z + e^T e \leq \gamma^2 d^T d$$

then $\sup_{\rho \in \mathcal{A}} \|G_\rho\|_{2\to 2} \leq \gamma$.



Proof: Integrate the dissipation inequality

$$\underbrace{V(x(T))}_{\geq 0} + \underbrace{V(x(0))}_{=0} + \underbrace{\int_0^T z(t)^T M z(t) dt}_{\geq 0} + \int_0^T e(t)^T e(t) dt \leq \gamma^2 \int_0^T d(t)^T d(t) dt$$

■

Comment

- Dissipation inequality can be expressed/solved as LMIs.
- Extends standard D/G scaling but requires selection of basis functions for IQC.

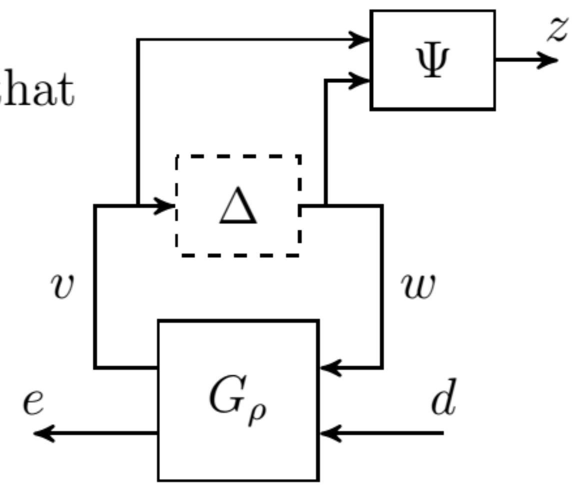
Less Conservative IQC Result

Theorem

If $\Delta \in IQC(\Psi, M)$ and there exists $V(x,\rho) \geq 0$ such that

$$\dot{V} + z^T M z + e^T e \leq \gamma^2 d^T d$$

then $\sup_{\rho \in \mathcal{A}} \|G_\rho\|_{2\to 2} \leq \gamma$.



Technical Result

- Positive semidefinite constraint on *V* and time domain IQC constraint can be dropped.
- These are replaced by a freq. domain requirement on $\Pi = \Psi^* M \Psi$.
- Some energy is “hidden” in the IQC.

Refs:

P. Seiler, Stability Analysis with Dissipation Inequalities and Integral Quadratic Constraints, IEEE TAC, 2015.

H. Pfifer & P. Seiler, Less Conservative Robustness Analysis of Linear Parameter Varying Systems Using Integral Quadratic Constraints, submitted to IJRNC, 2015.

Time-Domain Dissipation Inequality Analysis

Summary: Under some technical conditions, the frequency-domain conditions in (M/R, ’97 TAC) are equivalent to the time-domain dissipation inequality conditions.

Applications:

1. LPV robustness analysis (Pfifer, Seiler, IJRNC)
2. General LPV robust synthesis (Wang, Pfifer, Seiler, submitted to Aut)
3. LPV robust filtering/feedforward (Venkataraman, Seiler, in prep)
   - Robust filtering typically uses a duality argument. Extensions to the time domain?
4. Exponential rates of convergence (Hu,Seiler, submitted to TAC)
   - Motivated by optimization analysis with ρ-hard IQCs (Lessard, Recht, & Packard)
5. Nonlinear analysis using SOS techniques

Item 1 has been implemented in LPVTools. Items 2 & 3 parallel results by (Scherer, Köse, and Veenman) for LFT-type LPV systems.

Outline

- Applications
  - Flexible Aircraft
  - Wind Farms
- Numerical Tools
  - LPVTools
- **LPV Theory**
  - Lower Bounds
  - Analysis with IQCs
  - **Model Reduction**
  - Jacobian Linearization

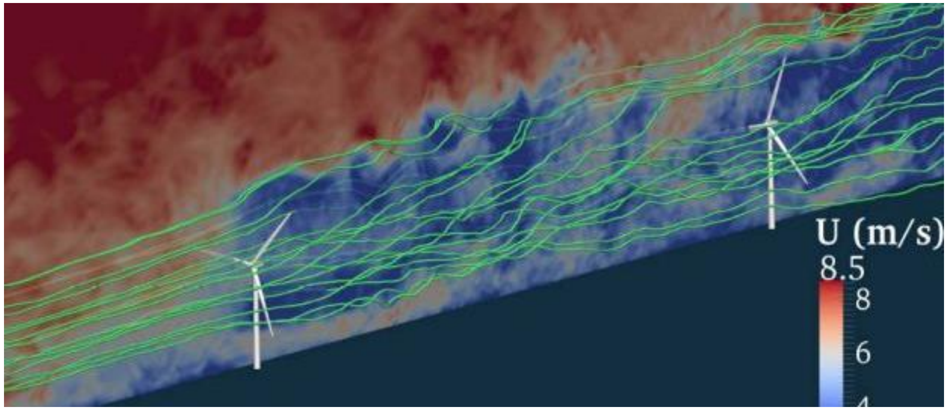
LPV Model Reduction

- Both flexible aircraft and wind farms can be modeled with high fidelity fluid/structural models.
- LPV models can be obtained via Jacobian linearization:

$$\dot{x}(t) = A(\rho(t))\ x(t) + B(\rho(t))\ d(t)$$

$$e(t) = C(\rho(t))\ x(t) + D(\rho(t))\ d(t)$$

- State dimension can be extremely large ($>10^6$)
- LPV analysis and synthesis is restricted to ≈50 states.
- Model reduction is required.



LPV Balancing and Truncation

Extends balanced truncation model reduction to gridded LPV

- Solve LMIs to construct parameter-varying Gramians $X_c(\rho)$ & $X_o(\rho)$
- The Hankel operator of the plant is bounded by the largest generalized Hankel singular value.

$$\|P_\rho\|_H \leq \max_\rho \sqrt{\lambda_1(X_c(\rho)X_o(\rho))}$$

- Compute a parameter-varying transformation $T(\rho)$ to balance generalized Gramians. Apply coordinate transform:

$$\dot{z} = \left(T(\rho)A(\rho) + \dot{T}(\rho)\right) T^{-1}(\rho)z + T(\rho)B(\rho)d$$
$$e = CT^{-1}(\rho)z + D(\rho)d$$

- Reduced order model can be obtained via truncation.

References

- Beck, Doyle, Glover, Model Reduction of Multi-Dimensional and Uncertain Systems, TAC, 1995.
- Wood, Control of parameter-dependent mechanical systems, Ph.D., Univ. Cambridge, 1995.
- Wood, Goddard, Glover, Approximation of linear parameter-varying systems, IEEE CDC, 1996.
- Widowati, Bambang, Model Reduction of LPV Control with Bounded Parameter Variation Rates, Asian CC, 2006.

LPV Balancing and Truncation

Issues:

1. Solving LMIs for generalized Gramians restricts the method to systems with moderate state order (<200).
2. Parameter-varying coordinate transformations *T(ρ)*
   a. Introduces rate dependence in model
   b. Destroys state consistency across parameter domain

$$\dot{z} = \left(T(\rho)A(\rho) + \dot{T}(\rho)\right) T^{-1}(\rho)z + T(\rho)B(\rho)d$$
$$e = CT^{-1}(\rho)z + D(\rho)d$$

References

- Beck, Doyle, Glover, Model Reduction of Multi-Dimensional and Uncertain Systems, TAC, 1995.
- Wood, Control of parameter-dependent mechanical systems, Ph.D., Univ. Cambridge, 1995.
- Wood, Goddard, Glover, Approximation of linear parameter-varying systems, IEEE CDC, 1996.
- Widowati, Bambang, Model Reduction of LPV Control with Bounded Parameter Variation Rates, Asian CC, 2006.

High Order Model Reduction

Large literature with recent results for LPV and Param. LTI

- Antoulas, Amsallem, Carlberg , Gugercin, Farhat, Kutz, Loeve, Mezic, Poussot-Vassal, Rowley, Schmid, Willcox, …

Two new results for LPV:

1. Input-Output Dynamic Mode Decomposition
   - Combine subspace ID with techniques from fluids (POD/DMD).
   - No need for adjoint models. Can reconstruct full-order state.
2. Parameter-Varying Oblique Projection
   - Petrov-Galerkin approximation with constant projection space and parameter-varying test space.
   - Constant projection maintains state consistency avoids rate dependence.

References

1A. Annoni & Seiler, A method to construct reduced-order parameter varying models, submitted to IJRNC, 2015.

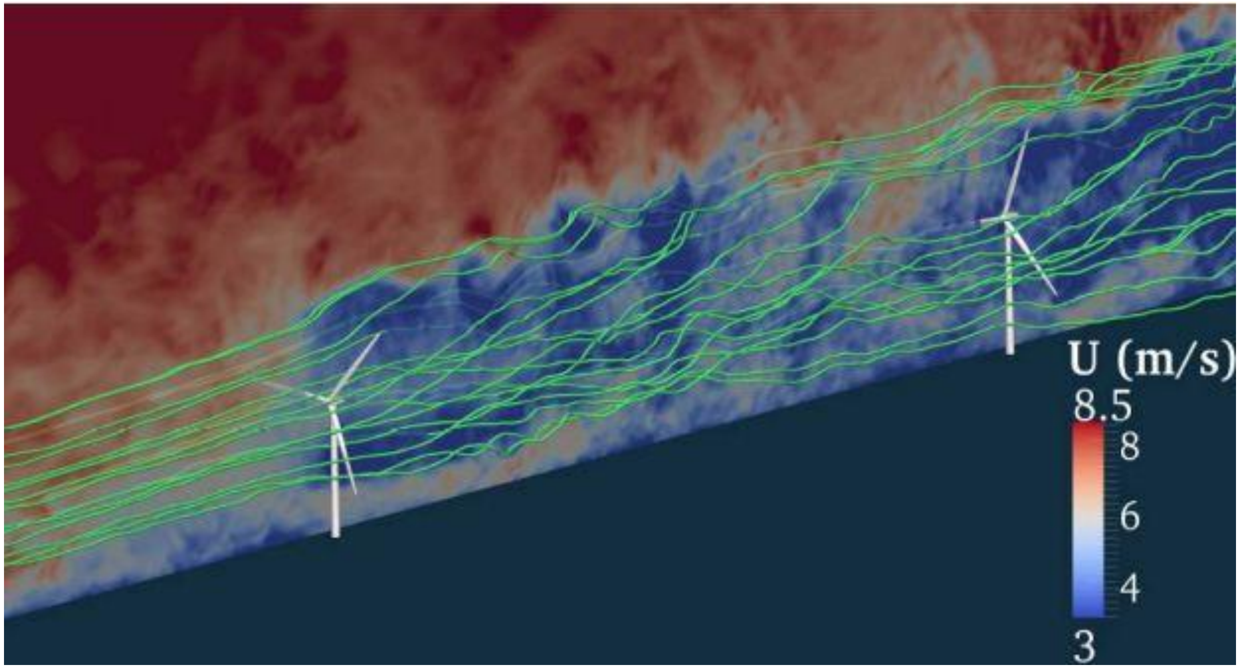
1B. Singh & Seiler, Model Reduction using Frequency Domain Input-Output Dynamic Mode Decomposition, sub. to '16 ACC.

2. Theis, Seiler, & Werner, Model Order Reduction by Parameter-Varying Oblique Projection, submitted to 2016 ACC.

Example: Wind Farm Modeling

SOWFA: Simulator for On/Offshore Wind Farm Applications

- 3D unsteady spatially filtered Navier-Stokes equations
- Two 5MW turbines with 126 m diam separated by 5 diams.
- # States≈3.6 Million (=1.6M grid points x 3 vel components)

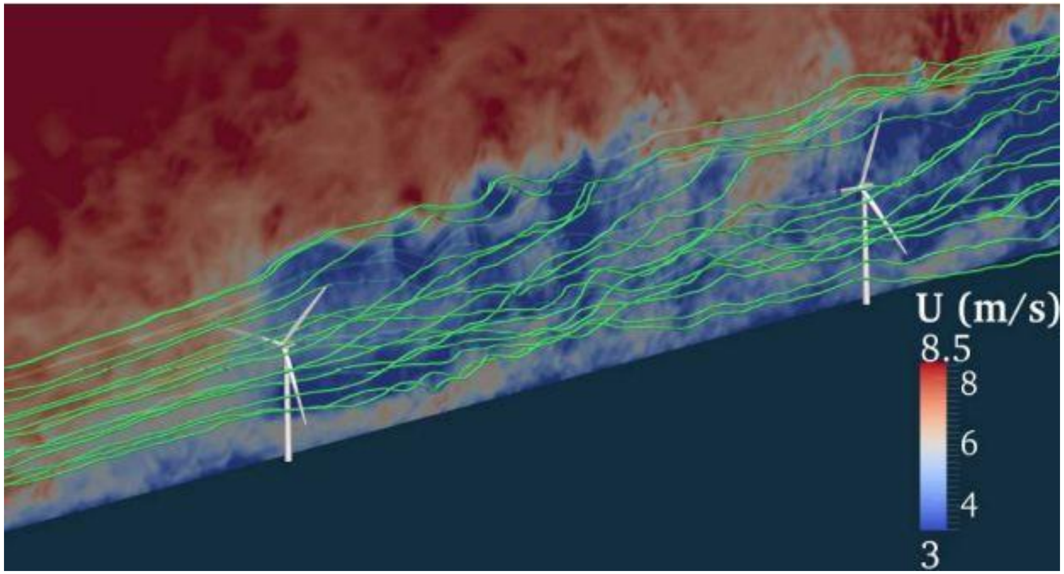


Churchfield, Lee, https://nwtc.nrel.gov/SOWFA

Results

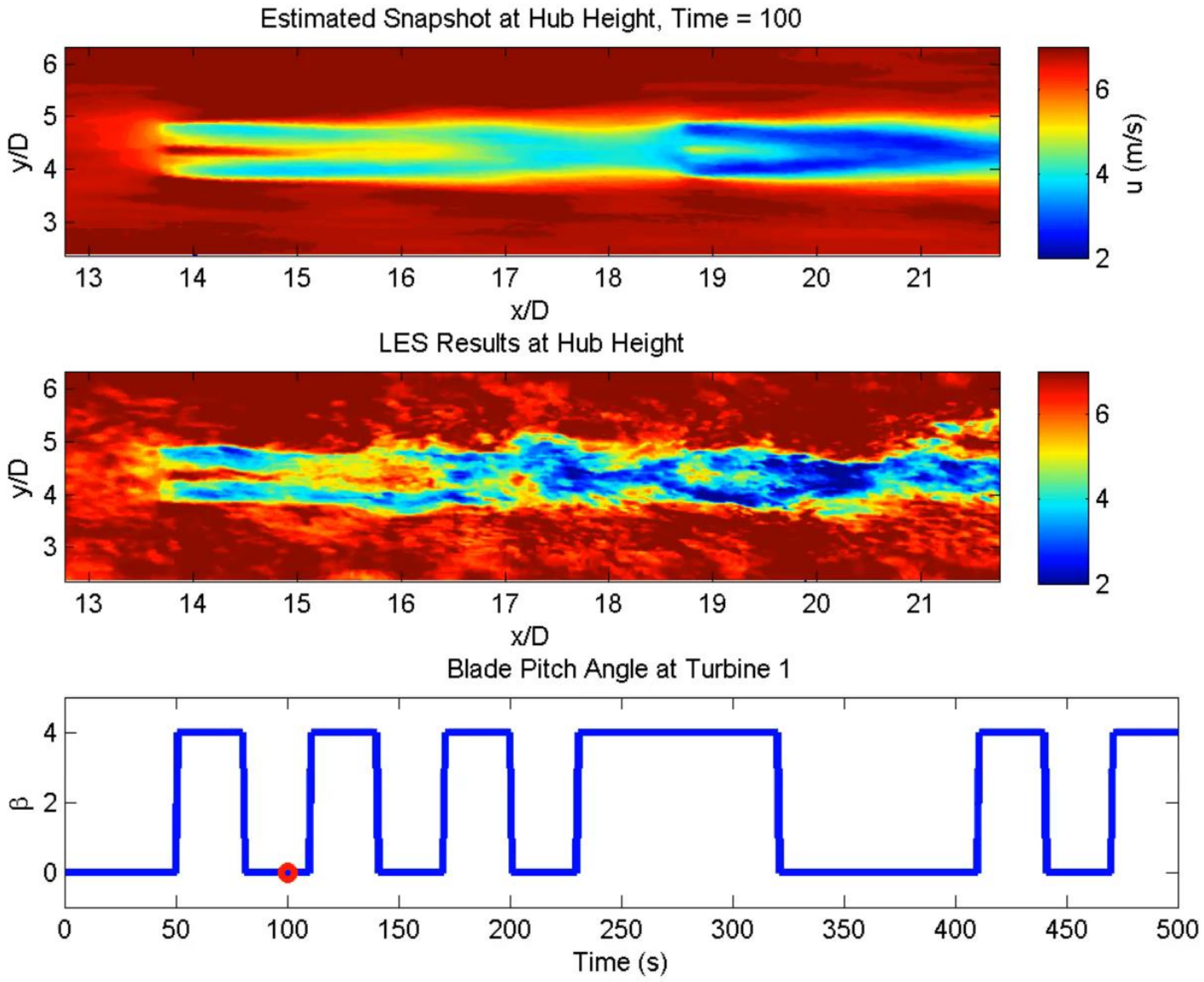
- Simulated at 7 m/s with 6% turb. in neutral boundary layer
- Excited upstream blade pitch and collected measurements of fluid flow and key turbine inputs/outputs
- Used IODMD to construct 20th order model.

Ref: Annoni, Gebraad, Seiler, Wind farm flow modeling using input-output dynamic mode decomposition, sub. to '16 ACC.

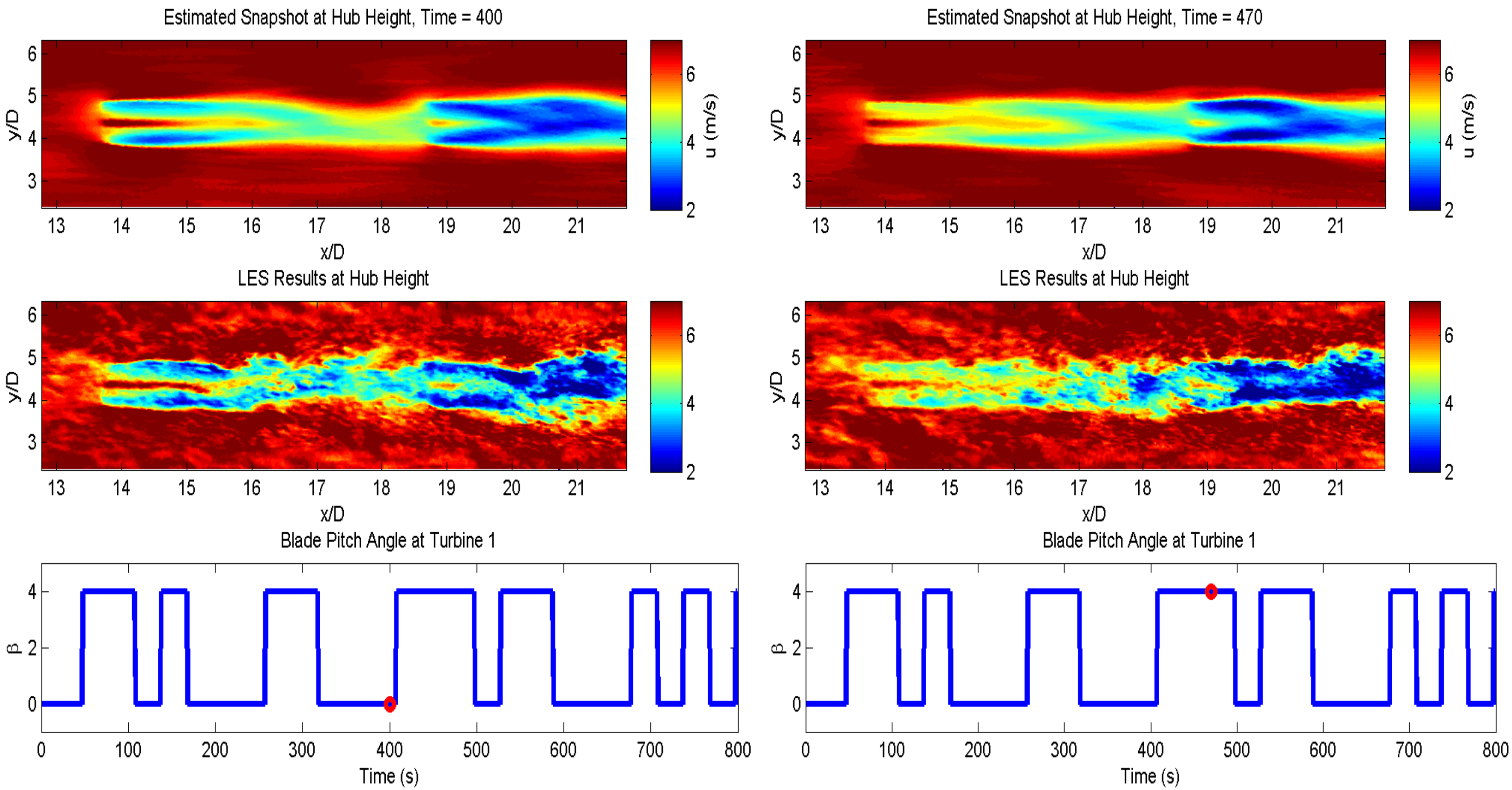


Churchfield, Lee, https://nwtc.nrel.gov/SOWFA

Flow Simulation



Compare Individual Snapshots



Outline

- Applications
  - Flexible Aircraft
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- Numerical Tools
  - LPVTools
- **LPV Theory**
  - Lower Bounds
  - Analysis with IQCs
  - Model Reduction
  - **Jacobian Linearization**

LTI Jacobian Linearization

- Autonomous Nonlinear System

$$\dot{x}(t) = f(x(t), \rho)$$

- Let $\{\bar{x}(\rho), \rho\}$ be a parameterized collection of eq. points

$$0 = f(\bar{x}(\rho), \rho)$$

- Assume frozen (constant) ρ and define $\delta_x(t) := x(t) - \bar{x}(\rho)$

$$\begin{aligned} \dot{\delta}_x(t) &= \dot{x}(t) - \dot{\bar{x}}(\rho) \\ &= f(x(t), \rho) \\ &\approx \underbrace{f(\bar{x}(\rho), \rho)}_{=0} + \underbrace{\left[\left. \frac{\partial f}{\partial x} \right|_{(\bar{x}(\rho), \rho)} \right]}_{:=A(\rho)} \delta_x(t) \end{aligned}$$

LTI Jacobian Linearization

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- Assume frozen (constant) ρ and define $\delta_x(t) := x(t) - \bar{x}(\rho)$

$$\dot{\delta}_x(t) = A(\rho)\delta_x(t)$$

Linearization is valid if solution remains near equilibrium point specified by ρ.

LPV Jacobian Linearization

- Autonomous Nonlinear System

$$\dot{x}(t) = f(x(t), \rho(t))$$

- Let $\{\bar{x}(\rho), \rho\}$ be a parameterized collection of eq. points

$$0 = f(\bar{x}(\rho), \rho)$$

- Assume time-varying ρ(t) and define $\delta_x(t) := x(t) - \bar{x}(\rho(t))$

$$\begin{aligned}
\dot{\delta}_x(t) &= \dot{x}(t) - \dot{\bar{x}}(\rho(t)) \\
&= f(x(t), \rho(t)) - \frac{\partial \bar{x}}{\partial \rho}(\rho(t))\ \dot{\rho}(t) \\
&\approx \underbrace{f(\bar{x}(\rho(t)), \rho(t))}_{=0} + \underbrace{\left[\left. \frac{\partial f}{\partial x} \right|_{(\bar{x}(\rho(t)), \rho(t))} \right]}_{:=A(\rho(t))} \delta_x(t) - \frac{\partial \bar{x}}{\partial \rho}(\rho(t))\ \dot{\rho}(t)
\end{aligned}$$

LPV Jacobian Linearization

- Autonomous Nonlinear System

$$\dot{x}(t) = f(x(t), \rho(t))$$

- Let $\{\bar{x}(\rho), \rho\}$ be a parameterized collection of eq. points

$$0 = f(\bar{x}(\rho), \rho)$$

- Assume time-varying ρ(t) and define $\delta_x(t) := x(t) - \bar{x}(\rho(t))$

$$\dot{\delta}_x(t) = A(\rho(t))\ \delta_x(t) \boxed{- \frac{\partial \bar{x}}{\partial \rho}(\rho(t))\ \dot{\rho}(t)}$$

Linearization is valid if solution remains near equilibrium manifold specified by ρ(t).

Summary: Jacobian Linearization

- Linearization for Non-autonomous Systems

$$\dot{x}(t) = A(\rho(t))\ x(t) + B(\rho(t))\ d(t) - \frac{\partial \bar{x}}{\partial \rho}(\rho(t))\ \dot{\rho}(t)$$

$$e(t) = C(\rho(t))\ x(t) + D(\rho(t))\ d(t)$$

- Parameter variation appears as an input forcing.
- Can we develop analysis/synthesis conditions that exploit knowledge of this forcing?

Ref: B. Takaric and P. Seiler, Gain Scheduling for Nonlinear Systems via Integral Quadratic Constraints, ACC, 2015.

- Initial synthesis results assuming forcing is measurable disturbance.
- Also exploits IQCs to bound the effect of Taylor series errors.

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- NASA
  - NRA NNX14AL36A: "Lightweight Adaptive Aeroelastic Wing for Enhanced Performance Across the Flight Envelope," Tech. Monitor: J. Bosworth.
  - NRA NNX12AM55A: “Analytical Validation Tools for Safety Critical Systems Under Loss-of-Control Conditions.” Tech. Monitor: C. Belcastro.
  - SBIR contract #NNX12CA14C: “Adaptive Linear Parameter-Varying Control for Aeroservoelastic Suppression.” Tech. Monitor. M. Brenner.
- Eolos Consortium and Saint Anthony Falls Laboratory
  - http://www.eolos.umn.edu/ & http://www.safl.umn.edu/

Conclusions

Gary had a significant technical impact in many areas.

- Applications
- Numerical Tools
- LPV Theory

Gary's impact extended beyond his technical contributions. He enjoyed the collaborations and friendships of the controls community.

http://www.aem.umn.edu/~SeilerControl/

Appendix

Body Freedom Flutter