Disintegration of moving liquid sheets using viscous potential analysis

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The breakup of a thin liquid film moving in a second fluid utilizing viscous potential analysis and considering disjoining pressure is studied. The occurrence of cavitation due to local pressure drop in the film has been considered. For a large Weber number and density ratio, cavitation might occur and it could be the main reason for breakup.

1. Introduction

Instability of moving film occurs in liquid atomizers where films breakup into threads and later into droplets. Dorman (1952) and Fraser & Eisenklam (1953) and later Dombrowski & Fraser (1954) were the first to describe the break-up and drop formation of plane fan sheets. Shea (1955) and Squire (1953) studied temporal (spatially periodic) behavior on an infinite liquid sheet. Sirignano & Mehring (2000) extended these studies to examine spatially developing distortion on a semi-infinite sheet flowing from an injector nozzle. Lin (1981) conducted a linear temporal and spatial stability analysis of a viscous liquid sheet falling in gravity. A complete review article by Sirignano & Mehring (2000) on disintegration of liquid streams provided relevant researches.

Stability analysis of stationary films for the critical thickness at which spontaneous local thinning first occurs have been given by Scheludko (1967), Ruckenstein & Jain (1974) and for films on solids, Homsy (1975) and Williams & Davis (1982). The term"disjoining pressur" was introduced by Deryagin to designate the excess pressure in a thin layer compared with a thick one. If film thickness h is small enough, the stabilizing effects of surface tension are insufficient and the film will thin under the influence of the large disjoining pressure. The stability analysis gives the critical h, the wave length and maximum growth rate of the most unstable disturbance. Lucassen *et al.* (1970); Vrij *et al.* (1970) considered instability of free liquid films utilizing Helmholtz decomposition.

In present study, the instability of a moving thin film in a still fluid by Squire (1953) is extended to include viscous effects and disjoining pressure utilizing viscous flow analysis.

2. Theoretical Development

Consider a two dimensional film of fluid of density ρ_1 and viscosity μ_1, surface tension γ and thickness $2h_0$ moving with velocity U through a fluid of density ρ_2 and viscosity μ_2 which is at rest (see figure 1). The film oscillation can divided into symmetric and antisymmetric modes. An investigation of antisymmetric disturbances can be carried out

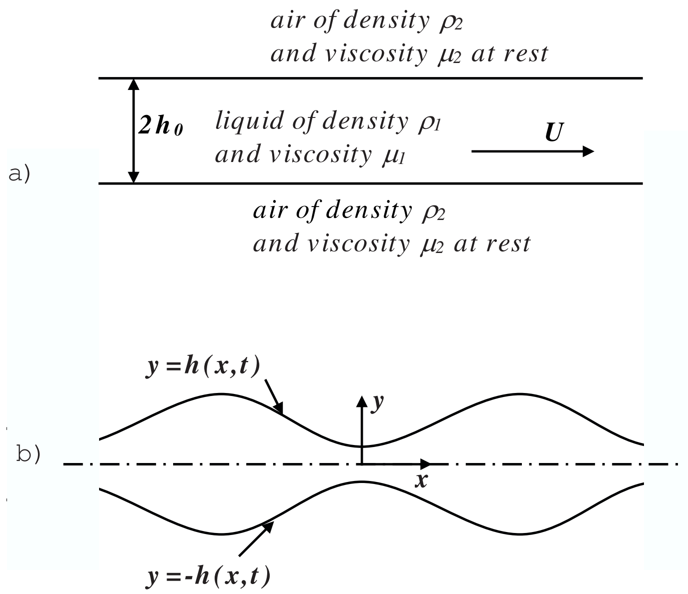


Figure 1: Sketch of symmetrical waves

by the same procedure explained below. $\lambda = 2\pi/k$ is the wavelength, t is time, and σ is the growth rate. $\phi = -Ux + \phi_1$ is the velocity potential of the motion of inner liquid where ϕ_1 is the disturbance potential and let ϕ_2 be the velocity potential of the outer liquid. From kinematic boundary conditions at $y = \pm h$ we have

$$\frac{\partial h}{\partial t} + U\frac{\partial h}{\partial x} = -\frac{\partial \phi_1}{\partial y}, \quad \frac{\partial h}{\partial t} = -\frac{\partial \phi_2}{\partial y} \tag{2.1}$$

ϕ_1 and ϕ_2 satisfying Laplace's equation corresponding to symmetric oscillations can be written as

$$\phi_1 = ia\left(\frac{\sigma}{k} - U\right)\frac{\cosh ky}{\sinh kh_0}\exp i(kx - \sigma t) \tag{2.2}$$

$$\phi_2 = -ia\frac{\sigma}{k}\exp[-k(y - h_0)]\exp i(kx - \sigma t) \tag{2.3}$$

Satisfying normal stress at the free surface, we have

$$\tau_{yy2} - \tau_{yy1} + \gamma\frac{\partial^2 h}{\partial x^2} = 0 \tag{2.4}$$

where $\tau_{yyi} = -p_i + 2\mu_i\frac{\partial v_i}{\partial y}$ and i is 1 for inner liquid and 2 for the outer liquid. From linearized Bernoulli equation

$$-\rho_1\left(\frac{\partial \phi_1}{\partial t} + U\frac{\partial \phi_1}{\partial x}\right) + p_1 + \Phi = C_1, \quad -\rho_2\frac{\partial \phi_2}{\partial t} + p_2 = C_2 \tag{2.5}$$

Φ related to disjoining pressure is defined as

$$\Phi = \Phi_B + A/6\pi(2h)^3 \tag{2.6}$$

where A is Hamaker constant. Now we define η that $h = h_0 + \eta$. Thus, we have $\int_{-\infty}^{\infty} \eta dx = 0$. Linearizing equation 2.6 we have

$$\Phi(2h) = \Phi_B + \frac{A}{48\pi}\left[\frac{1}{h_0^3} - \frac{3\eta}{h_0^4}\right] \tag{2.7}$$

Combining 2.4, 2.5, and 2.7, one can write

$$\begin{aligned} \rho_1 \frac{\partial \phi_1}{\partial t} + \rho_1 U \frac{\partial \phi_1}{\partial x} + C_1 + 2\mu_1 \frac{\partial^2 \phi_1}{\partial y^2} - \rho_2 \frac{\partial \phi_2}{\partial t} - C_2 - 2\mu_2 \frac{\partial^2 \phi_2}{\partial y^2} \\ = -\gamma \frac{\partial^2 \eta}{\partial x^2} + \frac{A}{48\pi h_0^3} - \frac{A\eta}{16\pi h_0^4} \end{aligned} \tag{2.8}$$

The Basic (undisturbed) state can be written as

$$C_1 - C_2 = \Phi_B + \frac{A}{48\pi h_0^3} \tag{2.9}$$

Subtracting 2.9 from **??** gives rise to

$$\rho_1 \frac{\partial \phi_1}{\partial t} + \rho_1 U \frac{\partial \phi_1}{\partial x} + 2\mu_1 \frac{\partial^2 \phi_1}{\partial y^2} - \rho_2 \frac{\partial \phi_2}{\partial t} - 2\mu_2 \frac{\partial^2 \phi_2}{\partial y^2} = -\gamma \frac{\partial^2 \eta}{\partial x^2} - \frac{A\eta}{16\pi h_0^4} \tag{2.10}$$

Utilizing boundary condition at the free surface and substituting for ϕ_1 and ϕ_2 equations 2.2 and 2.3, one finds the dispersion relation as

$$\frac{\sigma}{kU} = \frac{-\tilde{b} \pm \sqrt{\tilde{b}^2 - \tilde{a}\tilde{c}}}{\tilde{a}} \tag{2.11}$$

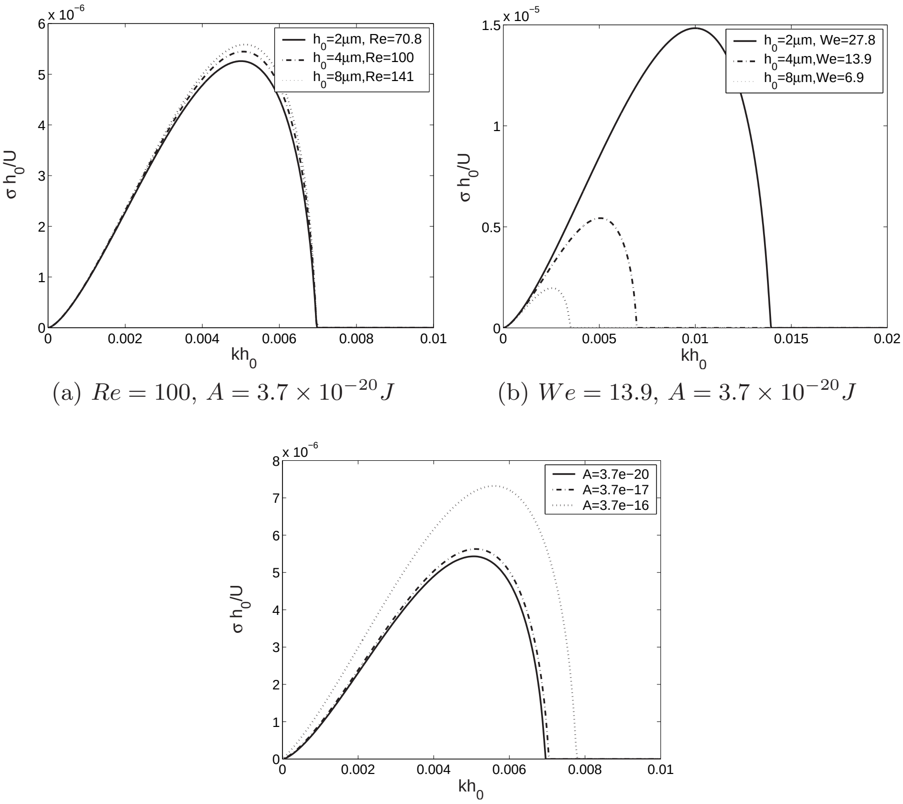
where

$$\begin{aligned} \tilde{a} &= 1 + \frac{\rho_2}{\rho_1} \tanh kh_0 \\ \tilde{b} &= -1 + \frac{i\mu_1 k}{\rho_1 U} + \frac{i\mu_2 k}{\rho_1 U} \tanh kh_0 \\ \tilde{c} &= 1 - \frac{2i\mu_1 k}{\rho_1 U} - \frac{\gamma k}{\rho_1 U^2} \tanh kh_0 + \frac{A}{16\pi h_0^4 k \rho_1 U^2} \tanh kh_0 \end{aligned} \tag{2.12}$$

We are also interested on the occurrence of cavitation. In the traditional criterion of cavitation, cavitation occurs when the pressure drops below the breaking strength of liquid, which we call critical pressure or critical stress, and in an ideal case is the vapor pressure at local temperature. Bair & Winer (1992) and, independently, Joseph (1998), proposed that the important parameter in cavitation is the total stress which includes both the pressure and viscous stress. Kottke *et al.* (2005) conducted an experiment on cavitation in creeping shear flow, where the reduction of hydrodynamic pressure does not occur. They observed the appearance of cavitation bubbles at pressures much higher than vapor pressure. Their data on cavitation inception agrees well with the total stress criterion for cavitation. Maximum total stress for a moving sheet can be calculated as follows

$$\boldsymbol{\tau} = i\rho_1 \sigma \phi_1 - iUk\phi_1 - C_2 - i\frac{Ak\phi_2}{16\pi h_0^4 \sigma} + 2\mu_1 \phi_1 \begin{bmatrix} -k^2 & ik^2 \tanh ky \\ ik^2 \tanh ky & k^2 \end{bmatrix} \tag{2.13}$$

$$\tau_1 = \frac{\tau_{xx} + \tau_{yy}}{2} + \sqrt{\left(\frac{\tau_{xx} - \tau_{yy}}{2}\right)^2 + \tau_{xy}^2} \tag{2.14}$$



(a) $Re = 100$, $A = 3.7 \times 10^{-20} J$

(b) $We = 13.9$, $A = 3.7 \times 10^{-20} J$

(c) $Re = 100$, $We = 13.9$

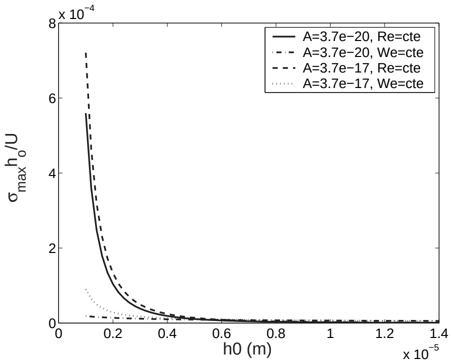
Figure 2: Growth rate as a function of wave number

In the next section, we observe that in some cases maximum total stress is large enough, pressure is low, that cavitation occurs. In this cases breakup occurs due to pulling apart of liquid sheet instead of necking down.

3. Results and Discussion

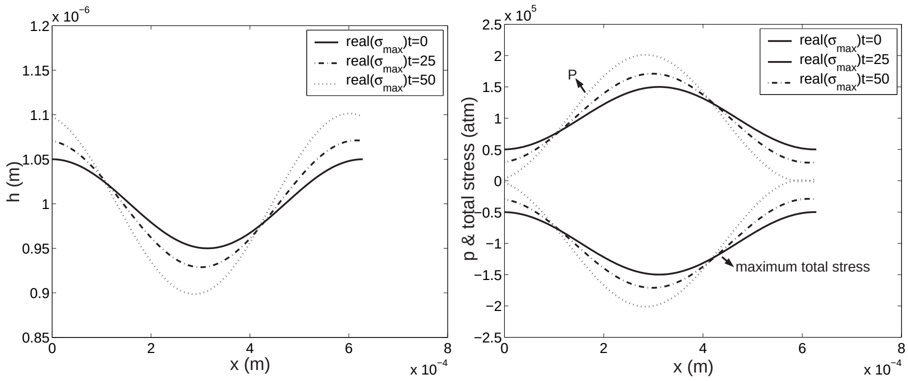
The dimensionless growth rate is plotted as a function of wave number for air and water in figure 2 where $m = \frac{\mu_2}{\mu_1} = 0.02$ and $r = \frac{\rho_2}{\rho_1} = 0.001$. As it can be seen as h_0 increases for a fix Reynolds number the maximum growth rate increases. However, the cutoff wave number remains constant. Increasing h_0 for a constant Weber number results in decrease of the maximum growth rate and cutoff wave number. The effect of Hamaker constant on the maximum growth rate is shown in figure 2(c). Both the maximum growth rate and the cutoff wave number increases as A increases. The relation between the maximum growth rate and h_0 is plotted in figure 3.

Using equation 2.13 and 2.14, we calculate pressure and total stress for the stream of water. Figure 4 shows sheet thickness h, centerline pressure and total stress, assuming C_2 is atmospheric pressure. The sheet of water is moving at $Re = 5000$, $We = 1.4 \times 10^5$ where $m = 0.02$, and 0.02, $A = 3.7 \times 10^{-20} J$. Initial disturbance, a, of $5\% h_0$ has been used. As it can be seen viscous stress is small and pressure and total stress criteria for



(a) $We = 13.9$

Figure 3: Maximum growth rate for different film thickness



(a) h (b) pressure and maximum total stress

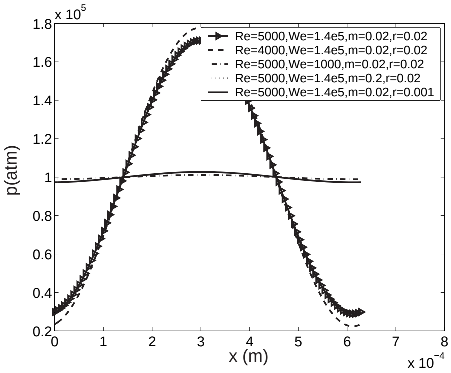
Figure 4: Sheet of water is moving at $Re = 5000$, $We = 1.4 \times 10^5$, $m = 0.02$, and 0.02, $A = 3.7 \times 10^{-20} J$

cavitation leads to the same results. Very low pressure occurs where h is maximum which could results in the cavitation.

Pressure at the centerline of moving sheet of water at $real(\sigma_{max})t = 25$ is shown in figure 5. The effect of Reynolds number, Weber number, density ratio and viscosity ratio are demonstrated. Lower pressure occurs at lower Reynolds number. The viscosity ratio does not change the pressure for fixed Re, We and r. No cavitation occurs for small density ratio and small We number.

4. Conclusions

The breakup of a thin liquid film moving in a fluid utilizing viscous potential analysis and considering disjoining pressure is studied. The occurrence of cavitation has been studied. For the large Weber number and density ratio, cavitation might occurs will be the main reason for breakup. In these cases breakup occurs due to pulling apart of liquid sheet instead of necking down.



(a)

Figure 5: Pressure at the centerline of moving sheet of water

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