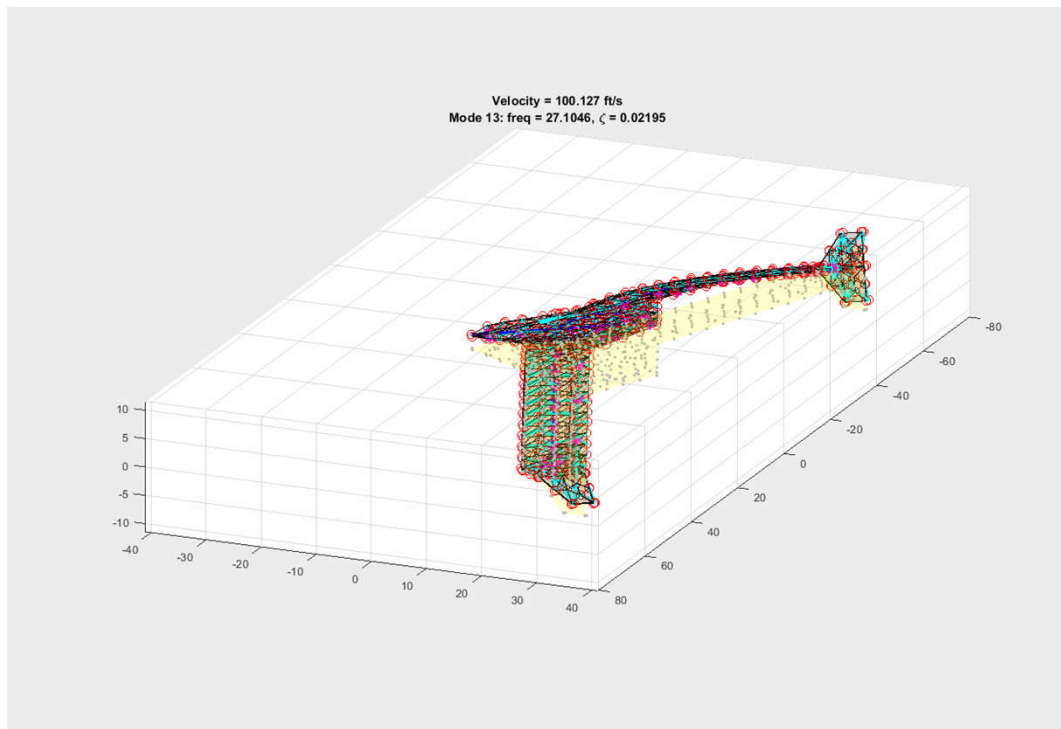


Enhancing Robustness in Reinforcement Learning

Peter Seiler
University of Minnesota



November 16, 2018
Seminar at United Technologies Research Lab



Research Overview

Jordan Hoyt
Parul Singh
Wind Energy



Raghu Venkataraman
Safety Critical Systems



Abhineet Gupta
Aeroelasticity



Robust Control Design and Analysis

Chris Regan
Curt Olson
Brian Taylor

Harish Venkataraman
Jyot Buch

Research Overview

Past: What is the impact of model uncertainty and nonlinearities on feedback system?

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Key contributions

1. Theoretical connections between frequency domain and time-domain (dissipation inequality) analysis methods
2. Tools for uncertain time-varying and gain-scheduled systems
3. Applications to wind energy, UAVs, flex aircraft, hard disk drives
4. Numerically reliable algorithms with transition to Matlab's Robust Control Toolbox

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Key contributions

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3. Applications to wind energy, UAVs, flex aircraft, hard disk drives
4. Numerically reliable algorithms with transition to Matlab's Robust Control Toolbox

Future: What is the impact of model uncertainty on control systems designed via data-driven methods?

Outline

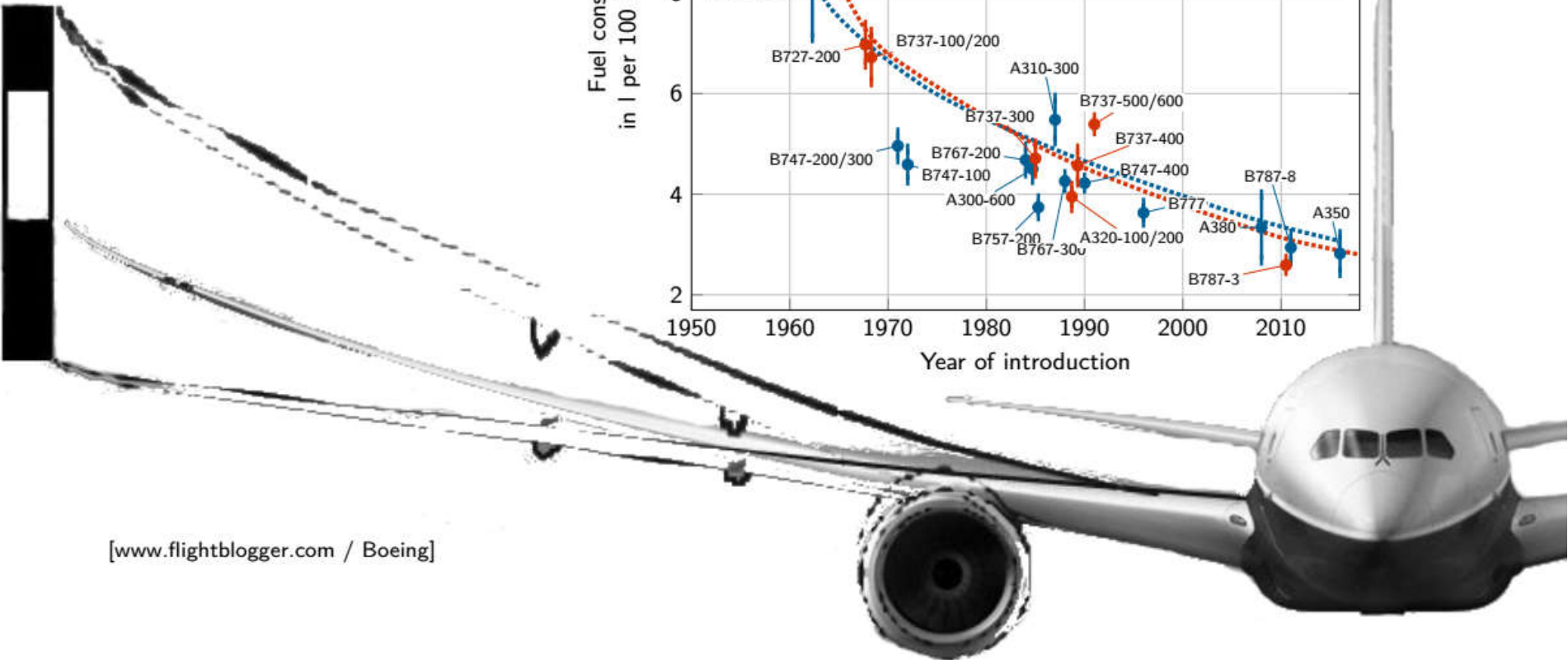
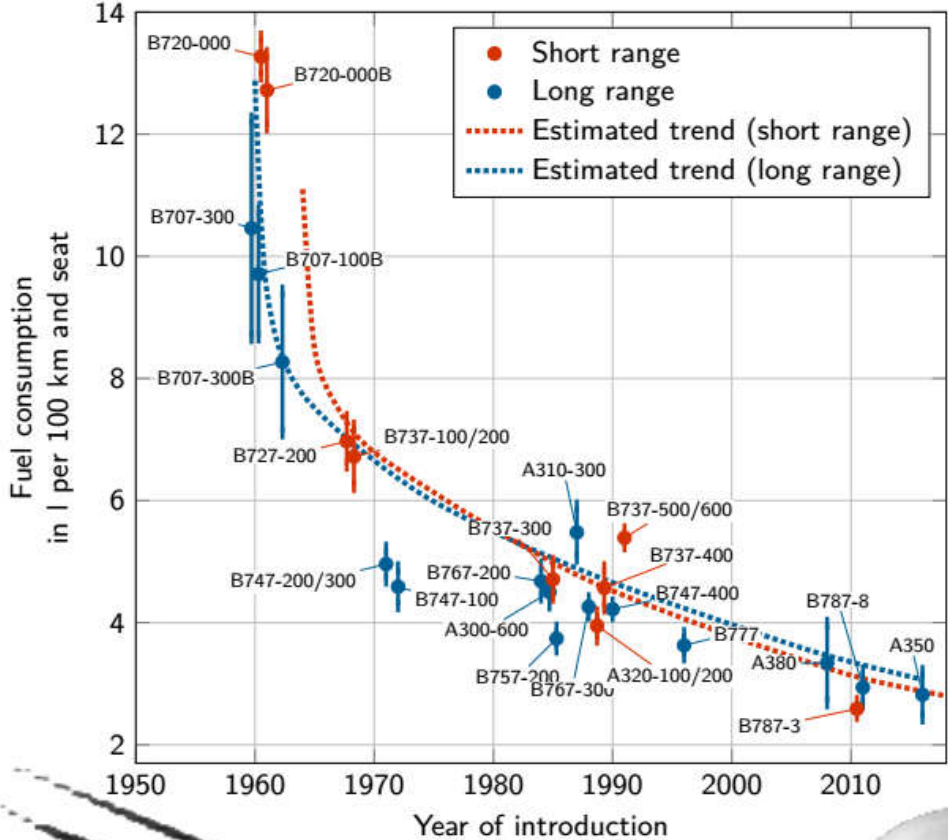
- Flutter Suppression on Flexible Aircraft
- Robustness of Time-Varying Systems
- Robustness in Reinforcement Learning

Outline

- **Flutter Suppression on Flexible Aircraft**
- Robustness of Time-Varying Systems
- Robustness in Reinforcement Learning

Fuel Consumption of Commercial Airlines

[Knoblach 2015]



[www.flightblogger.com / Boeing]

Fuel Efficient Aircraft Design

- Breguet Range Equation

$$\text{Range} = V I_{sp} \frac{L}{D} \ln \left(\frac{m_{\text{takeoff}}}{m_{\text{landing}}} \right)$$

- Improve fuel efficiency by
 - Reducing structural mass
 - Reducing drag with longer, more slender wings (high aspect ratio)
 - Improving engine efficiency

Fuel Efficient Aircraft Design

- Breguet Range Equation

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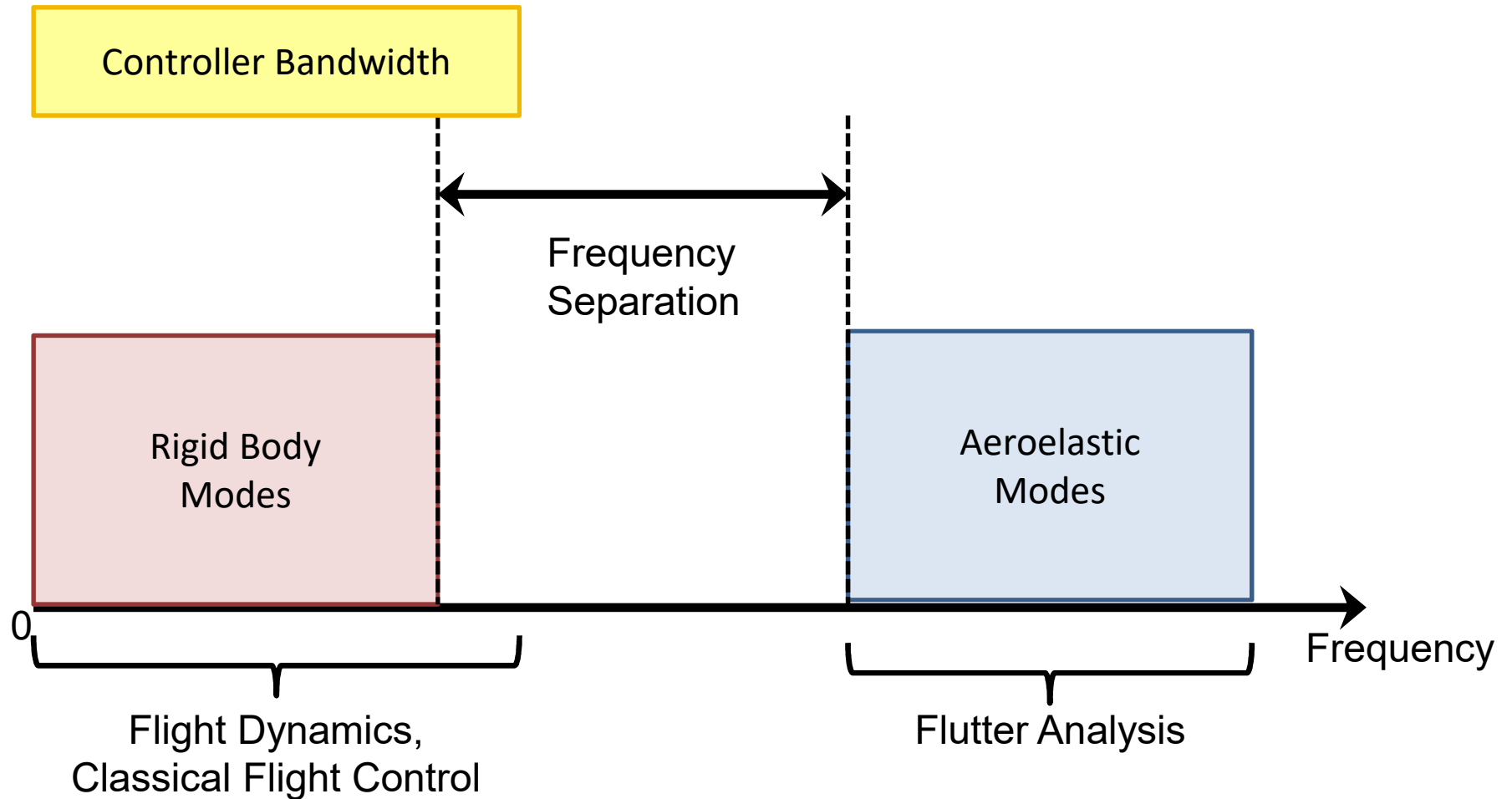
- Improve fuel efficiency by
 - Reducing structural mass
 - Reducing drag with longer, more slender wings (high aspect ratio)
 - Improving engine efficiency
- Adverse effects
 - increased coupling of structural dynamics and rigid body motion
 - increased coupling of aerodynamic loads and structural deformation
 - reduced flutter margins

Flutter

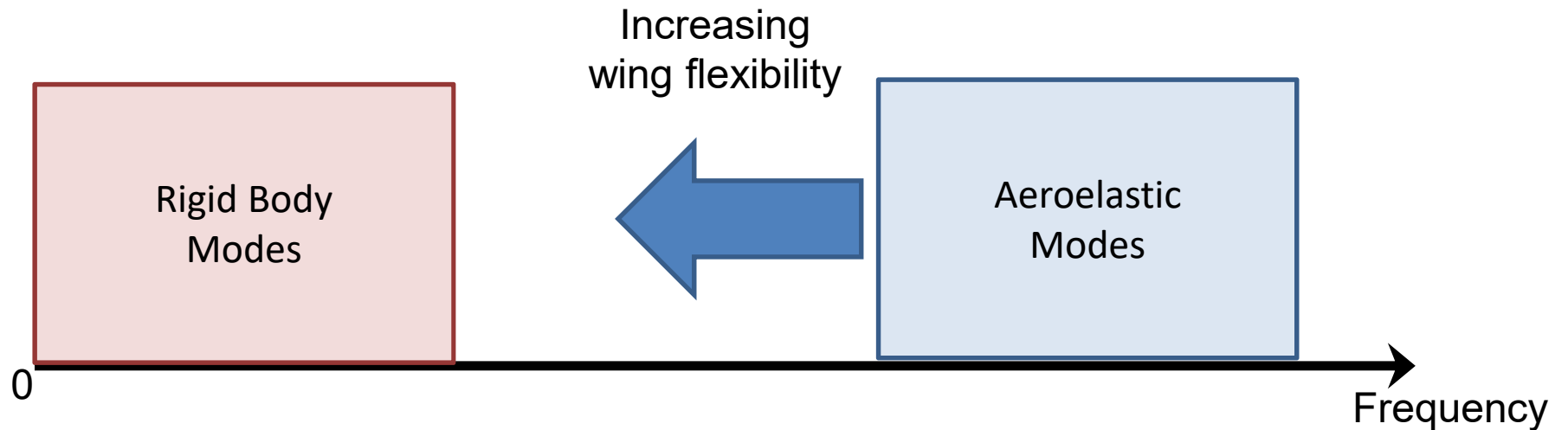


Source: NASA Dryden Flight Research

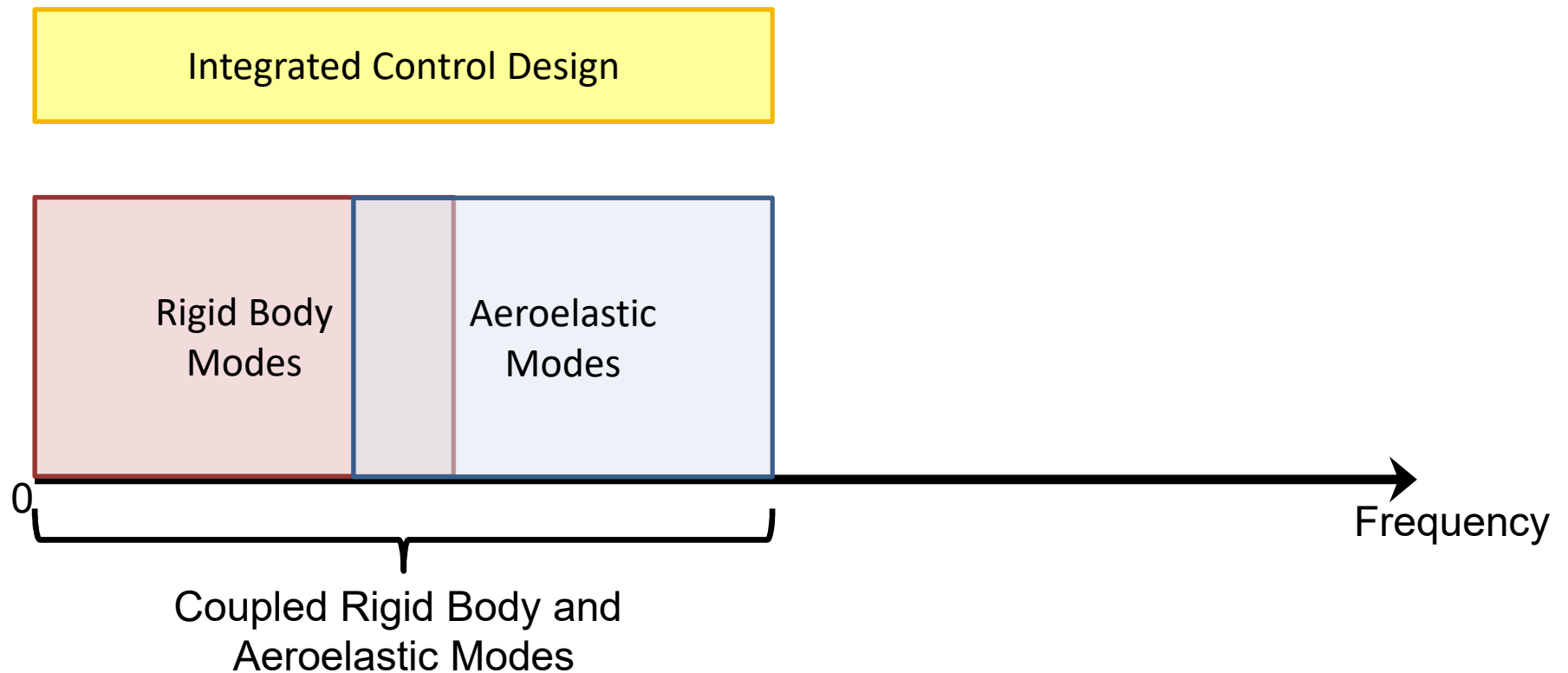
Classical Approach



Flexible Aircraft Challenges



Flexible Aircraft Challenges



Recent Flight Demonstrators

- **Lockheed Martin/AFRL: Body Freedom Flutter (BFF)**
 - Ref: Burnett, et al, AIAA MST Conference, 2010-7780
 - Ref: Holm-Hansen, et al, AUVSI, 2010



Recent Flight Demonstrators

- Lockheed Martin/AFRL: Body Freedom Flutter (BFF)
- **NASA/Lockheed Martin: X-56A Multi-Utility Tech. Testbed (MUTT)**
 - Ref: Schaefer, ACGSC, 2018



Recent Flight Demonstrators

- Lockheed Martin/AFRL: Body Freedom Flutter (BFF)
- NASA/Lockheed Martin: X-56A Multi-Utility Tech. Testbed (MUTT)
- Flutter Free FLight Envelope eXpansion for ecOnomical Performance improvement (FlexOp)
 - EU 2020 Horizon Project with B. Vanek as PI (MTA-Sztaki in Hungary)
 - Project Site: <https://flexop.eu/>

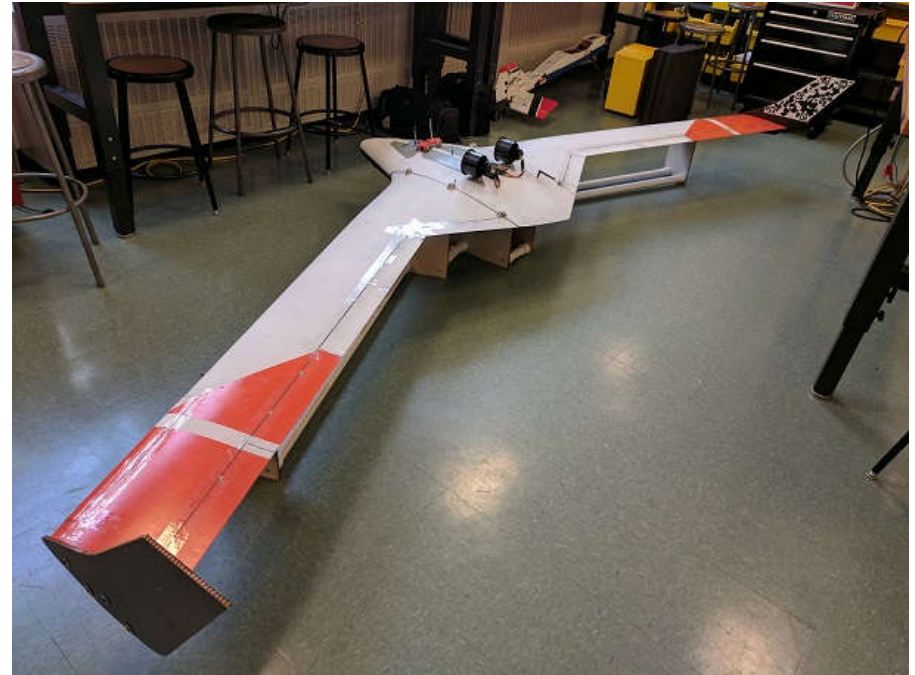


Performance Adaptive Aeroelastic Wing (PAAW)

- **Goal:** Suppress flutter, control wing shape and alter shape to optimize performance
 - Funding: NASA NRA with Dr. Jeffrey Ouellette as Tech. Monitor
 - Team: UMN, VT, STI, CMSoft, Aurora, Schmidt & Assoc.
 - Project Site: paaw.net

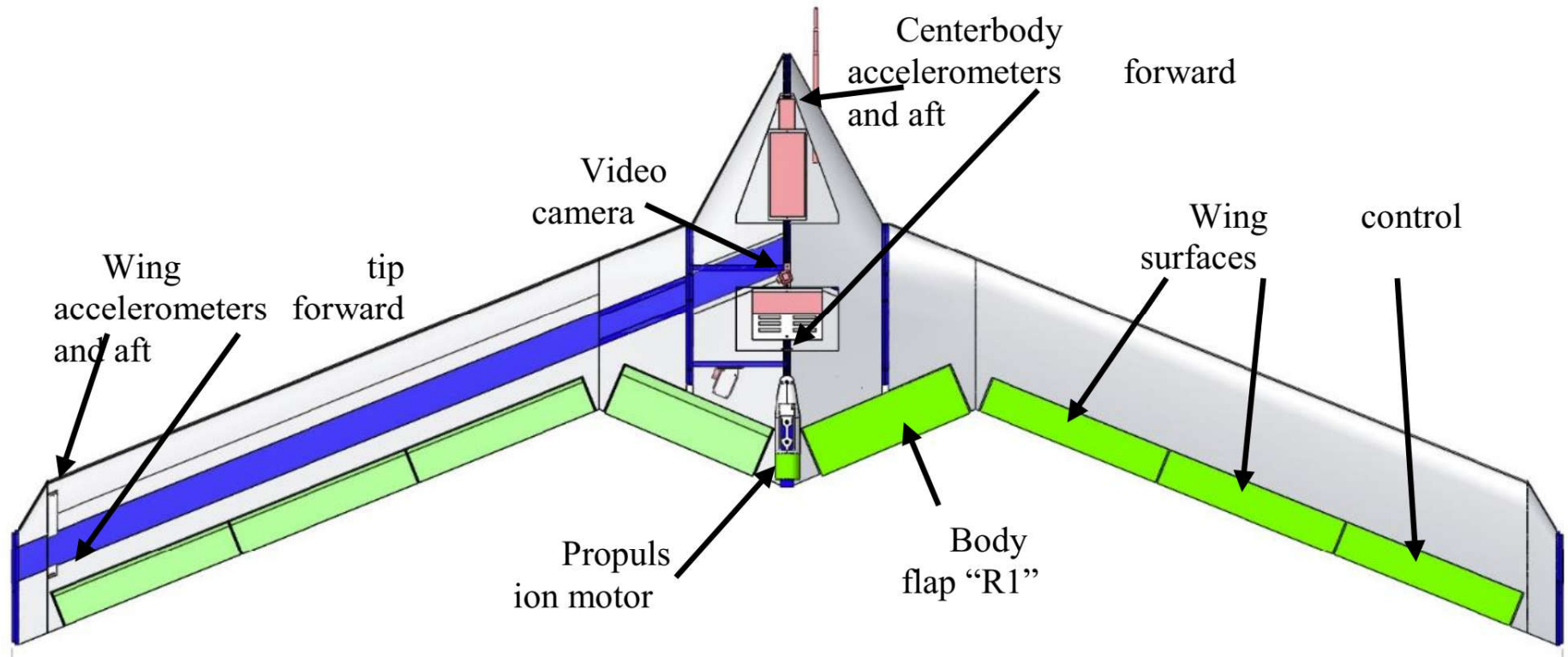


mAEWing1: BFF Replica



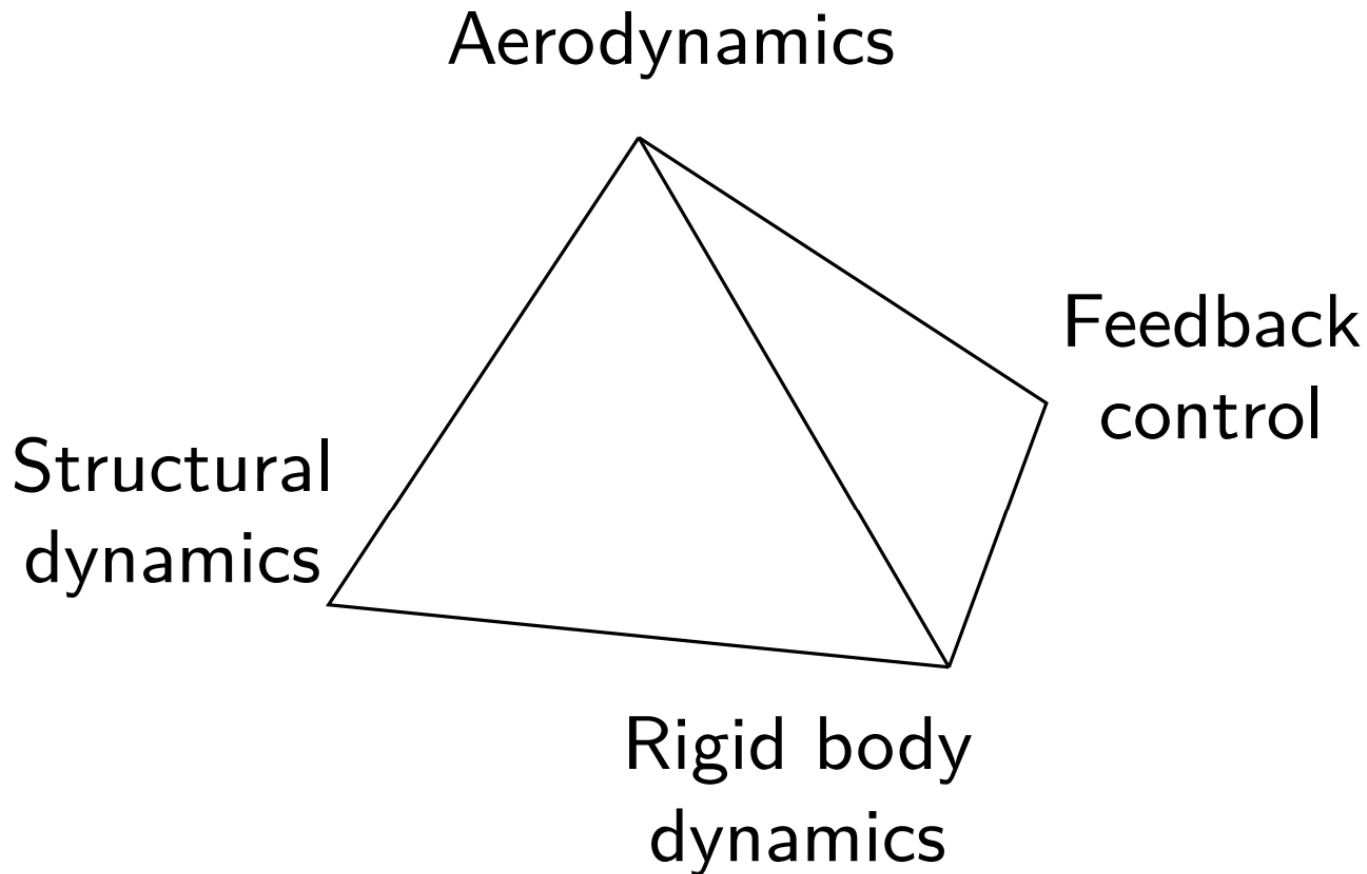
mAEWing2: Half-scale X-56A

mAEWing1 Sensor/Actuator Configuration



Ref: Regan & Taylor, AIAA 2016-1747

Modeling for Aeroservoelastic Systems



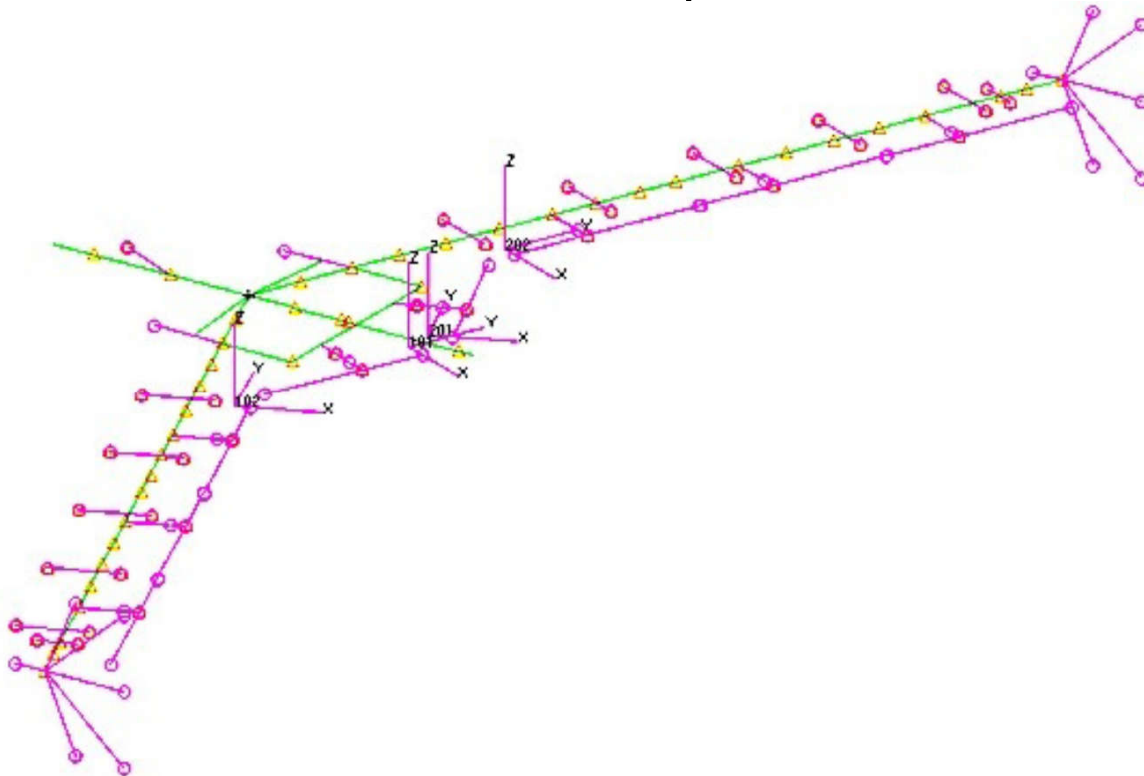
[Cooper & Wright 2015; Collar 1946]

mAEWing1 Models: NASTRAN (VT), CFD/CSD (CMSoft), IO Reduced Order Model (STI/CMSoft), Flight Dynamics (Schmidt)

Control-Oriented Modeling

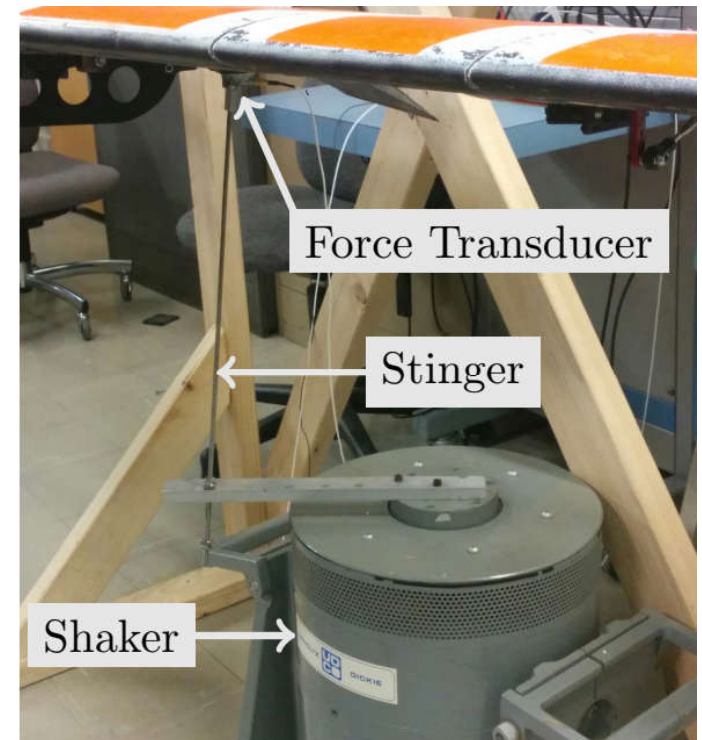
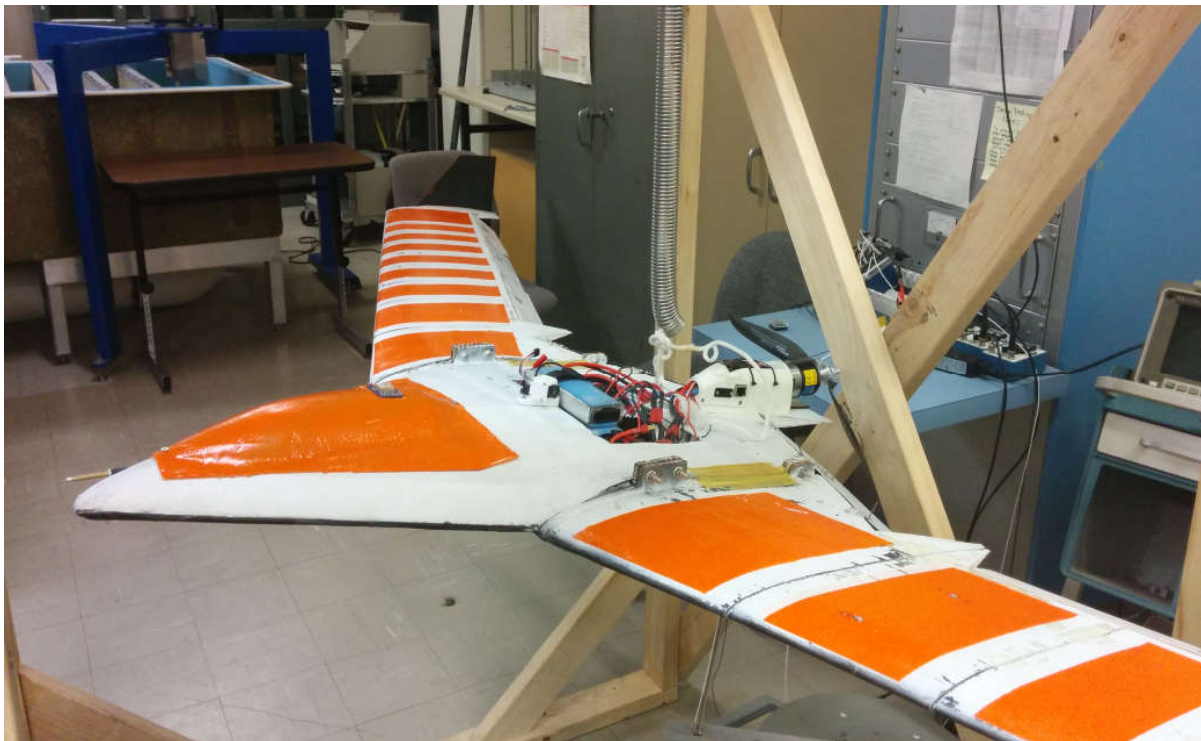
1. VT: Construct MSC NASTRAN model

- Ref: Schmidt, Zhao, Kapania, AIAA 2016-1748
- Finite-element model with rod / beam elements & unsteady aerodynamic model with double lattice.
- Initial model from CAD and simple static test data from UMN



Control-Oriented Modeling

1. VT: Construct MSC NASTRAN model
2. VT/UMN: Update NASTRAN FEM with ground test data
 - Ref: Gupta, Seiler, Danowsky, AIAA 2016-1753
 - Matlab Demo: “Modal Analysis of a Flexible Flying Wing Aircraft”, Demonstrates frequency domain fitting in System ID Toolbox



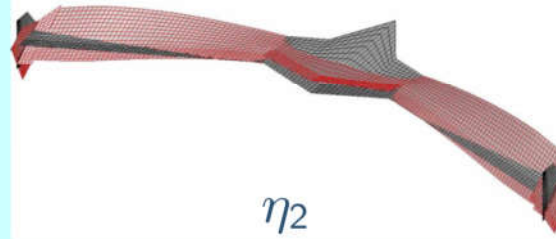
Control-Oriented Modeling

1. VT: Construct MSC NASTRAN model
2. VT/UMN: Update NASTRAN FEM with ground test data
3. VT: Obtain mode shapes & frequencies from NASTRAN
 - Ref: Schmidt, Zhao, Kapania, AIAA 2016-1748

Structural modes [Schmidt et al 2016 AIAA]



*Symmetric 1st
Bending ~6Hz*



*Symmetric 1st
Torsion ~12Hz*



*Symmetric 2nd
Bending ~19.5Hz*

Control-Oriented Modeling

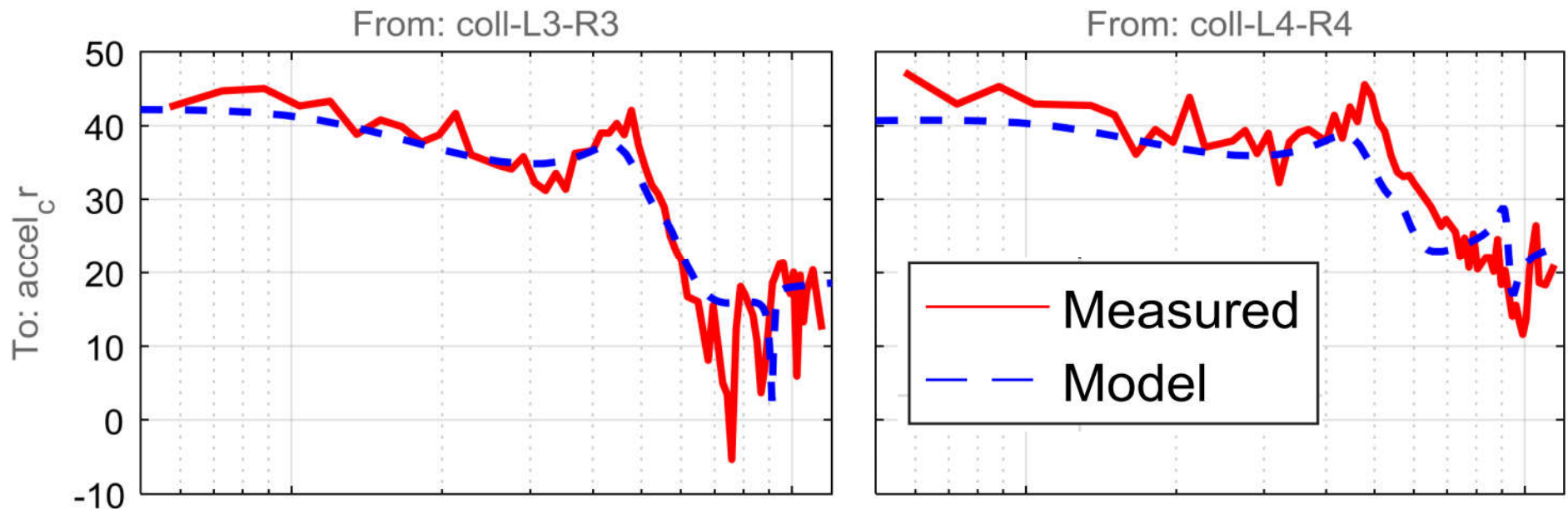
1. VT: Construct MSC NASTRAN model
2. VT/UMN: Update NASTRAN FEM with ground test data
3. VT: Obtain mode shapes & frequencies from NASTRAN
4. Schmidt: Construct low-order flight dynamics model
 - Ref: Schmidt, Zhao, Kapania, AIAA 2016-1748
 - Ref: Schmidt, Journal of Aircraft, 2016.
 - Parameter-varying model constructed using mean-axes
 - Model has longitudinal rigid body dyn. & three elastic modes

$$\begin{aligned}\dot{x} &= A(V_\infty)x + B(V_\infty)u \\ y &= C(V_\infty)x + D(V_\infty)u\end{aligned}$$

$$\text{where } x = [u \quad \alpha \quad \theta \quad q \mid \eta_1 \quad \dot{\eta}_1 \quad \eta_2 \quad \dot{\eta}_2 \quad \eta_3 \quad \dot{\eta}_3]^T$$

Control-Oriented Modeling

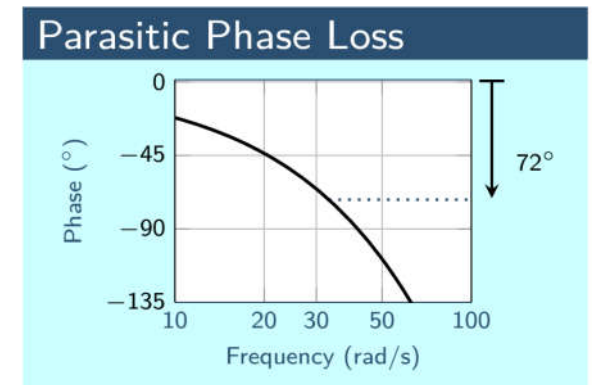
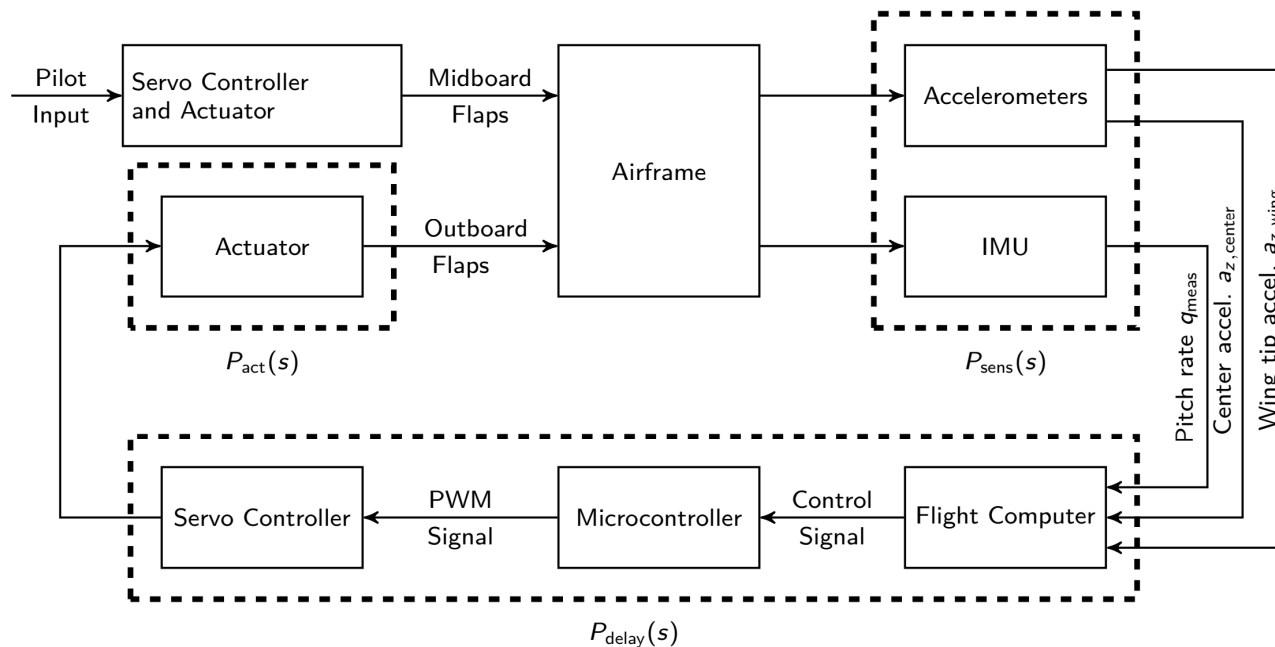
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3. VT: Obtain mode shapes & frequencies from NASTRAN
4. Schmidt: Construct low-order flight dynamics model
5. STI/UMN/Schmidt: Grey-box ID from flight tests
 - Ref: Danowsky, Schmidt, Pfifer, AIAA 2017-1394



Bode mag (dB) from symmetric L3/R3 and L4/R4 to center body accel

Control-Oriented Modeling

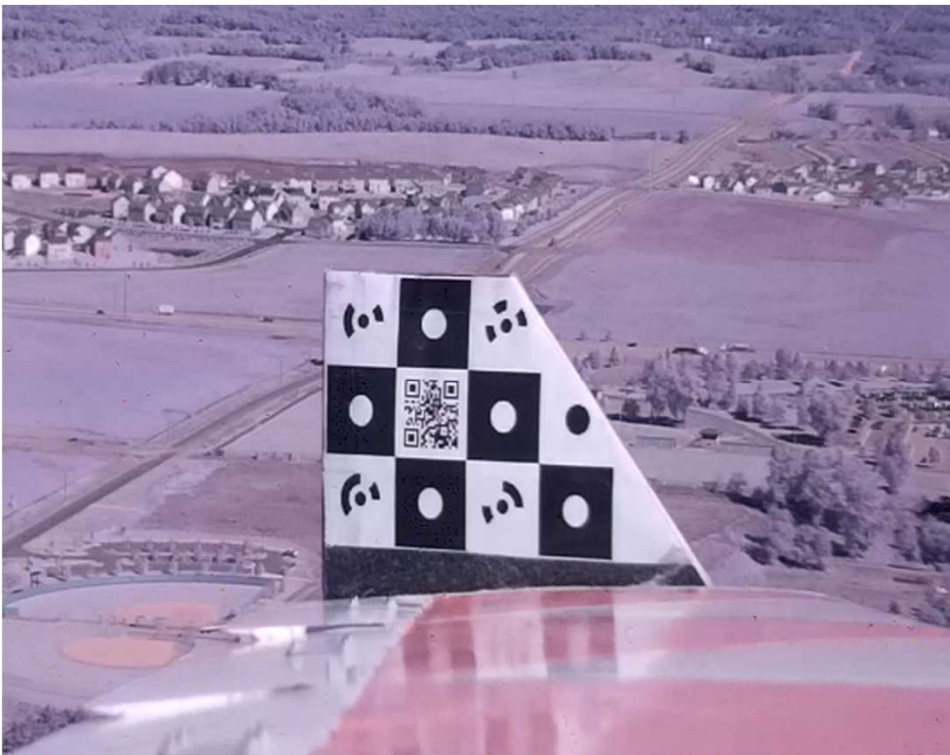
1. VT: Construct MSC NASTRAN model
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4. Schmidt: Construct low-order flight dynamics model
5. STI/UMN/Schmidt: Grey-box ID from flight tests
6. UMN: Component Modeling
 - Ref: Theis, Pfifer, Seiler, AIAA 2016-1751



Open-Loop Flutter at $\sim 30\text{m/s}$

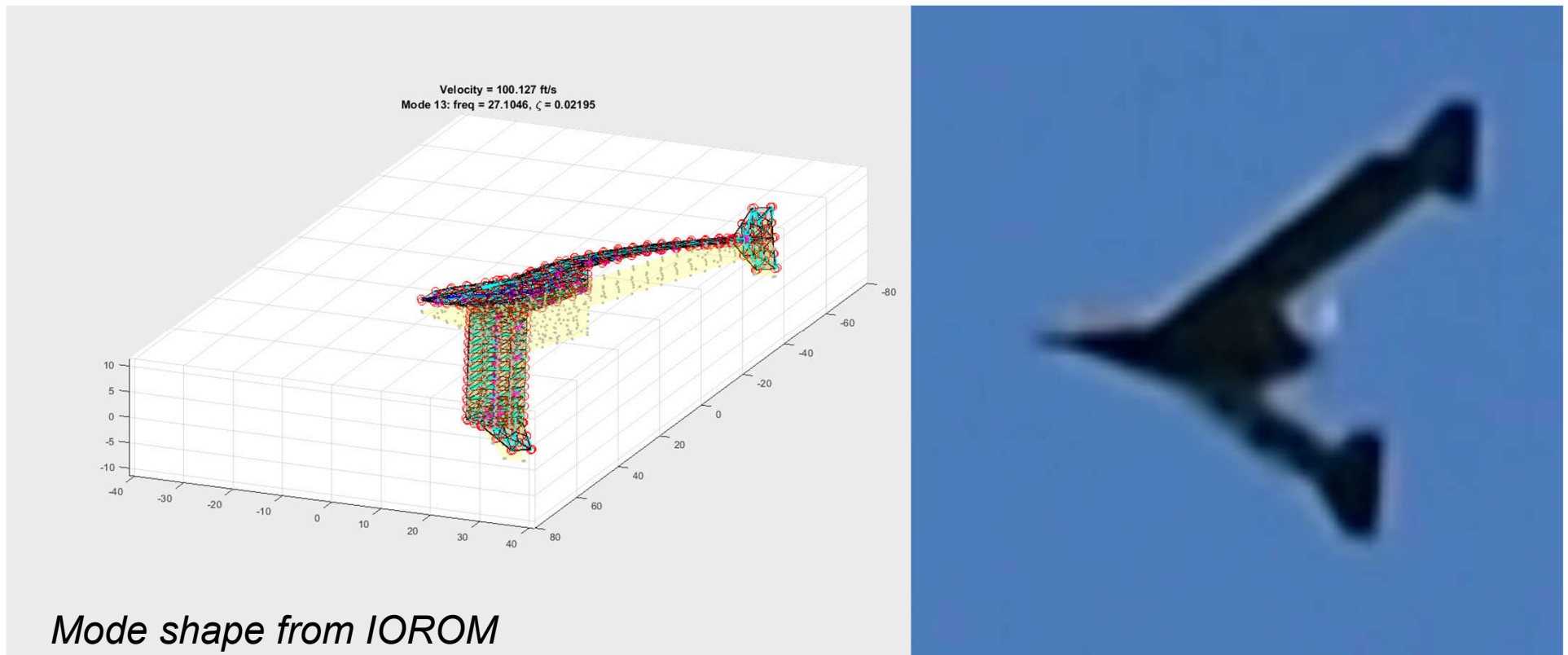
Model Predictions:

- Flight Dynamics (Schmidt): 29.1 m/s
- NASTRAN (VT): 29.5 m/s
- CFD/CSD (CMSOft): 30.8 m/s
- Input/Output Reduced Order Model (STI/CMSOft): 31.7m/s



Open-Loop Flutter at $\sim 30\text{m/s}$

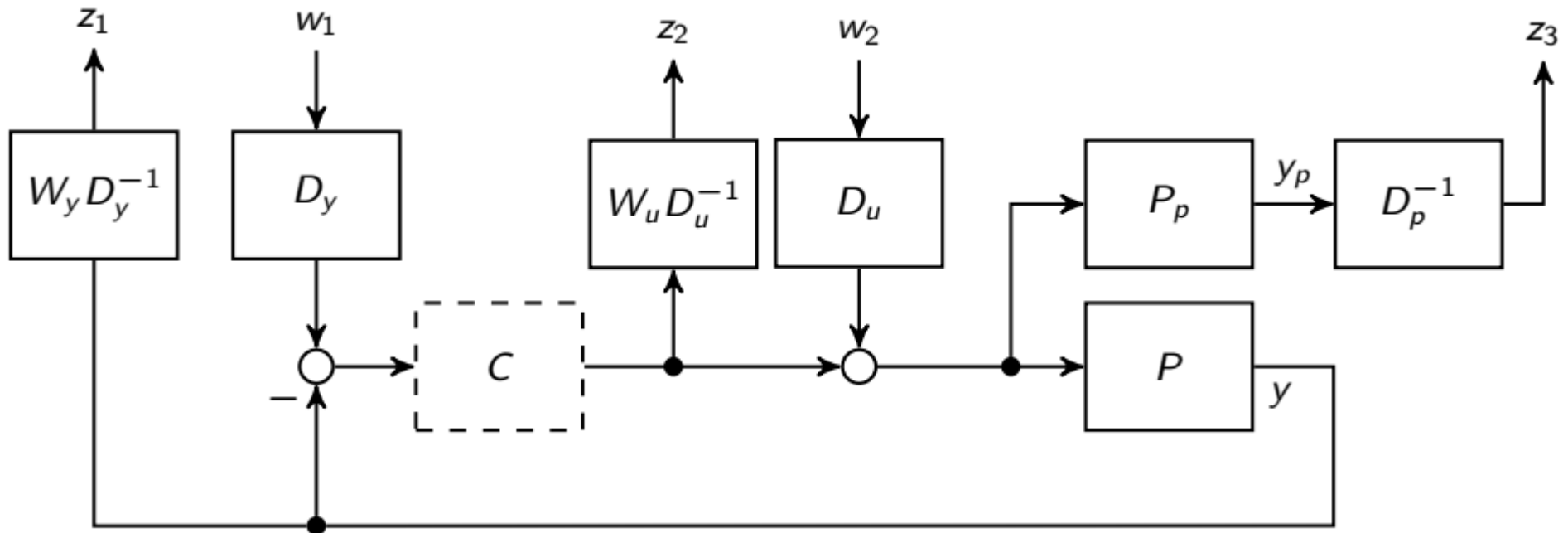
Mode Shape: Coupling of rigid body short period and 1st symmetric wing bending



Mode shape from IOROM

Active Flutter Suppression

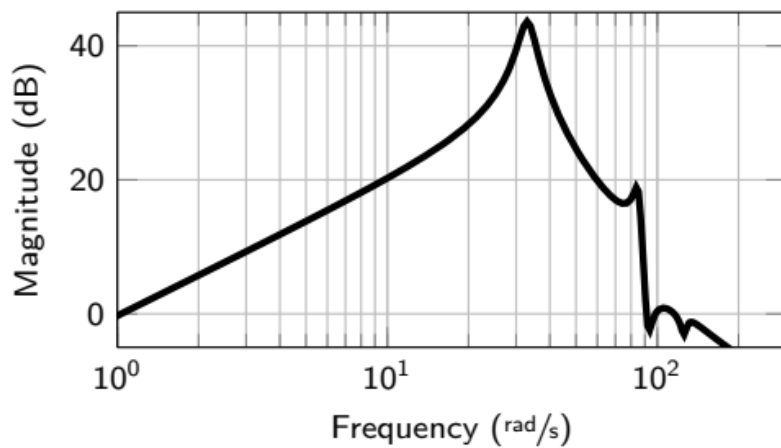
- Mixed sensitivity H_∞ Loopshaping
 - Ref: Theis, Pfifer, Seiler, AIAA 2016-1751
 - Ref: Theis, Ph.D., 2018



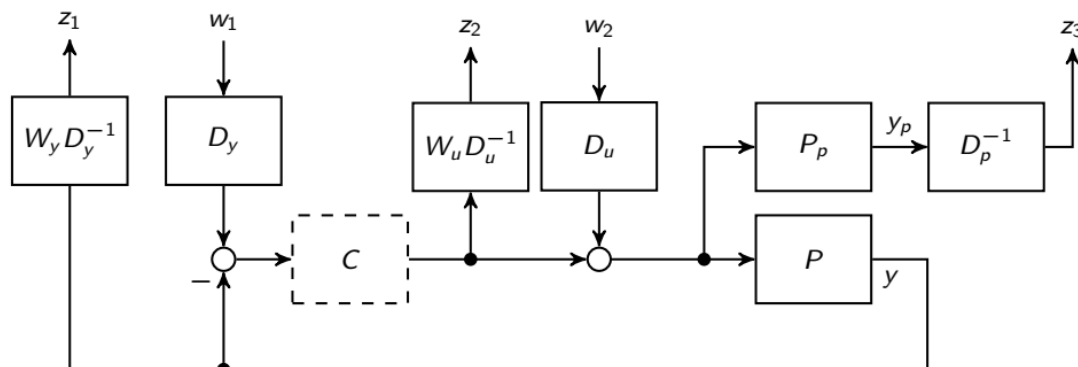
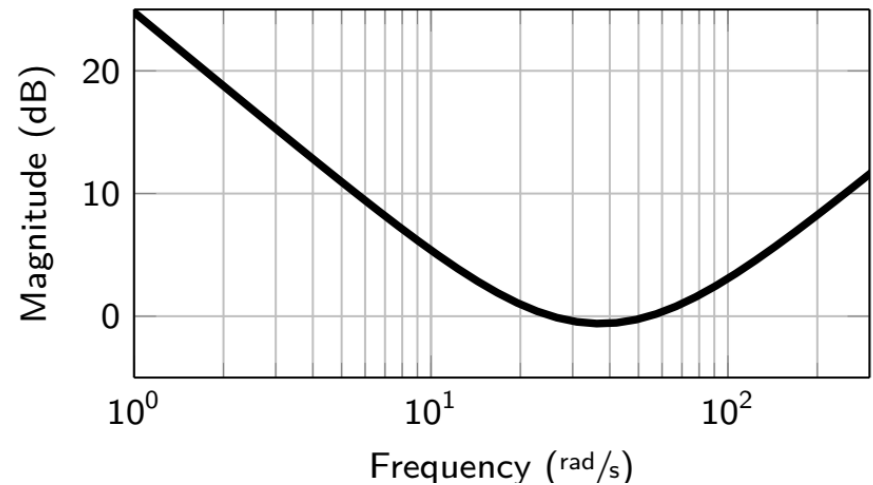
Active Flutter Suppression

- Modal velocity as performance output
- Bandpass penalty on control effort

Model P_p for structural mode $\dot{\eta}_1$



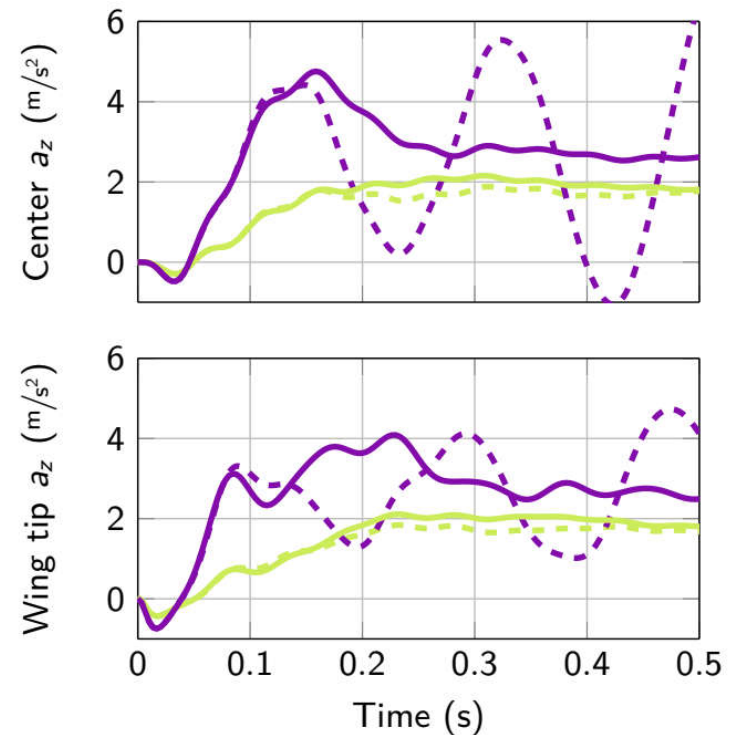
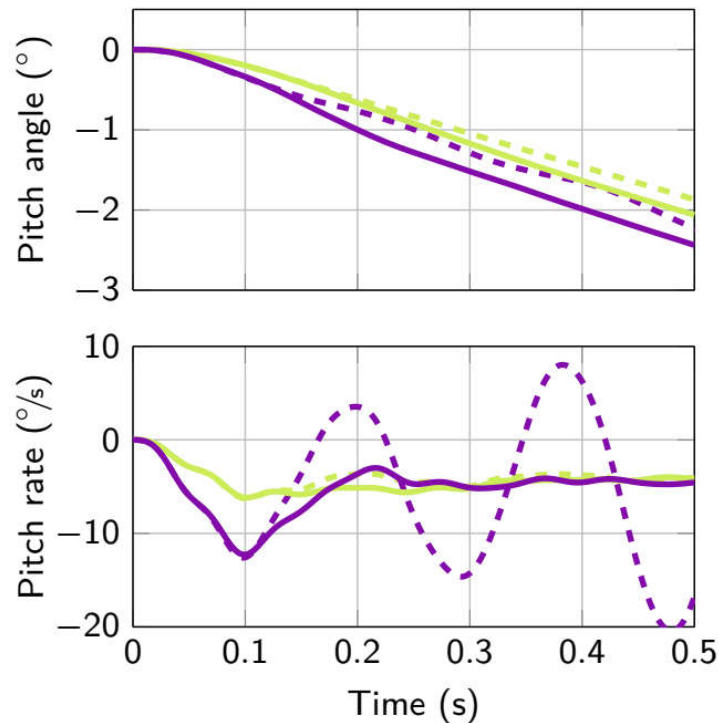
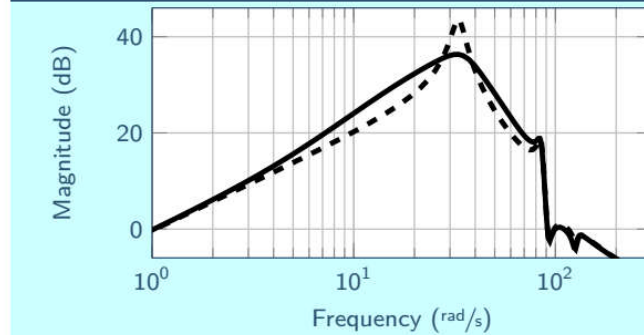
Weight W_u for control effort



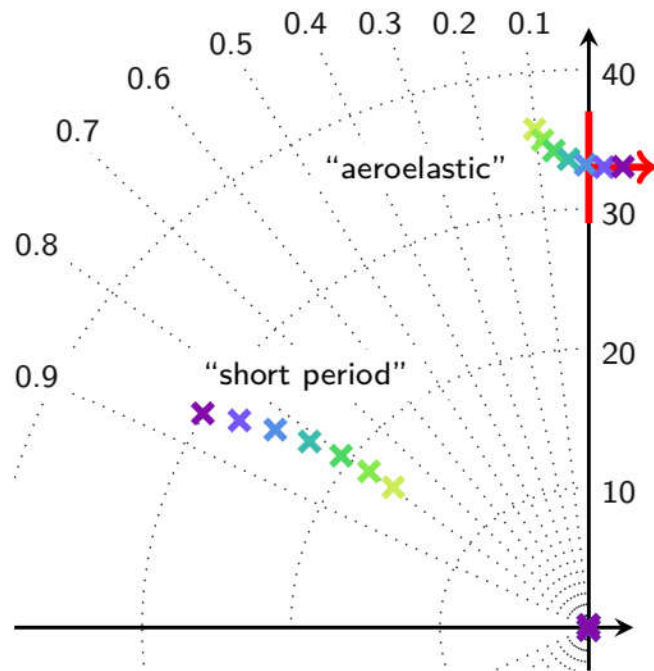
Closed-Loop Evaluation

- 24 m/s airspeed, open loop
- 24 m/s airspeed, closed loop
- 33 m/s airspeed, open loop
- 33 m/s airspeed, closed loop

Structural Sensitivity ($P_p S_i$)

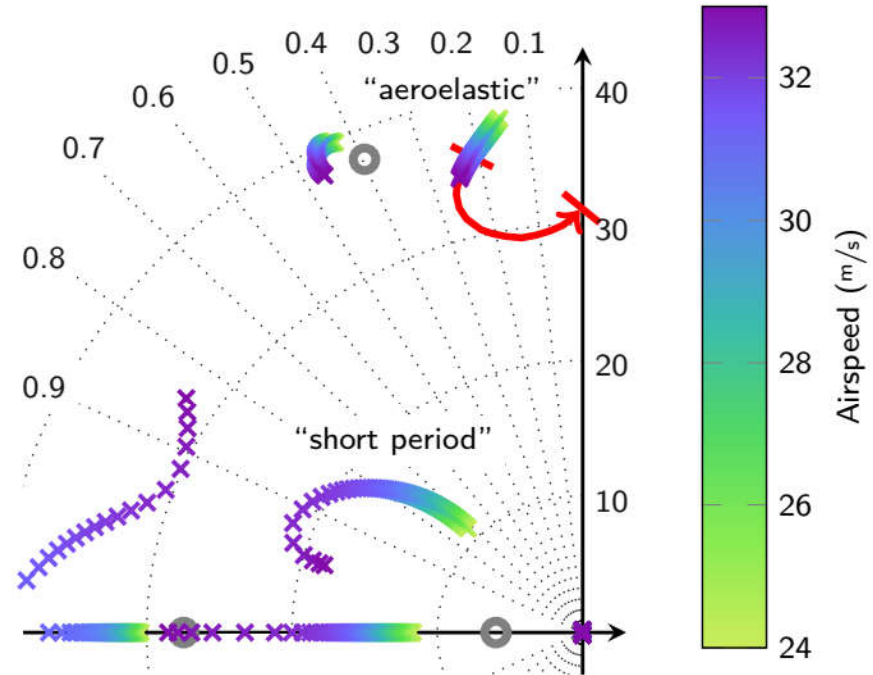


Closed-Loop Evaluation



Open Loop

flutter beyond 30 m/s airspeed



Closed Loop

envelope expansion to 43 m/s airspeed

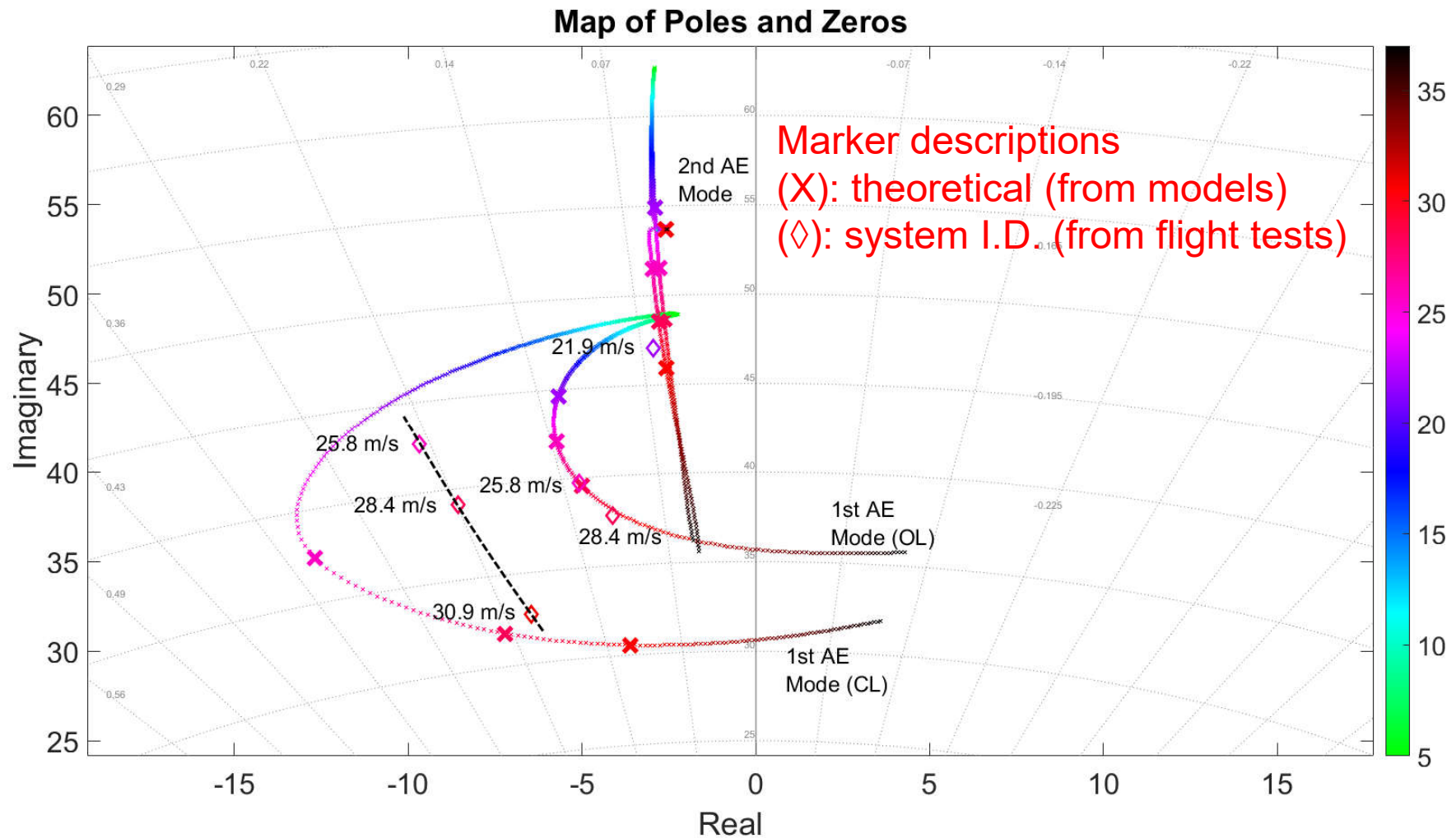
Closed-Loop Flight Tests

- Three controllers designed to increase damping to BFF mode at 23m/s
 - Hinf Controller (Retuned): Kotikalpudi, et al, AIAA 2018-3426)
 - MIDAAS: Danowsky (STI), 2017-4353
 - Classical Controller: Schmidt, Journal of GCD, 2016.

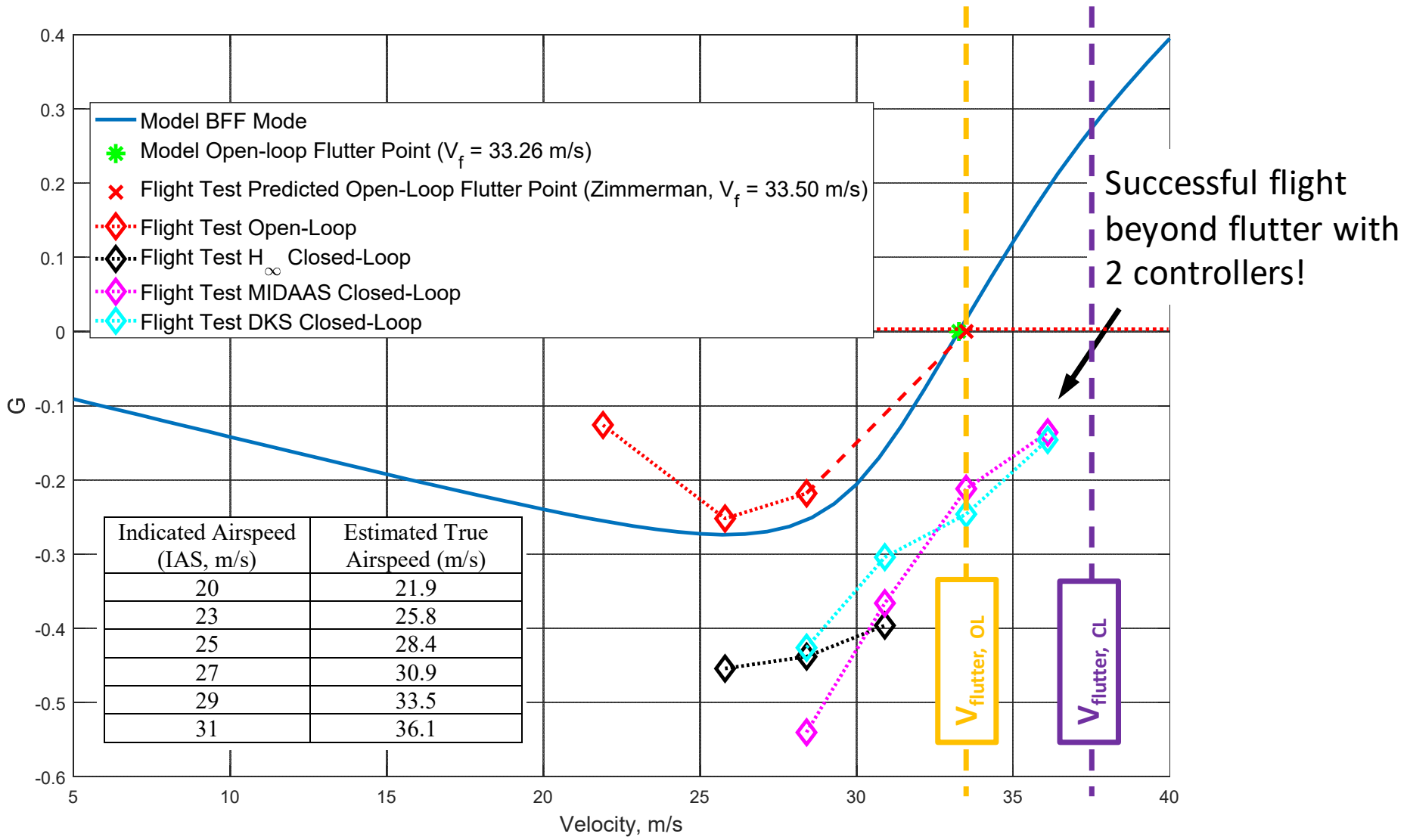
Closed-Loop Flight Tests

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 - MIDAAS: Danowsky (STI), 2017-4353
 - Classical Controller: Schmidt, Journal of GCD, 2016.
- Flight Tests
 - Ref: Danowsky, Kotikalpudi, Schmidt, Regan, Seiler, AIAA 2018-3427
 - Controllers tested at and above the designed airspeed.
 - All controllers added damping at the designed speed.
 - MIDAAS & classical designs flown above open-loop flutter speed.
 - Hinf controller did not increase flutter speed but this was an artifact of our design objective and flight test plan.

Pole Map for Retuned H_∞ Controller



Flight Test Summary



Next Steps

- **Robust Flutter Speed (RFS):** Airspeeds where active flutter control has 6dB/45deg margins on all inputs & outputs
 - Metric for safe flight envelope with active flutter suppression
 - Current: Restrict envelope to 20% below (open-loop) flutter speed

Next Steps

- **Robust Flutter Speed (RFS):** Airspeeds where active flutter control has 6dB/45deg margins on all inputs & outputs
 - Metric for safe flight envelope with active flutter suppression
 - Current: Restrict envelope to 20% below (open-loop) flutter speed
- Redesign all 3 controllers to maximize robust flutter speed
 - Design complicated by second bending mode at higher speeds
- **Preliminary Results:**
 - H_∞ control achieves robust/absolute flutter speeds of 43/41 m/s
 - Similar but slightly lower speeds for MIDAAS & classical designs
 - Tested in linear parameter varying sim with actuator limits.
 - Flight tests planned for spring 2019

Outline

- Flutter Suppression on Flexible Aircraft
- **Robustness of Time-Varying Systems**
 - **Joint work with M. Arcak, A. Packard, M. Moore, and C. Meissen at UC, Berkeley.**
 - **Funded by ONR BRC with B. Holm-Hansen at Tech. Monitor**
- Robustness in Reinforcement Learning

Time-Varying Systems



Wind Turbine
Periodic /
Parameter-Varying



Flexible Aircraft
Parameter-Varying



Vega Launcher
Time-Varying
(Source: ESA)



Robotics
Time-Varying
(Source: ReWalk)

Issue: Few numerically reliable methods to assess the robustness of time-varying systems.

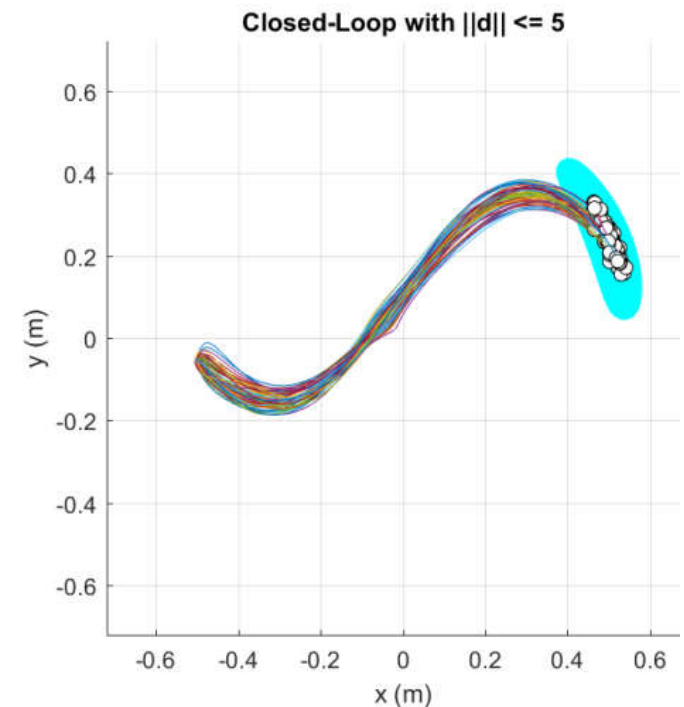
Finite Horizon Analysis

Goal: Assess the robustness of linear time-varying (LTV) systems on finite horizons.

Issue: Classical Gain/Phase Margins focus on (infinite horizon) stability and frequency domain concepts.

Instead focus on:

- Finite horizon metrics, e.g. induced gains and reachable sets.
- Effect of disturbances and model uncertainty (D-scales, IQCs, etc).
- Time-domain analysis conditions.



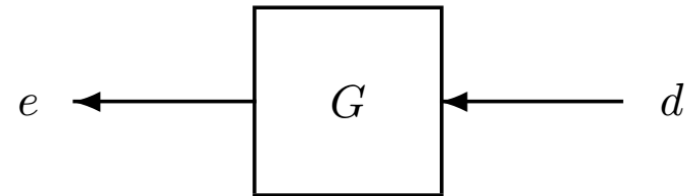
(Nominal) Finite Horizon Analysis

Nominal System

$$\dot{x}(t) = A(t)x(t) + B(t)d(t)$$

$$e(t) = C(t)x(t)$$

$$x(0) = 0$$



Analysis Objective

Derive bound on $\|e(T)\|_2$ that holds for all disturbances $\|d\|_{2,[0,T]} \leq 1$ on the horizon $[0,T]$.

Nominal Analysis with Dissipation Inequalities

Theorem [1]

If there exists $P(t) = P(t)^T$ such that $P(T) = C(T)^T C(T)$ and $V(x, t) := x^T P(t)x$ satisfies

$$\frac{d}{dt}V(x, t) - \gamma^2 d(t)^T d(t) \leq 0 \quad \forall t \in [0, T]$$

then $\|e(T)\|_2 \leq \gamma$

Proof

Integrate from $t = 0$ to $t = T$:

$$\underbrace{V(x, T)}_{e(T)^T e(T)} - \underbrace{V(x, 0)}_{=0} \leq \gamma^2 \int_0^T d(t)^T d(t) dt \quad \blacksquare$$

[1] Moore, Finite Horizon Robustness Analysis using IQCs, MS Thesis, Berkeley, 2015.

[2] Willems, Dissipative Dynamical Systems: Parts i and ii, 1972.

[3] Green & Limebeer, Linear Robust Control, 1995.

Nominal Analysis with Dissipation Inequalities

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Comments

- Time-varying matrix inequality can be “solved” via convex optimization using basis functions for $P(t)$, gridding on time.
- Extensions to parameter varying [4] and periodic [5] systems.

[1] Moore, Finite Horizon Robustness Analysis using IQCs, MS Thesis, Berkeley, 2015.

[2] Willems, Dissipative Dynamical Systems: Parts i and ii, 1972.

[3] Green & Limebeer, Linear Robust Control, 1995.

[4] Wu. Control of Linear Parameter Varying Systems. PhD thesis, Berkeley, 1995.

[5] Bittanti & Colaneri, Periodic Systems. Springer, 2009.

(Robust) Finite-Horizon Analysis

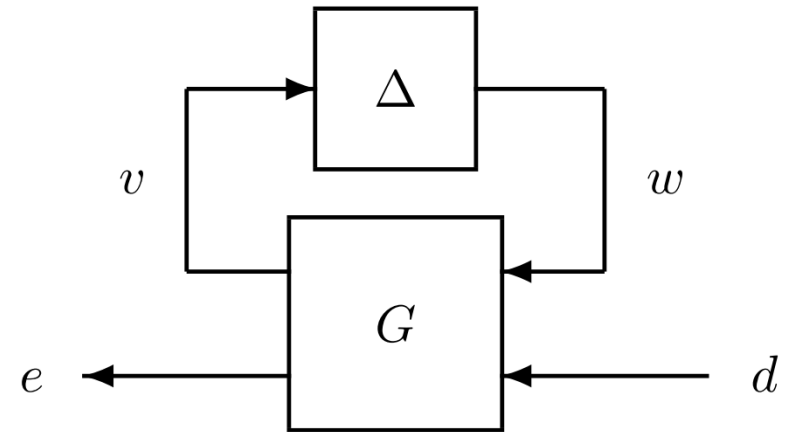
Uncertain Time-Varying Model

$$\dot{x}(t) = A(t)x(t) + B_1(t)w(t) + B_2(t)d(t)$$

$$v(t) = C_1(t)x(t)$$

$$e(t) = C_2(t)x(t)$$

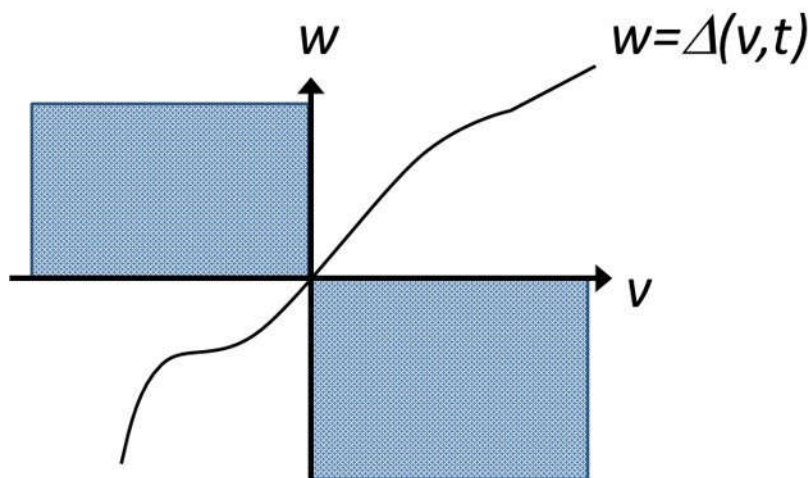
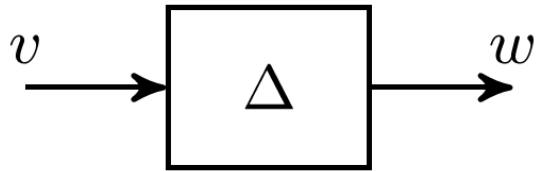
$$x(0) = 0$$



Analysis Objective

Derive bound on $\|e(T)\|_2$ that holds for all disturbances $\|d\|_{2,[0,T]} \leq 1$ and uncertainties $\Delta \in \mathbf{\Delta}$ on the horizon $[0,T]$.

Example: Passive Uncertainty



$w = \Delta(v, t)$ is a passive system
(pointwise in time).



$$2v(t)^T w(t) \geq 0 \quad \forall t$$



$$\begin{bmatrix} v(t) \\ w(t) \end{bmatrix}^T \begin{bmatrix} 0 & I \\ I & 0 \end{bmatrix} \begin{bmatrix} v(t) \\ w(t) \end{bmatrix} \geq 0 \quad \forall t$$

Pointwise Quadratic Constraint

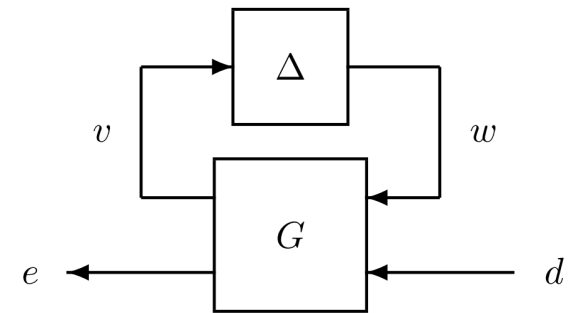
Robust Finite Horizon Analysis

Theorem [1]

Assume $x(0) = 0$, $\|d\|_{2,[0,T]} \leq 1$, and Δ is pointwise passive. If there exists $P(t) = P(t)^T$ such that $P(T) = C(T)^T C(T)$ and $V(x, t) = x(t)^T P(t)x(t)$ satisfies:

$$\frac{d}{dt}V(x, t) - \gamma^2 d(t)^T d(t) + \begin{bmatrix} v(t) \\ w(t) \end{bmatrix}^T \begin{bmatrix} 0 & I \\ I & 0 \end{bmatrix} \begin{bmatrix} v(t) \\ w(t) \end{bmatrix} \leq 0 \quad \forall t \in [0, T]$$

then $\|e(T)\|_2 \leq \gamma$.



Proof

Integrate from $t = 0$ to $t = T$ to show:

$$\underbrace{\underbrace{V(x, T)}_{e(T)^T e(T)} - \underbrace{V(x, 0)}_{=0}}_{\geq 0} + \underbrace{\int_0^T \begin{bmatrix} v(t) \\ w(t) \end{bmatrix}^T \begin{bmatrix} 0 & I \\ I & 0 \end{bmatrix} \begin{bmatrix} v(t) \\ w(t) \end{bmatrix} dt}_{\geq 0} \leq \gamma^2 \int_0^T d(t)^T d(t) dt$$

[1] Moore, Finite Horizon Robustness Analysis using IQCs, MS Thesis, Berkeley, 2015.

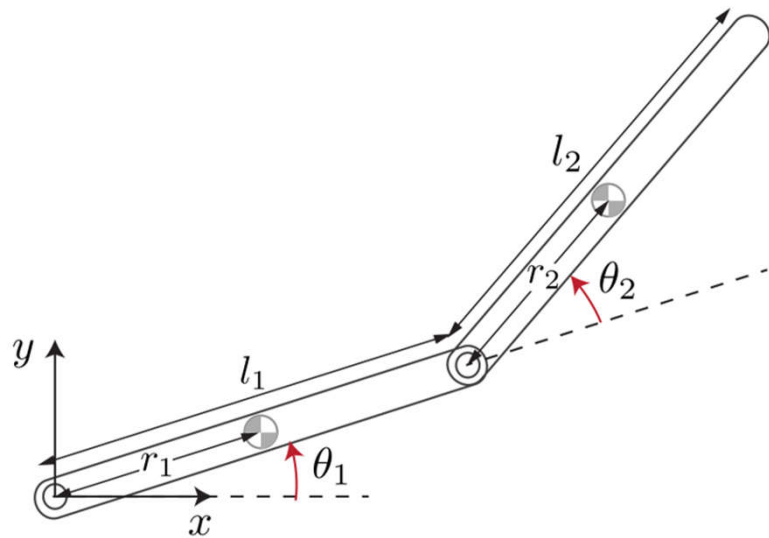
More General Results

- **Approach**
 - Combine dissipation inequalities (Willems) and more general integral quadratic constraint framework (Megretski & Rantzer)
 - Requires technical factorization result to connect incorporate frequency domain IQCs into this time-domain analysis [1,2].
 - Developed numerical algorithms that combine differential linear matrix inequalities and Riccati differential equations [3].
- **Extensions**
 - Similar methods can be used to assess robustness of linear parameter varying (LPV) [4,5] and periodic systems [6].

References

- [1] Seiler, Stability Analysis with Dissipation Inequalities and Integral Quadratic Constraints, TAC, 2015.
- [2] Hu, Lacerda, Seiler, Rob. Analysis of Uncertain Discrete-Time Sys. with Dissipation Ineq. and IQCs, IJRNC, 2016.
- [3] Seiler, Moore, Meissen, Arcaç, Packard, Finite Horizon Robustness Analysis of LTV Systems Using Integral Quadratic Constraints, arXiv / Automatica, 2018.
- [4] Pfifer & Seiler, Less Conservative Robustness Analysis of LPV Systems Using IQCs, IJRNC, 2016.
- [5] Hjartarson, Packard, & Seiler, LPVTools: A Toolbox for Modeling, Analysis, and Synthesis of LPV Systems, '15.
- [6] Fry, Farhood, & Seiler, IQC-based robustness analysis of discrete-time LTV systems, IJRNC 2017.

Two-Link Robot Arm



Two-Link Diagram [MZS]

Nonlinear dynamics [MZS]:

$$\dot{\eta} = f(\eta, \tau, d)$$

where

$$\eta = [\theta_1, \dot{\theta}_1, \theta_2, \dot{\theta}_2]^T$$

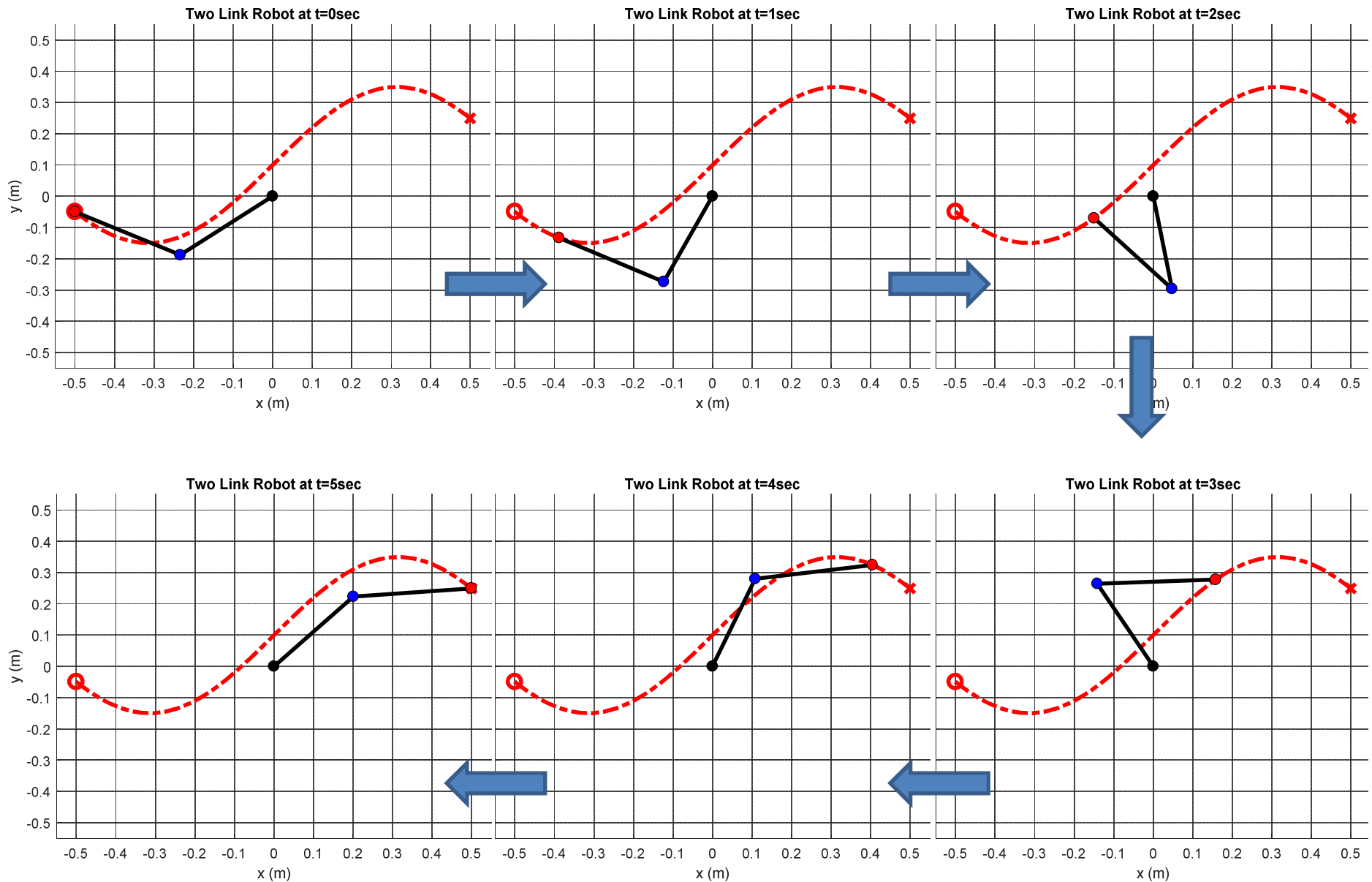
$$\tau = [\tau_1, \tau_2]^T$$

$$d = [d_1, d_2]^T$$

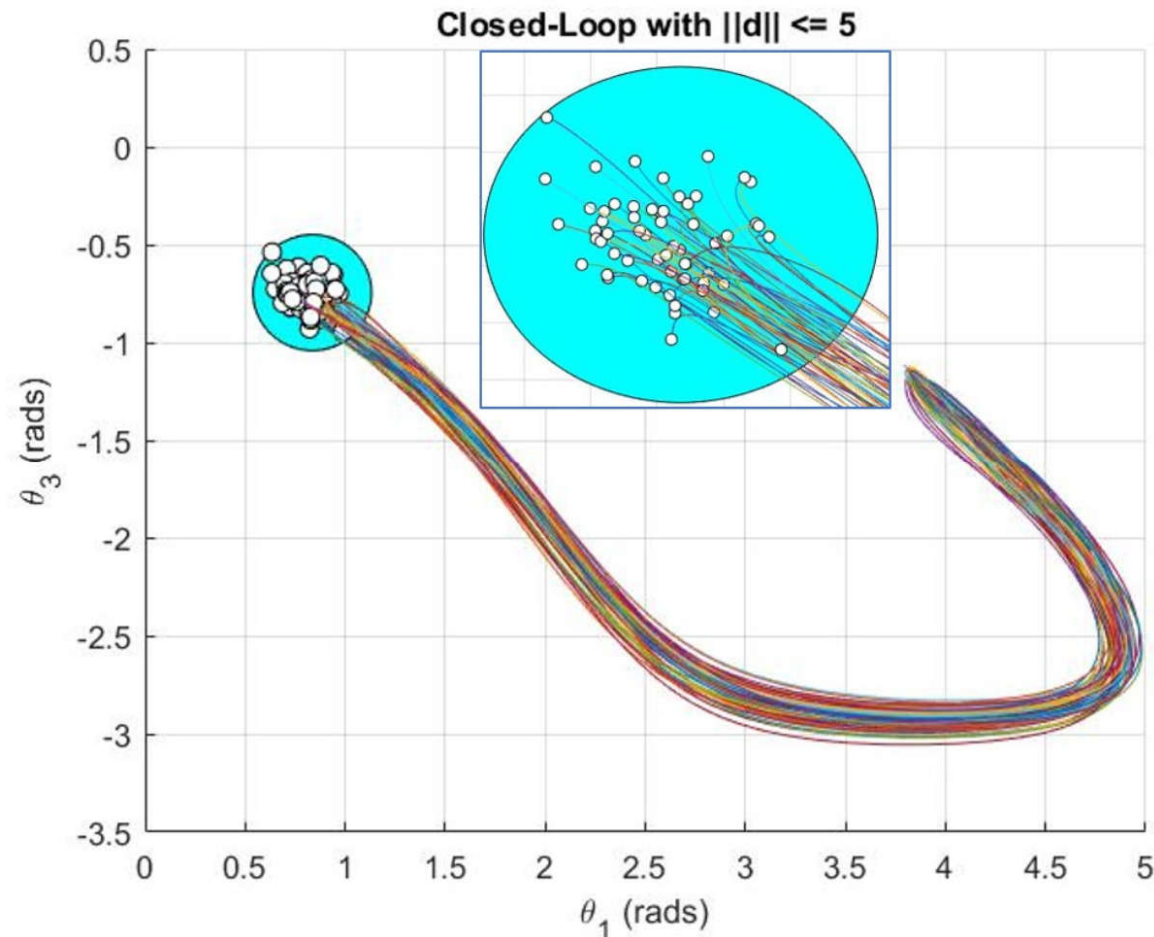
τ and d are control torques and disturbances at the link joints.

[MZS] R. Murray, Z. Li, and S. Sastry. *A Mathematical Introduction to Robot Manipulation*, 1994.

Nominal Trajectory (Cartesian Coords.)



Effect of Disturbance / Uncertainty



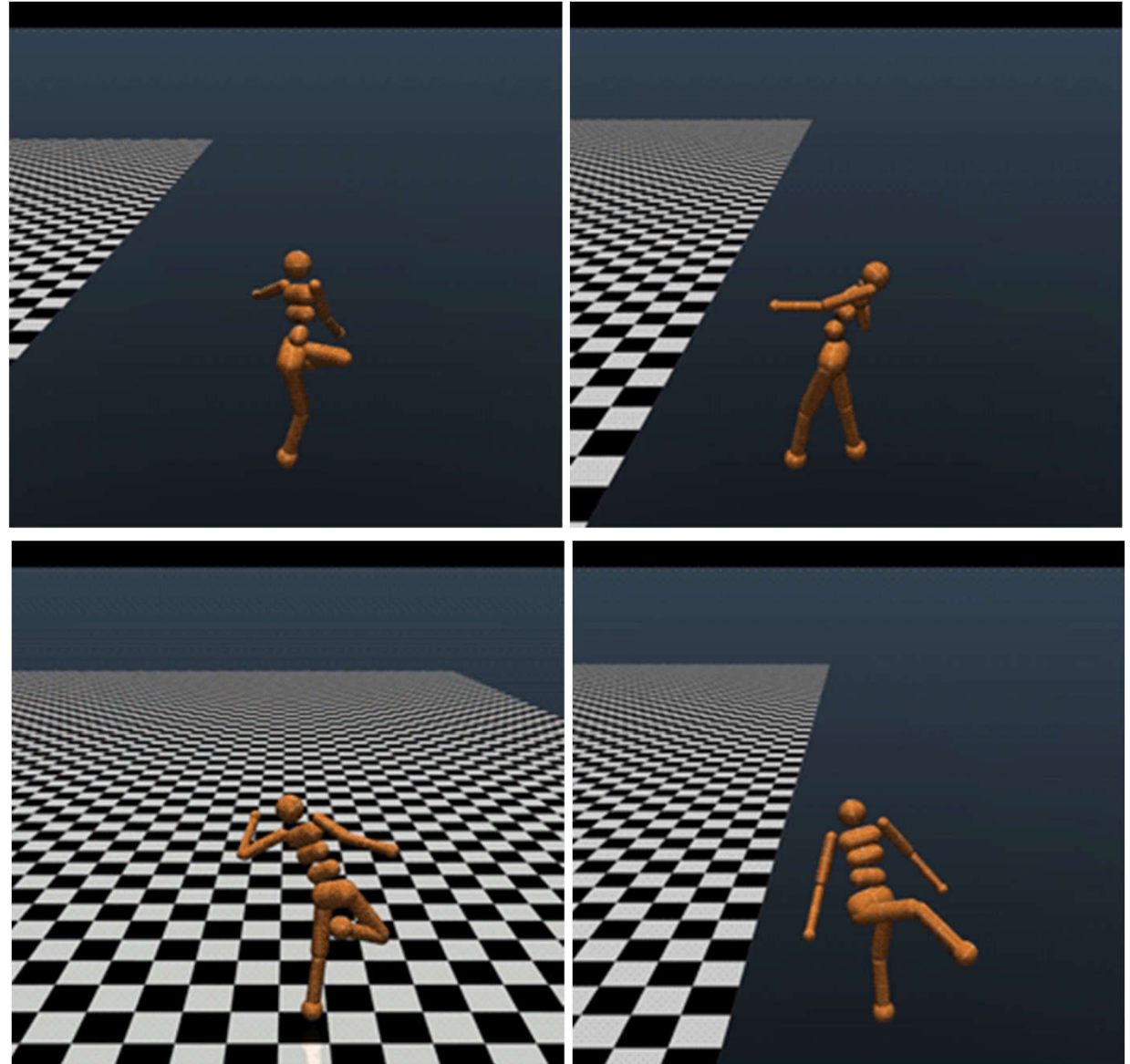
- Bound on final position computed in 102sec.
- Numerically robust algorithm to construct the worst-case disturbance
- Ref: Iannelli, Seiler & Marcos, submitted to AIAA JGCD.

Outline

- Flutter Suppression on Flexible Aircraft
- Robustness of Finite Horizon Systems
- **Robustness in Reinforcement Learning**

Benchmark: Robotic Walking on MuJoCo

- *Training can exploit flaws in the simulator.*
 - *B. Recht, arXiv, 2018*
- *There are many model-based methods to ensure robustness.*
- **Goal:** *Develop model-free methods to ensure robustness.*
 - *Venkataraman & Seiler, Recovering Robustness in Model-Free Reinforcement Learning, submitted to 2019 ACC.*



Linear Quadratic Gaussian (LQG)

Minimize $J_{LQG}(u) := \lim_{N \rightarrow \infty} \frac{1}{N} E \left[\sum_{t=0}^N x_t^T Q x_t + u_t^T R u_t \right]$

Subject To: $x_{t+1} = A x_t + B u_t + B_w w_t$
 $y_t = C x_t + v_t$

The optimal controller has an observer/state-feedback form

$$\hat{x}_{t+1} = A \hat{x}_t + B u_t + L (y_t - C \hat{x}_t)$$
$$u_t = -K \hat{x}_t$$

Gains (K, L) computed by solving two Riccati equations.

This solution is model-based, i.e. it uses data A, B, C , etc

Reinforcement Learning

- Partially Observable Markov Decision Processes (POMDPs)
 - Set of states, S
 - Set of actions, A
 - Reward function, $r: S \times A \rightarrow \mathbb{R}$
 - State transition probability, T
 - Set of observations and observation probability, O
- Many methods to synthesize a control policy from input/output data to maximize the cumulative reward

$$J_{RL}(a) := E \left[\sum_{t=0}^N r(s_t, a_t) \right]$$

- The LQG problem is a special case of this RL formulation

Doyle's Example ('78 TAC)

- LQR state-feedback regulators have provably good margins.
- Doyle's example shows that LQG regulators can have arbitrarily small input margins.

Doyle's Example ('78 TAC)

- LQR state-feedback regulators have provably good margins.
- Doyle's example shows that LQG regulators can have arbitrarily small input margins.
- Doyle's example can also be solved within RL framework using direct policy search:

$$\begin{aligned}z_{t+1} &= A_K(\theta)z_t + B_K(\theta)y_t \\ u_t &= C_K(\theta)z_t\end{aligned}$$

where

$$A_K(\theta) := \begin{bmatrix} 0 & \theta_1 \\ 1 & \theta_2 \end{bmatrix}, B_K(\theta) := \begin{bmatrix} 1 \\ 0 \end{bmatrix}, C_K^T(\theta) := \begin{bmatrix} \theta_3 \\ \theta_4 \end{bmatrix}$$

- RL will converge to the optimal LQG control with infinite data collection. Thus RL can also have poor margins.

Implications?

Recovering Robustness

- Increase process noise during training?
 - This causes margins to decrease on Doyle's example
 - Process noise is not model uncertainty

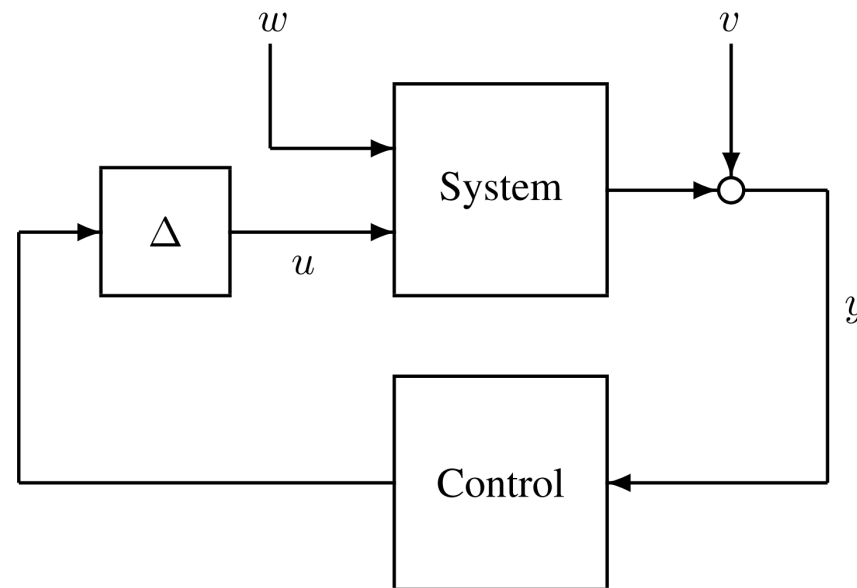
Recovering Robustness

- Increase process noise during training?
- Modify reward to increase state penalty or decrease control penalty?
 - Again, this causes margins to decrease on Doyle's example
 - Trading performance vs. robustness via the reward function can be difficult or counter-intuitive

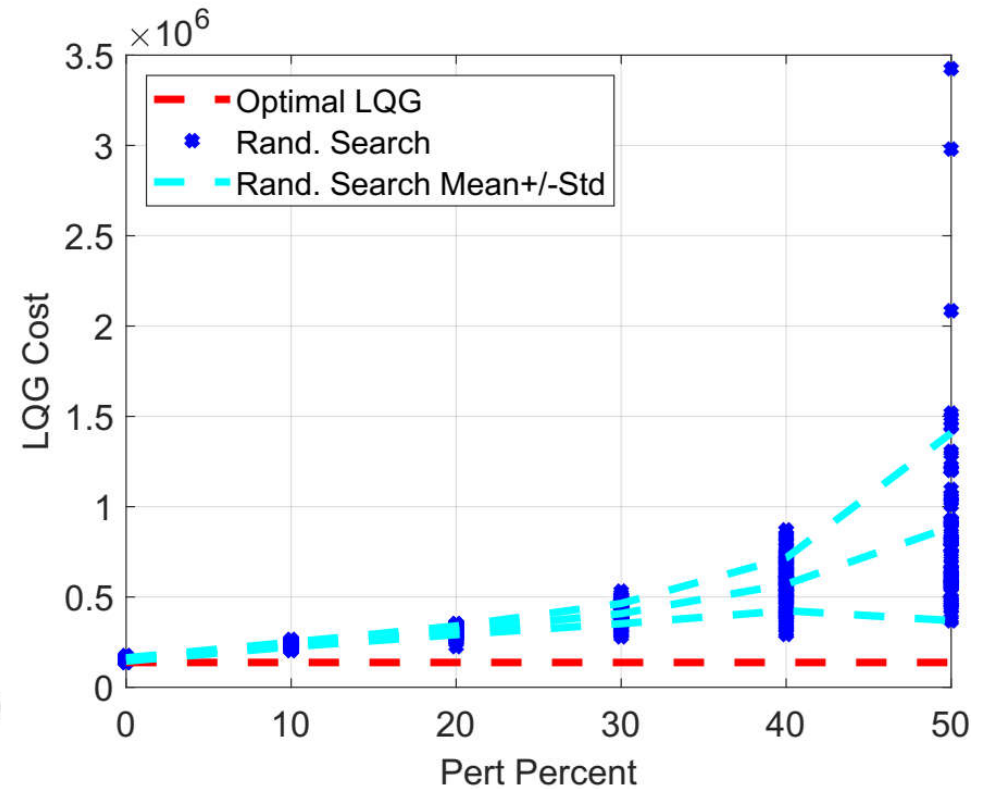
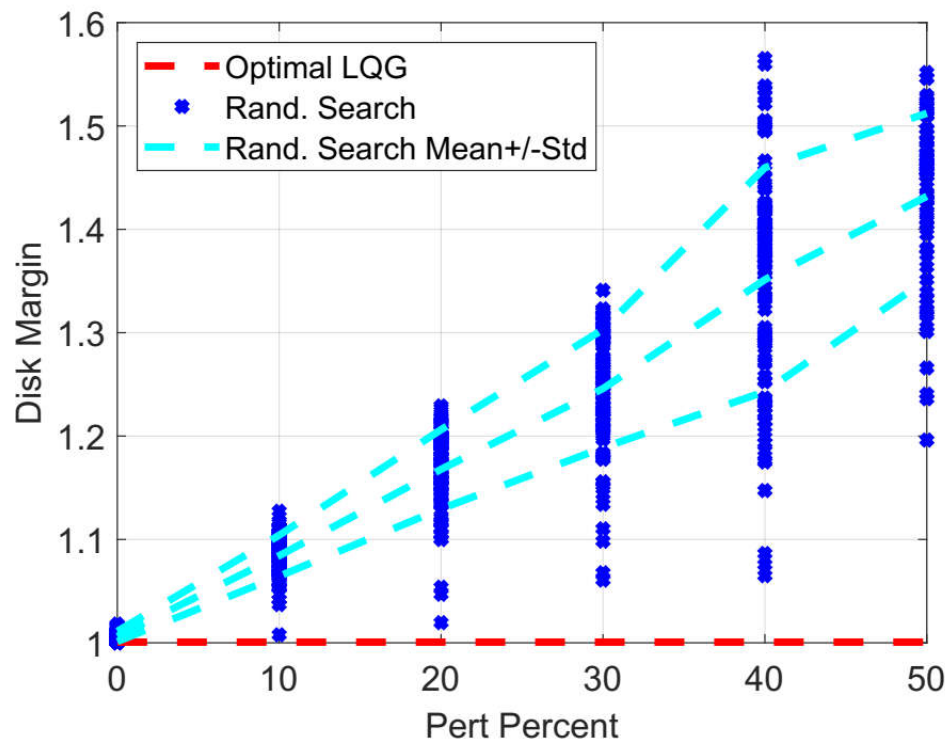
Recovering Robustness

- Increase process noise during training?
- Modify reward to increase state penalty or decrease control penalty?
- Inject synthetic gain/phase variations at the plant input (and output?) during the training phase.

$\Delta=1+\delta$ where
 δ is $U[-b,b]$

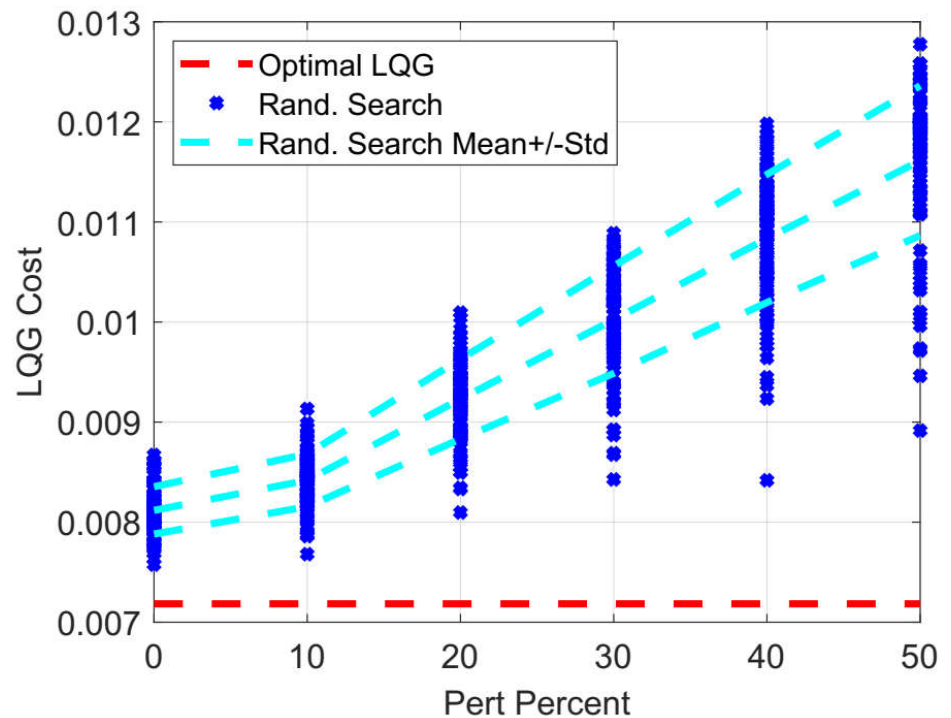
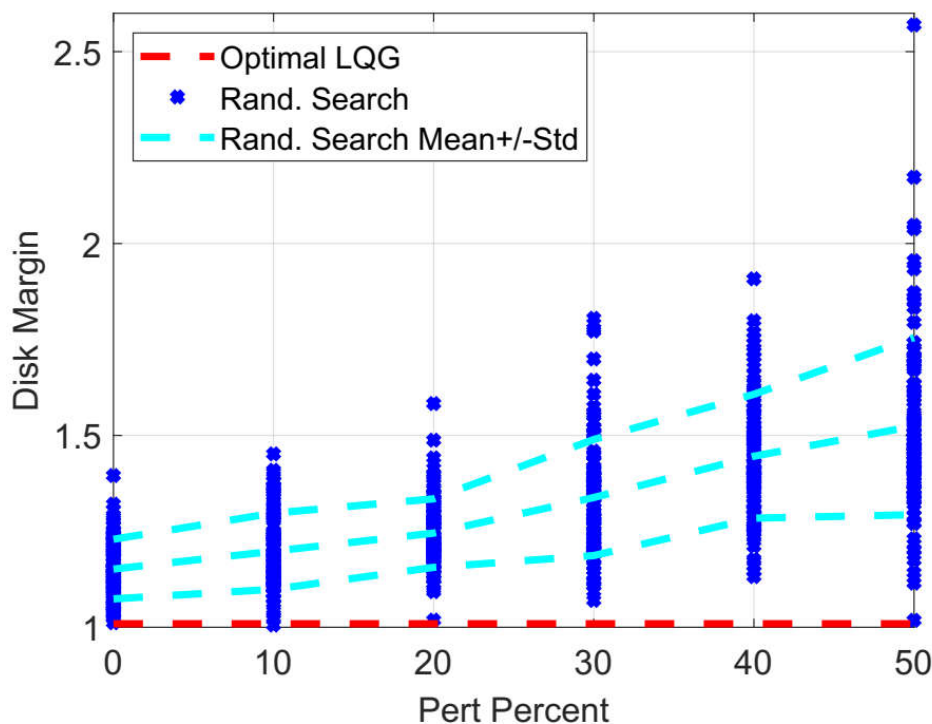


Results On Doyle's Example



Results on Simplified Flex System

- Model has 4-states (Rigid body and lightly damped modes)
- RL applied to 3-state controller parameterization
 - LQG controller is not in the control policy parameterization
 - Still converges to policy with small margins
 - Robustness recovered with synthetic perturbations during training



Next Steps

- How should synthetic perturbations be introduced during training?
- Can we make any rigorous claims about the proposed method?
- Attempt experimental tests on a simple system

Research Overview

Past: What is the impact of model uncertainty and nonlinearities on feedback system?

Key contributions

1. Theoretical connections between frequency domain and time-domain (dissipation inequality) analysis methods
2. Tools for uncertain time-varying and gain-scheduled systems
3. Applications to wind energy, UAVs, flex aircraft, hard disk drives
4. Numerically reliable algorithms with transition to Matlab's Robust Control Toolbox

Future: What is the impact of model uncertainty on control systems designed via data-driven methods?

<https://www.aem.umn.edu/~SeilerControl/>

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- ONR
 - ONR BRC: “Finite-Horizon Robustness: Moving Beyond Traditional Stability Analysis.” Tech Monitor: B. Holm-Hansen.
- NASA
 - NRA NNX14AL36A: "Lightweight Adaptive Aeroelastic Wing for Enhanced Performance Across the Flight Envelope," Tech. Monitor: J. Ouelette.
 - NRA NNX12AM55A: “Analytical Validation Tools for Safety Critical Systems Under Loss-of-Control Conditions.” Tech. Monitor: C. Belcastro.
- Eolos Consortium and Saint Anthony Falls Laboratory
 - <http://www.eolos.umn.edu/> & <http://www.safl.umn.edu/>