Rocket Stability

Center of Gravity (CG)

(also called the Center of Mass (CM))

Center of Pressure (CP)

and Static Margin (SM)

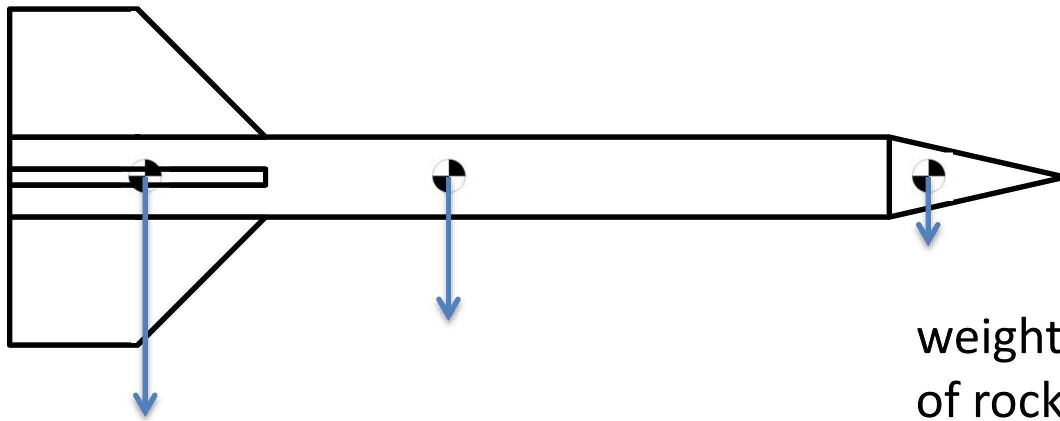
CP Reference: James Barrowman
Technical Information Report 33
Centuri Engineering Company, 1966

Center of Gravity (CG)

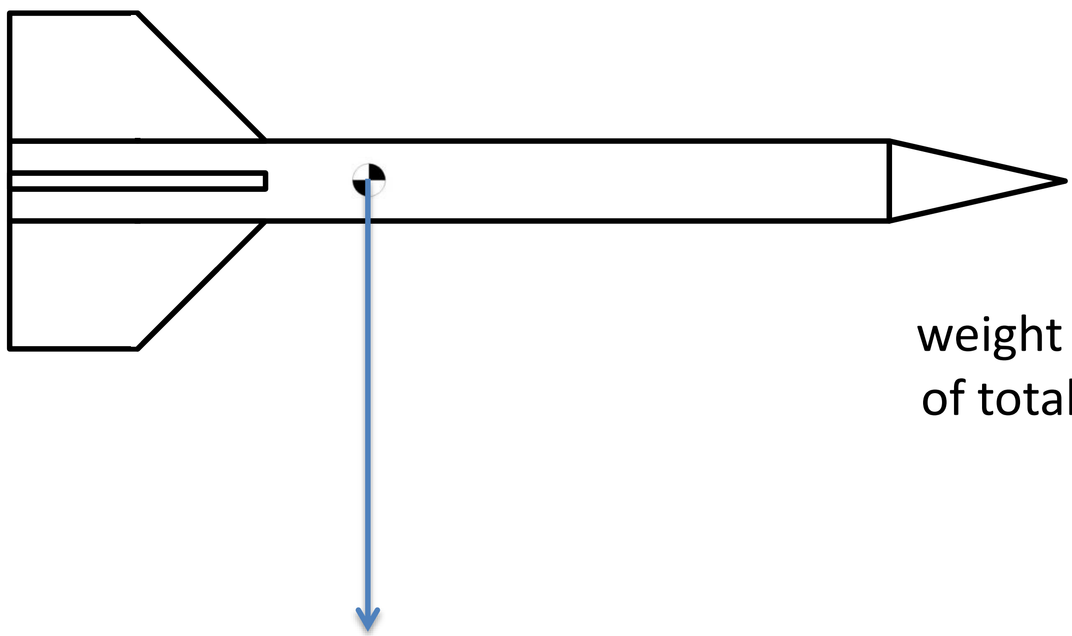
- the CG is the average location of all the mass of an object (or the average location of all the weight forces on an object when in a uniform gravitational field)
- CG is useful because we pretend the total gravitational force (on all the pieces) applies just at the CG
- for fully symmetrical objects the CG will be at the geometric center, but we need to be able to locate the CG for all objects, including asymmetrical ones

- important to us because a rocket in free flight may only wobble about (i.e. may only pitch about) its CG
- on figures we use the symbol  (in any orientation) to indicate the location of the CG

Center of Gravity – representing the distributed downwards force due to the Earth's gravity as a single downward force acting through the CG.



weight and CG
of rocket parts



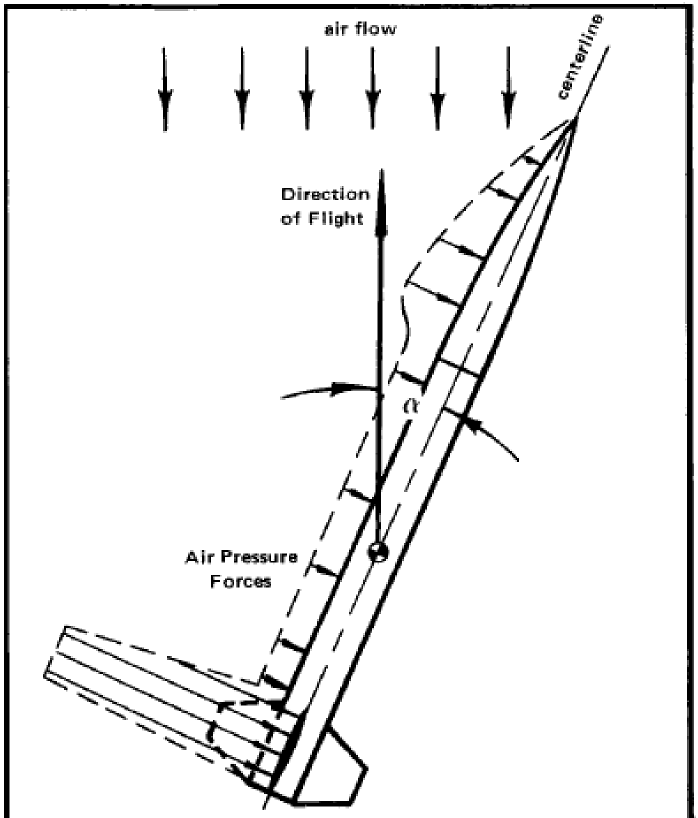
weight and CG
of total rocket

Center of Pressure (CP)

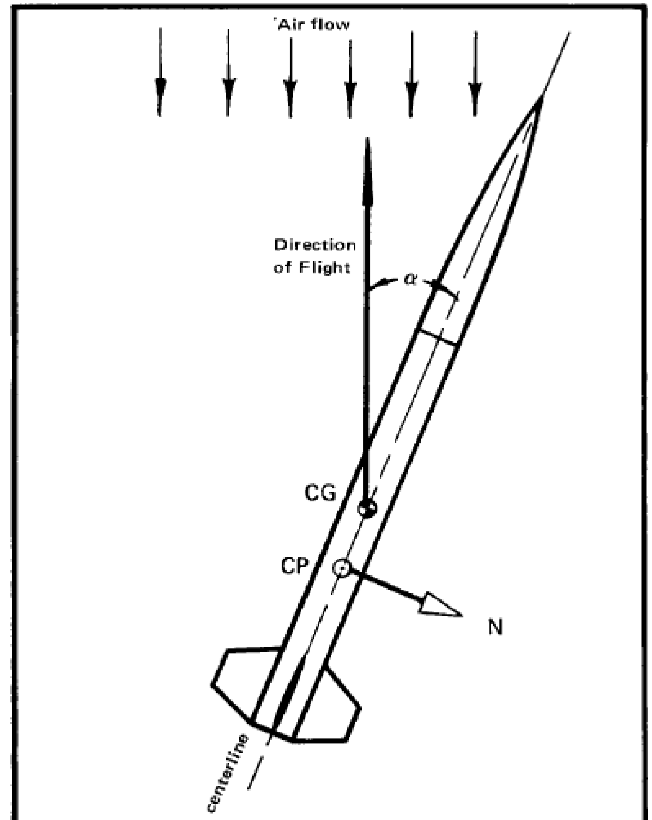
- the CP is the average location of all the aerodynamic forces acting on an object as it travels through the air
- we will focus just on components of aerodynamic forces that are "normal" to the body (i.e. perpendicular to the direction it is pointing), as opposed to the drag forces which point backward, parallel to the rocket body
- CP is useful because we pretend the total aerodynamic normal force (on all the pieces) applies just at the CP

- important to us because a rocket in free flight will be stable or unstable, depending on the relative positions of CP and CG
- on figures, use the symbol $\odot$ to indicate the CP point

Center of Pressure – representing the distributed normal component of force due to air flow into a single normal force acting through the CP.



distributed normal forces due to air flow acting on various rocket parts



summed normal force due to air flow acting at the CP

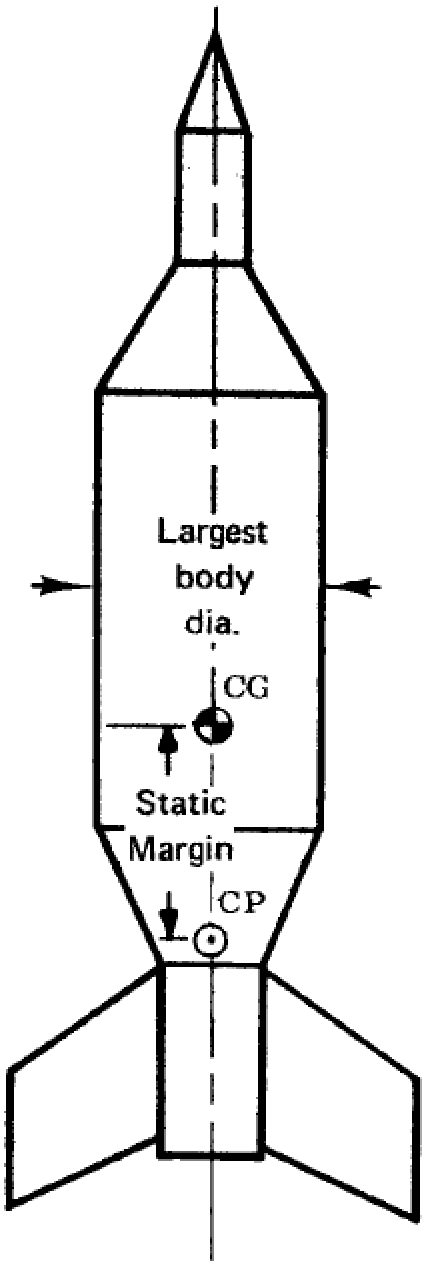
Here the CP lies <u>behind</u> the CG so the normal aerodynamic force tends to <u>correct</u> the pitch angle.

Static Margin (SM)

- the SM characterizes the tendency of a rocket to self-correct its direction of travel back towards nose-first if it is disturbed for any reason (i.e. if it develops a non-zero angle of attack) while in flight
- the key is to use aerodynamic (normal) forces to reduce rather than enhance any non-zero angle of attack; for this to work the CP must be behind (i.e. aft of) the CG
- in general, a rocket will be “stable” in flight if the CP is at least 1 body diameter (AKA “1 caliper”) behind the CG – keep that in mind as you design!

Static Margin – the distance between the CG and the CP (often reported in “calipers” which are units of largest-body-diameter).

$$SM = (\bar{X}_{CP} - \bar{X}_{CG})/D$$



Ways to locate the CG

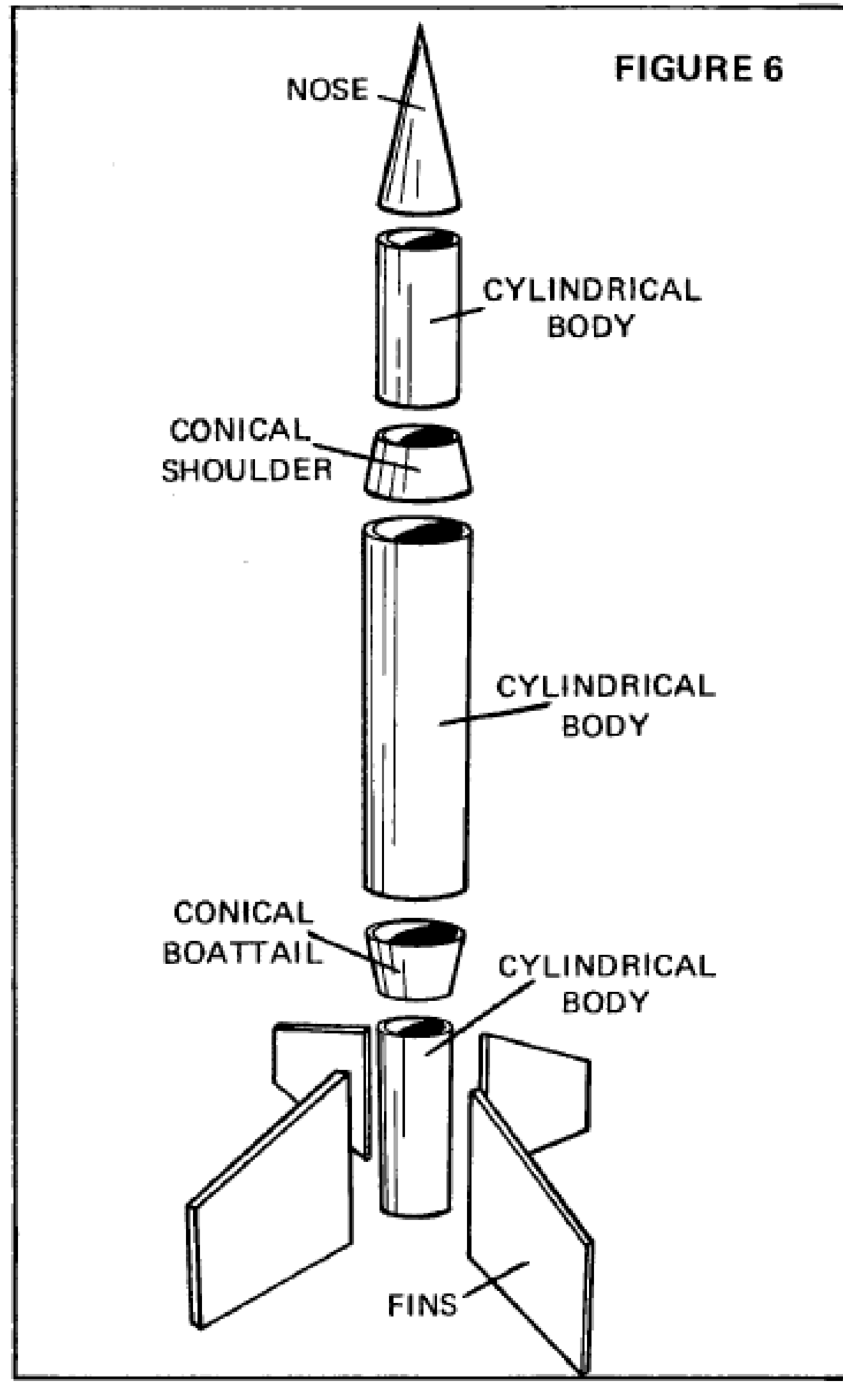
- experimental balance test – this can be done for small rockets (once built) but it is impractical for large rockets and also we usually want to know CG <u>before</u> we build – can try to test scale models
- calculate it by hand or with a spreadsheet (next!)
- let simulation software like RockSim calculate it (later in the semester)

Ways to locate the CP

- experimental wind tunnel test – this can be done for small rockets (once built) but it is impractical for large rockets and also we usually want to know CP <u>before</u> we build – easier to test scale models
- calculate it by hand or with a spreadsheet (next!)
- let simulation software like RockSim calculate it (later in the semester)

Handling the various parts of the rocket for CG & GP calculations.

Note – not all rockets will have tapered shoulder or boattail sections.



Normal aerodynamic force coefficients for various parts of a typical rocket.

Note 1: since the CP coefficient $C_{N\alpha}$ is zero for the 3 tubular body sections, it is not drawn.

Note 2: the normal force from a conical boattail (a reducing taper) points the opposite direction of that from a conical shoulder (an expanding taper).

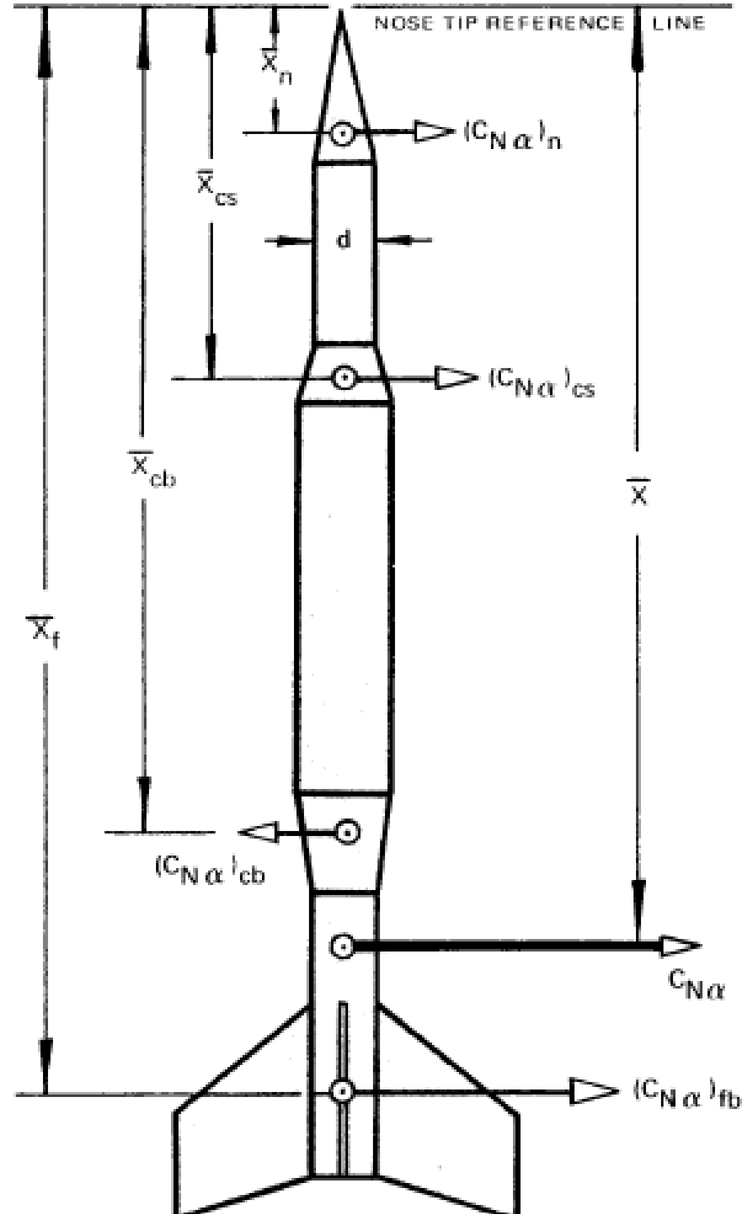


FIGURE 7
AERODYNAMIC NORMAL FORCES ACTING ON THE DIFFERENT REGIONS OF A TYPICAL ROCKET

Requirements to apply "Barrowman's equations" when calculating the location of the CP.

1) The angle-of-attack of the rocket is near zero (less than 10^{o}).

2) The speed of the rocket is much less than the speed of sound (not more than 600 feet per second).

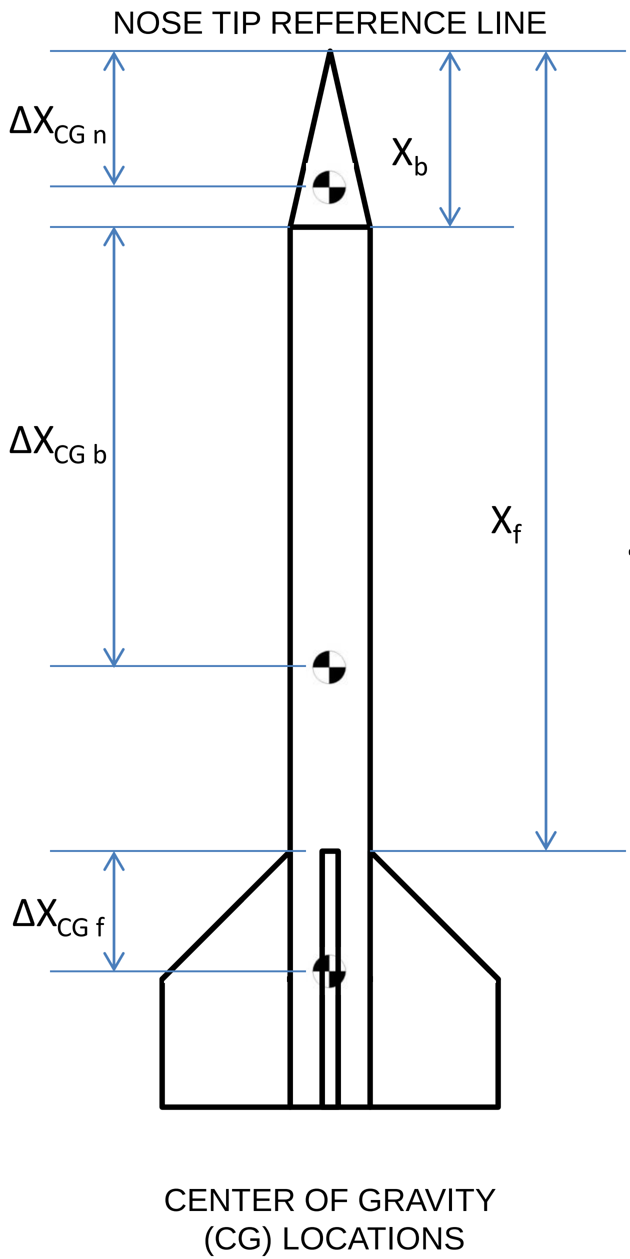
3) The air flow over the rocket is smooth and does not change rapidly.

4) The rocket is thin compared to its length.

5) The nose of the rocket comes smoothly to a point.

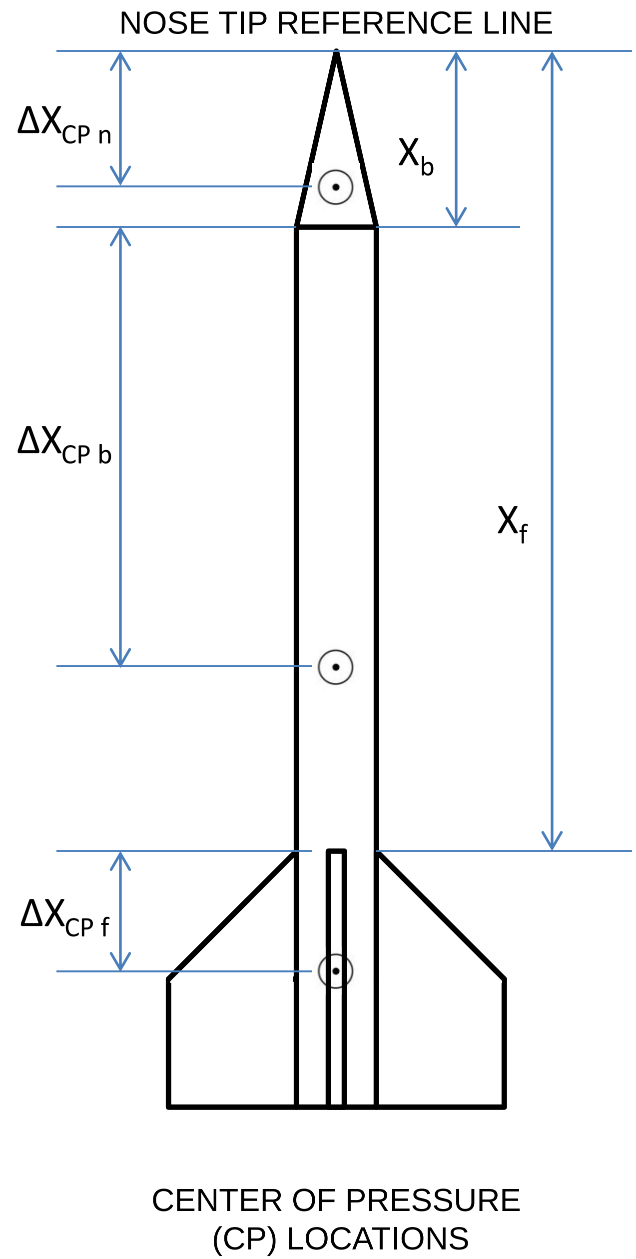
6) The rocket is an axially symmetric rigid body.

7) The fins are thin flat plates.



Defining variables for CG & GP calculations.

The offset ΔX for each component is measured from the leading edge (i.e. the top).



Steps for calculating Center of Gravity location.

(Consider organizing using an Excel spreadsheet.)

1. List the mass M_i of every component (note: this includes things inside the rocket as well).
2. Calculate the "CG station" of every component – $\bar{X}_{CGi}$ is its CG location with respect to a fixed origin (we will measure from the tip of the nose cone).
3. Calculate a sum of the masses.

$$\sum_i M_i = M_1 + M_2 + \cdots \stackrel{\text{def}}{=} M_{tot}$$

4. Calculate a sum of CG stations * masses.

$$\sum_i \bar{X}_{CGi} * M_i = \bar{X}_{CG1} * M_1 + \bar{X}_{CG2} * M_2 + \cdots \stackrel{\text{def}}{=} \bar{X}_{CG} * M_{tot}$$

5. Find the CG location $\bar{X}_{CG}$ by dividing by M_{tot}.

Steps for calculating Center of Pressure location.

(Consider organizing using an Excel spreadsheet.)

1. List the normal force coefficient $(C_{N\alpha})_i$ of every exposed component. Insides don't feel air forces.
2. Calculate the "CP station" of every component – $\bar{X}_{CPi}$ is its CP location with respect to a fixed origin (we will measure from the tip of the nose cone).
3. Calculate a sum of the normal force coefficients.

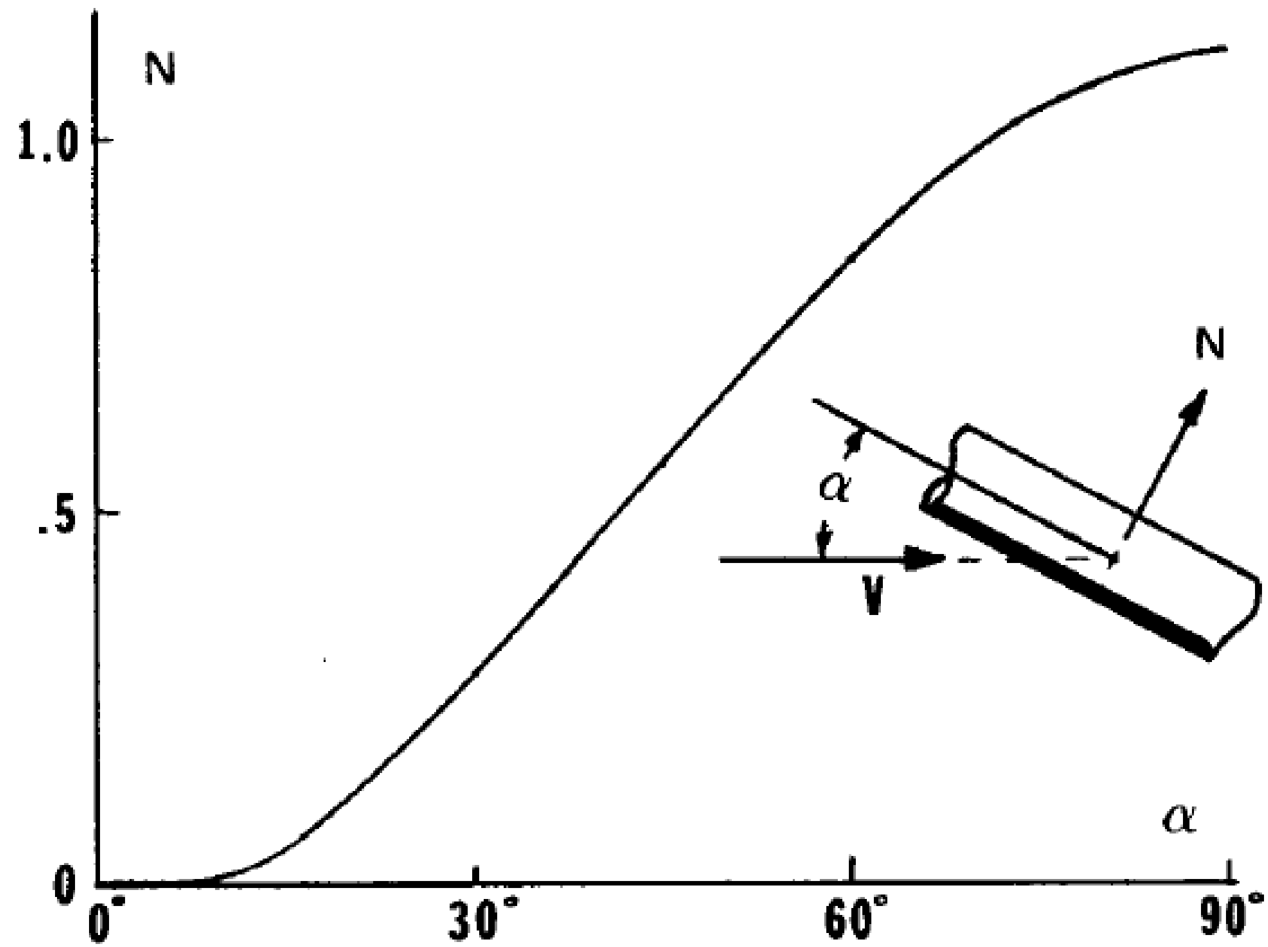
$$\sum_i (C_{N\alpha})_i = (C_{N\alpha})_1 + (C_{N\alpha})_2 + \cdots \stackrel{\text{def}}{=} (C_{N\alpha})_{tot}$$

4. Calculate a sum of CP stations * coefficients.

$$\sum_i \bar{X}_{CPi} * (C_{N\alpha})_i = \bar{X}_{CP1} * (C_{N\alpha})_1 + \bar{X}_{CP2} * (C_{N\alpha})_2 + \cdots \stackrel{\text{def}}{=} \bar{X}_{CP} * (C_{N\alpha})_{tot}$$

5. Find the CP location $\bar{X}_{CP}$ by dividing by $(C_{N\alpha})_{tot}$.

Plot of the normal aerodynamic force due to air flow versus angle of attack on circular cylinders (like a rocket body tube). Note that this normal force is negligible for angles below about 10°.

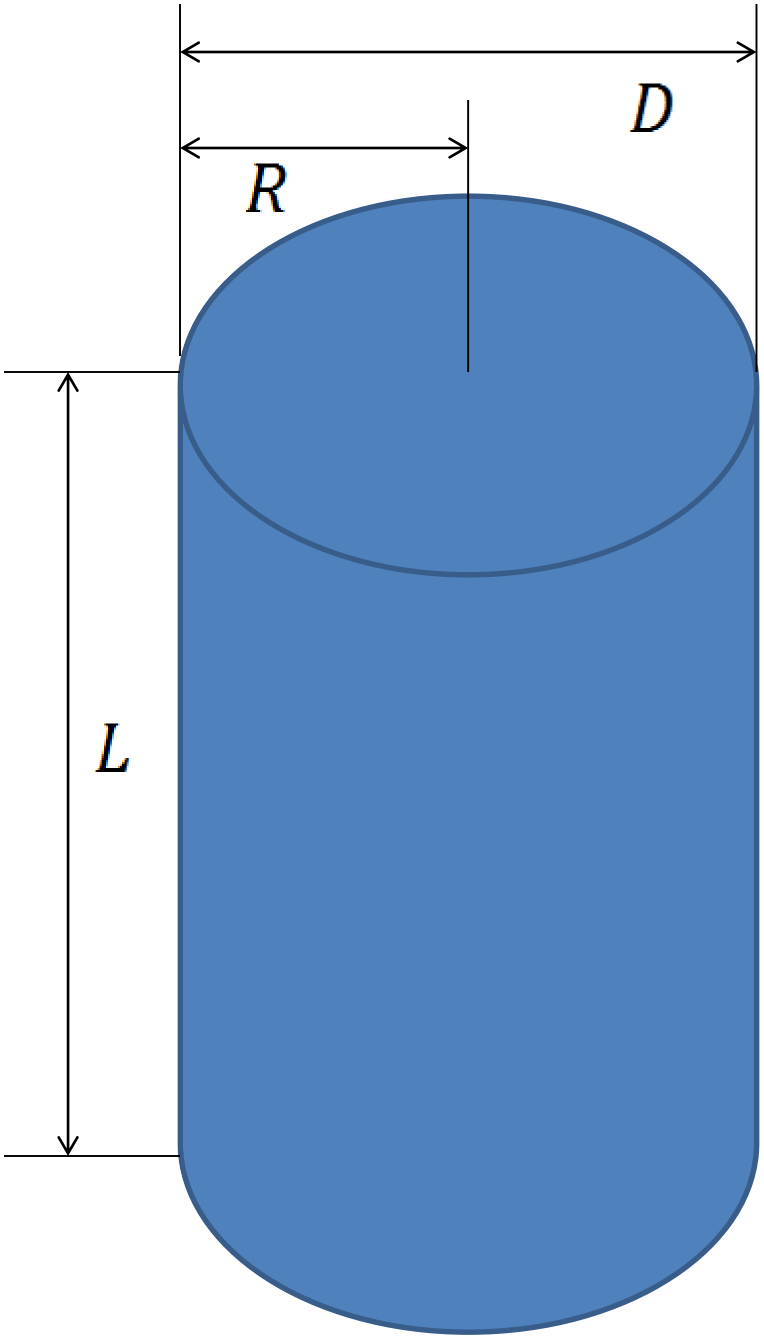


Equations for solid or hollow cylinders (like body tubes, centering rings, motors, etc.)

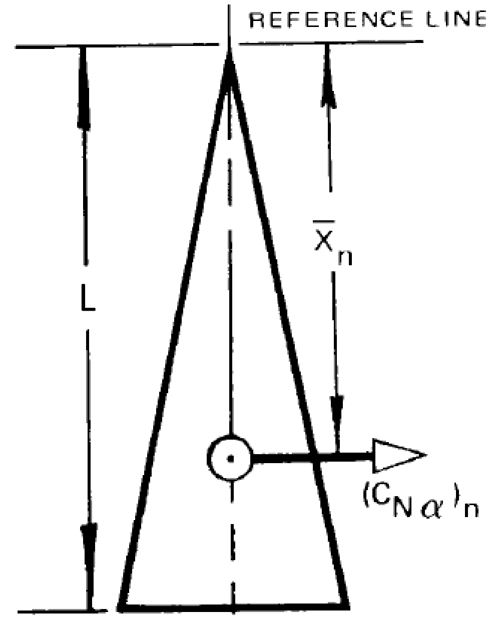
$$\Delta X_{CG\ c} = \frac{1}{2}L$$

$$\Delta X_{CP\ c} = \frac{1}{2}L$$

$$(C_{N\alpha})_c = 0$$



Equations for a solid conical nose cone.

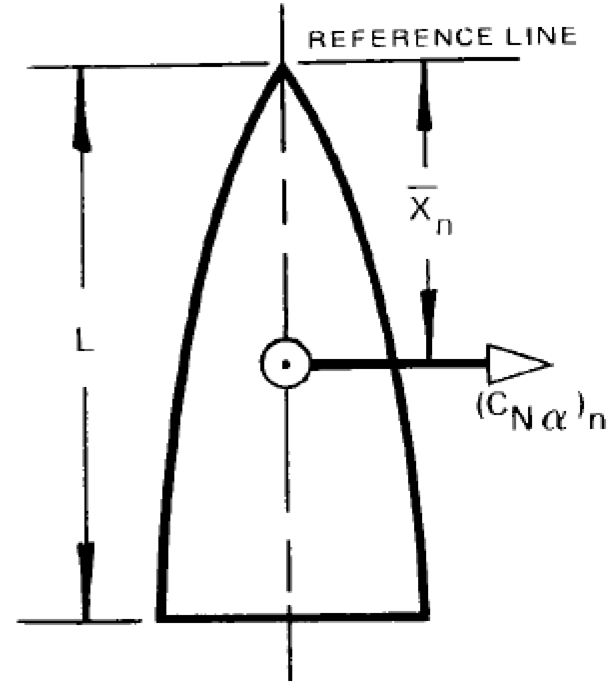


$$\Delta X_{CG\,n} = \frac{3}{4}L$$

$$\Delta X_{CP\,n} = \frac{2}{3}L$$

$$(C_{N\alpha})_n = 2$$

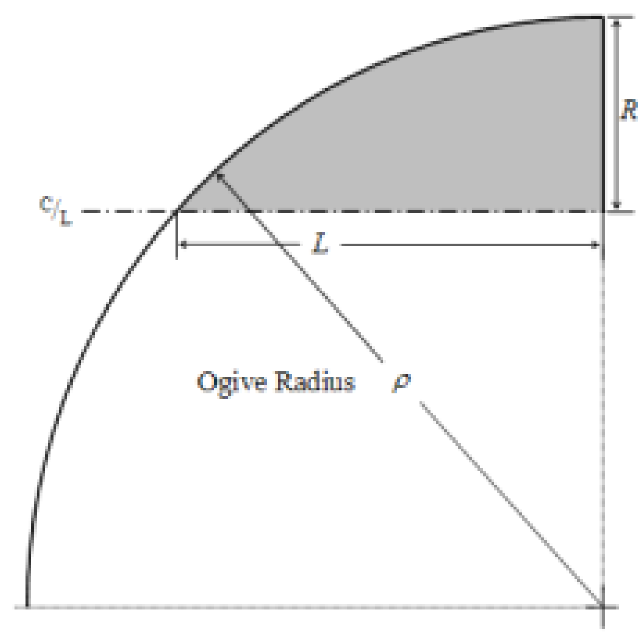
Equations for a solid "tangent Ogive" (oh-jive) nose cone.



$$\Delta X_{CG\,n} = 0.685 * L$$

$$\Delta X_{CP\,n} = 0.466 * L$$

$$(C_{N\alpha})_n = 2$$



Equations for a "clipped delta" fins.

$$\Delta X_{CG\ f} = \frac{b^2 + \frac{2}{3}m^2 + 2mb}{2b + m}$$

$$\Delta X_{CP\ f} = \frac{m(a + 2b)}{3(a + b)} + \frac{1}{6}\left(a + b - \left(\frac{ab}{a + b}\right)\right)$$

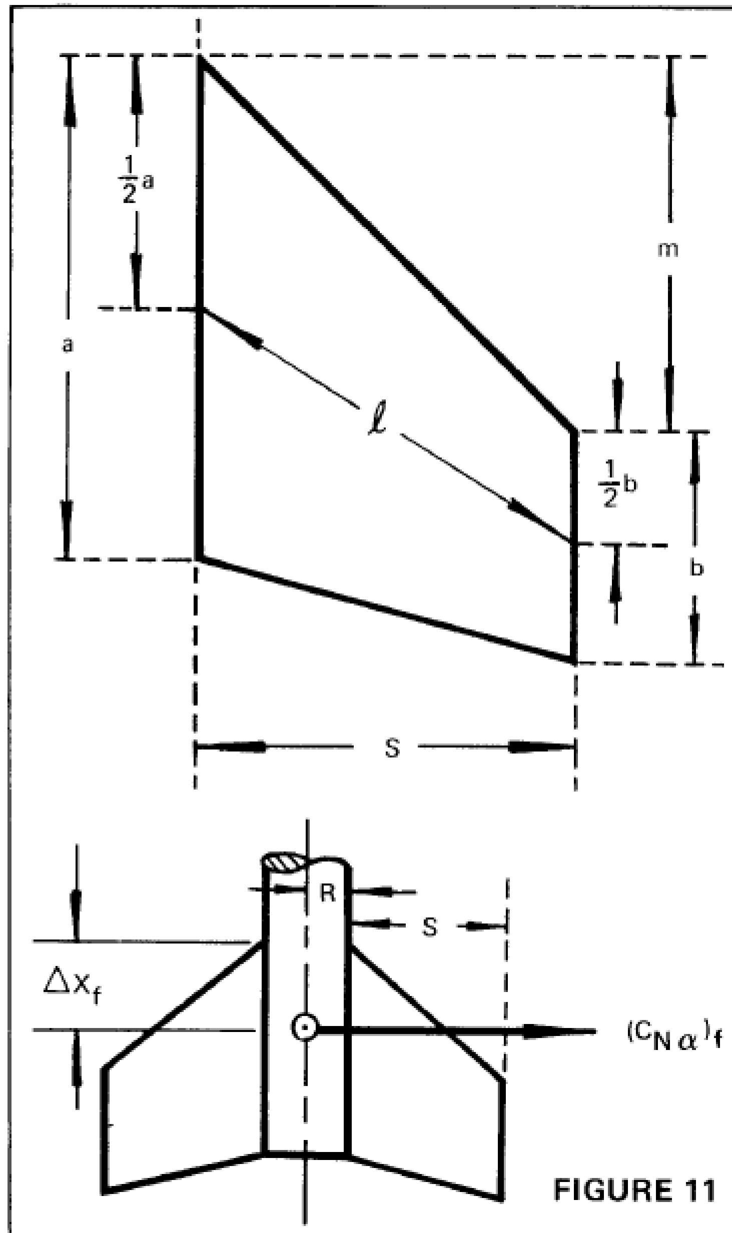


FIGURE 11

See next slide for normal coefficient equations.

Normal coefficients for n "clipped delta" fins where n = 3 or 4 fins.

$$(C_{N\alpha})_f = \frac{4n\left(\frac{s}{d}\right)^2}{1+\sqrt{1+\left(\frac{2l}{a+b}\right)^2}}$$

Taking the body interference factor K_{fb} into account

$$(C_{N\alpha})_{fb} = K_{fb} * (C_{N\alpha})_f$$

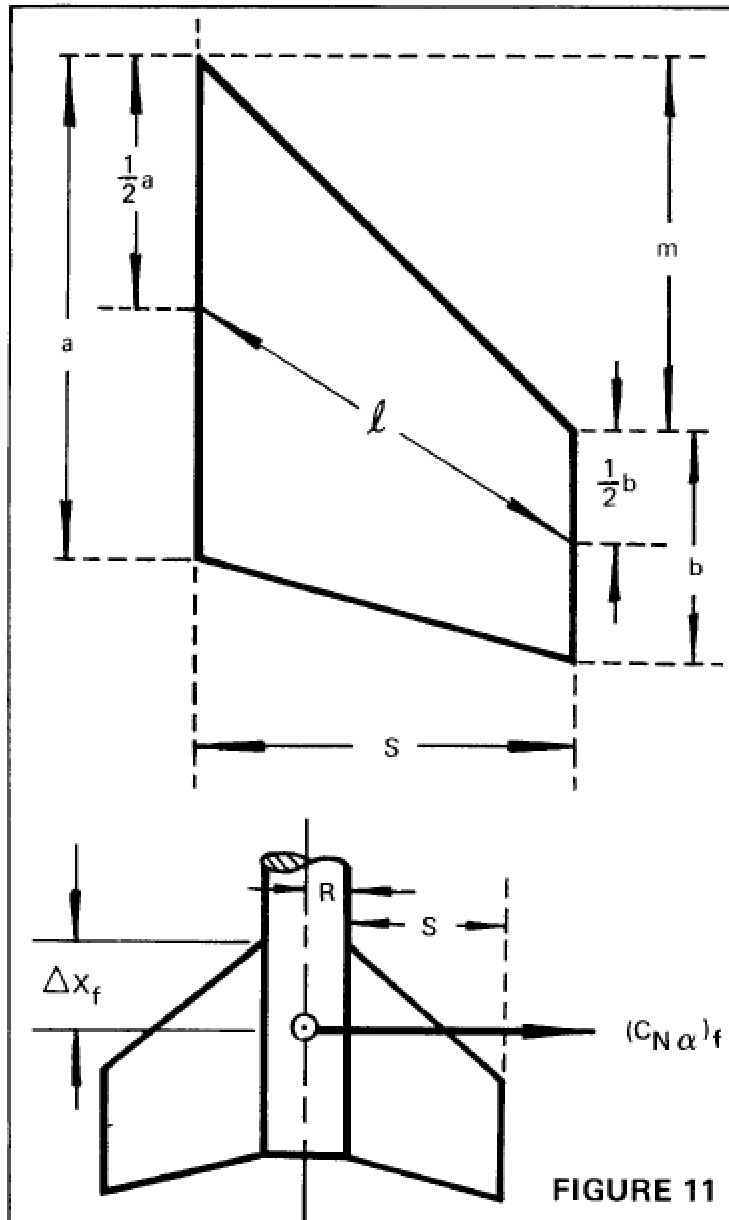


FIGURE 11

where $K_{fb} = 1 + \frac{r}{s+r}$ (r is the rocket body radius).