



Figure 3: Surfaces separating at high speed, $\nu > \nu_c$, showing FECO fringes (top), schematic side-view (middle), and optical microscope view (bottom) of the surfaces. Homogeneous nucleation of a vapor cavity is shown in D ($t = 10.01$ sec), after which the cavity grows rapidly and then collapses (D to F). Note that in picture F (1 sec after inception) the cavity has still not totally disappeared (evaporated or collapsed).

9 Outgassing and cavitation

It is generally believed that vapor bubbles may be initiated from small gas nuclei (Plesset & Prosperetti [1977]). Here we are looking at the situation in which a gas (say air) rather than vapor of the solvent goes in and comes out of solution. The cavitation of pure liquid into vapor is a phase change. When the pressure in the liquid drops below the saturation pressure at a given temperature the liquid is superheated (say, supersaturated) and vaporizes; if the pressure is raised above the saturation pressure, the vapor condenses. Cavitation bubbles are found at places where the liquid is supersaturated.

We may also model the dissolution and cavitation of foreign gases in the solvent as phase changes. Dissolved gases also pass into solution when the pressure is raised above a saturation value; the dissolved gas has condensed into a liquid and the condensed liquid is miscible.

Gases dissolved in liquids experience intermolecular forces between liquid and gas molecules. We could think of a gas molecule as a bridge between two liquid molecules. Of course, you can liquefy gases, but the thermodynamic conditions under which a substance is gas or liquid in isolation need not, indeed should not, apply when the substance is dissolved in another liquid.

Unlike miscible liquids, condensed gas cannot be mixed into the solvent in all proportions; there is a saturation concentration given as “solubility” which is expressed as a weight or volume fraction of gas in solution. Solubility is a thermodynamic variable which depends on temperature and pressure. In the case of condensed gas, saturation conditions are expressed in terms of three quantities, temperature, pressure and gas fraction, rather than the two quantities, temperature and pressure which define saturation in a pure liquid. Naturally, if a solution is degassed, it is starved of gas and will not cavitate.

The thermodynamic of solubility is meant to apply to pure liquids. Such liquids must be carefully prepared; natural liquids are “dirty” and contain dust particles and other impurities which act as nucleation sites for cavitation. Tap water and crude oil are impure in this sense. A precise characterization of the impurities in “dirty” liquids, especially water is problematic. According to Batchelor [1967]

Tests on liquids at rest show that the tendency to form cavities when the pressure is reduced nearly to zero is associated with the continual presence of nuclei which are believed to be tiny pockets of undissolved gas; some liquid vapour is certain to exist also in any small pockets, but the gas, usually air, appears to be the more essential element for cavity formation. It is not known with certainty how these pockets of gas are able to persist in a liquid under normal conditions.

The inward force at the boundary of a small spherical bubble due to surface tension is very strong, much too strong to be balanced by vapour pressure, and gas subjected to this pressure would quickly pass into solution in the liquid. A common postulate is that the pockets of gas and vapour are able to persist in equilibrium under normal conditions by being trapped in crevices in small hydrophobic (non-wetting) solid particles such as dust particles, which are usually present in liquids; the liquid surface in such cracks and crevices can be concave outwards, in which case the direction of the surface tension force is outward. Then when the pressure in the surrounding liquid is reduced sufficiently below the vapour pressure (which for water at 15°C is 1.704×10^4 dyn/cm², or about 0.017 atmosphere), the gaseous pocket grows and, despite the fact that for a cavity larger than the host solid particle the surface tension force is inward, will not be able to find a new equilibrium radius. For tap water and sea water, the critical steady ambient pressure, below which cavities grow indefinitely in size, is found to be different from the vapour pressure of the water by only a small margin which is usually neglected. On the other hand, water which has been compressed for a few minutes at about 700 atmospheres and which is saturated with air can withstand *tensions* of about 25 atmospheres ... presumably because all except the smallest pockets of undissolved air have been eliminated. Water which has been ‘degassed’ in this way likewise does not boil at atmospheric pressure until the temperature is considerably above 100°C; the phenomena of boiling and growth of cavities in liquids at low pressures are of course mechanically similar.

In Batchelor’s cartoon of impure water just cited we have condensed air and very small pockets of undissolved gas attached to hydrophobic particles. The pockets of undissolved gas are postulated as nucleation sites. A degree of supersaturation is required for these pockets to open “... when the pressure in the surrounding is reduced sufficiently below the vapor pressure.” The required superstauration implies that the pocket opening would occur suddenly, with rapid filling of the gas bubble with gaseous air vaporizing from liquid air in solution.

The same discussion of cavitation of impure water can be applied to flowing water with the added caveat that cavitation is expected when one of the principal values of the stress at the gas pocket falls below the vapor pressure. Since the pocket is attached to a crack or crevice of hydrophobic dust and since supersaturation out of equilibrium is required, the cavitation event is to a degree like the fracture of impure solids with points of weakness at the boundaries of foreign inclusions.

The postulate of gas pockets on hydrophobic particles leaves certain questions unanswered. What is the nature of cavitation in water with no particles or hydrophilic particles? What kind of cracks and crevices can be expected on hydrophobic dust? What does it take to keep hydrophobic dust with gas pockets from floating out of solution? Can hydrophobic particles be floatated by bubbling water?

Impure liquids may be regarded as just another kind of (multiphase) liquid with its own thermodynamic properties. Solubility measurements can be carried in impure liquids and gas bubbles will nucleate when one of the principal stresses puts the liquid into supersaturation at a nucleation site.

Solubility measurement for impure liquids have been carried out extensively by Canadian researchers (see Peng et al. [1991]) in studies of “foamy oils” in which copious amounts of gases (Methane, Carbon dioxide, Nitrogen) may be dissolved in bitumen from the Alberta Oil Sands. These oils are a good laboratory for testing the idea that even in impure liquids the dissolved gases are condensed and miscible.

Svrcek and Mehrotra [1982] studied gas solubility, viscosity and density for mixtures of Athabasca bitumens and dissolved CO₂, CH₄ (methane) and N₂ (Nitrogen) gases at saturation. They found that the viscosity and density of saturated oils depends on the gas, bitumen, temperature and pressure and is such that more gas is dissolved at higher pressures and lower temperatures. The viscosity of the solution drops precipitously with the amount of dissolved gas. The viscosity of bitumen without gas is more or less independent of pressure and drops more rapidly with temperature than does the saturated bitumen. The saturation concentration of CO₂, CH₄, and NO₂ in bitumen increases with pressure at each fixed temperature; the increase is linear for pressures below 5 MPa and is almost linear above. The solubility and viscosity are greater at lower temperatures; at room temperature the viscosity of dissolved CO₂ in bitumen drops nearly two orders of magnitude from the value of degassed bitumen, apparently because more condensed CO₂ is added as the pressure is increased. The solubility and reduction in viscosity is greatest for carbon dioxide and least for nitrogen.

The reduction of viscosity of bitumen with increasing concentrations of liquefied gas and the fact that different condensed gases lead to different reductions is consistent with the miscible liquid model. The parameters governing solubility are not so easily modeled.

The miscible liquid model may also be applied to density measurements; assuming that the solution is an ideal mixture the density of the solution ought to be equal to $f_g\phi + f_b(1 - \phi)$ where ρ_b and ρ_g are densities of the bitumen and the condensed gas and ϕ is the volume fraction of gas. The volume fraction of condensed gas can be obtained from the measured increase in the solution volume with concentration (the swelling factor in figure 5 of Peng et al. [1991]). The measurements

of Svrcek & Mehrotra [1982] show that the density of carbon dioxide and nitrogen in bitumen is independent of pressure, hence concentration, whereas the methane in bitumen solution decreases linearly with pressure; that is, with concentration. The ideal mixture theory then implies that the density of carbon dioxide and nitrogen condensed in bitumen is nearly the same as the density of the bitumen, but the density of condensed methane is smaller.

10 The effect of polymeric additive on cavitation

It is generally thought that the effects of polymer additives on cavitation are due to the large-scale modification of flow structure due to polymers, as in the suppression of turbulence, rather than to a direct effect on the formation of cavities. However, consideration has not been given to effects that might be associated with the deviatoric part of the stress (rather than the “pressure”) due to motion in such non-linear fluids. Since these direct effects of motion on the formation of cavities is the precise focus of this paper, and they have not been considered before, it is of value to carry this further.

Unfortunately, most of the studies of cavitation in non-Newtonian fluids are restricted to the same extremely dilute solutions of polymer in water which reduce drag. Reviews of these studies have been given by Arndt, et al. [1976], Hoyt & Taylor [1981], Arndt [1981] and Trevena [1987]; they show that the polymer reduces the cavitation index (suppresses cavitation) in some flows, whereas in others the results are ambiguous and even depend on the material of construction of the flow obstacle used to create low pressure. The suppression of cavitation with polymers might be expected to coincide with a reduction in the breaking strength of the solution; this does not appear to be the case. Sedgewick and Trevena [1978] studied the effects of polyacrilamide additives on the breaking tension of water using a static Berthelot tube method and a dynamic bullet piston method. In the static case, the presence of the polymer additive did not noticeably change the breaking tension, but in the dynamic tests the breaking tension was *reduced*; the greater the concentration of polymer the greater was the reduction. To understand these contradictory results it is necessary to better understand how polymer additives modify the state of stress in a flowing fluid.

Mathematical studies of the effects of polymer additives on cavitation have not been published. There are two difficulties: the choice of the constitutive equation and the calculation of the pressure; the “pressure” in an Oldroyd B fluid and in a second order fluid are not in the same relation to the mean normal stress. The composition of the stress into a pressure and a part related a-priori to deformation is not unique, but the state of stress a point given by different constitutive equations

applied to the same problem can be compared.

Ellis and Ting [1974] constructed an argument based on the second order fluid noting that this fluid "... appears capable of explaining the observed cavitation pressure due to flow." Their idea is that cavitation suppression is due to an over-pressure in the region of the obstacle where the pressure would ordinarily be low enough to permit cavitation in the case of water. Further, they attribute the difference to the presence of a correction term in the Bernoulli equation for a second order fluid.

Lumley in an appendix to the aforementioned paper agreed with their arguments and conclusions but he objected to using a second order model "... since it includes only in a rudimentary way the effect of molecular extension..." which Lumley argued was responsible for drag reduction. He gets a similar result using a centerline Bernoulli equation for an Oldroyd B model. (Lumley [1972]).

Here, I am going to construct a mathematical analysis based mainly on the second order theory in which the idea of Ellis and Ting is put to test; we find there is a basis for the overpressure they mention, at the boundary of a rigid solid, and there is a Bernoulli equation in potential flow with an extra term. However, the extra term contributes to the promotion rather than to the suppression of cavitation. It is certain that the second order fluid is far from an ideal model for the fast flows usually studied in experiments on cavitation. On the other hand, the second order fluid is the only model of a viscoelastic fluid, other than the linear viscoelastic model, which admits a pressure function in potential flow (Joseph & Liao [1996]). Moreover, the second order fluid admits a pressure function of an entirely different origin, for perturbations of Stokes flow, which allows one to draw conclusions following from the no-slip or incomplete slip condition of real obstacles on which cavitation bubbles appear that cannot be studied in potential flows.

11 Potential flow cavitation of viscoelastic fluids

The constitutive equation for a second order fluid is usually given as

$$\mathbf{T} = -p\mathbf{1} + \eta\mathbf{A} + \alpha_1\mathbf{B} + \alpha_2\mathbf{A}^2, \quad (38)$$

$$\mathbf{B} = \frac{\partial\mathbf{A}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{A} + \mathbf{A}\mathbf{L} + \mathbf{L}^T\mathbf{A},$$

$$\mathbf{L} = \nabla\mathbf{u} \left(L_{ij} = \frac{\partial u_i}{\partial x_j} \right),$$

$$\mathbf{A} = \mathbf{L} + \mathbf{L}^T,$$

$$\alpha_1 = -\Psi_1/2, \alpha_2 = \Psi_1 + \Psi_2 \quad (39)$$

where $\Psi_1 > 0$ and $\Psi_2 < 0$ are the coefficients of the first and second normal stress differences. (A typical value for estimates is $\Psi_2 = -\Psi_1/10$)

The “pressure” p in (38) is not the mean normal stress; it is better for cavitation studies to write (38) as

$$\mathbf{T} = -\tilde{p}\mathbf{1} + \mathbf{S} \quad (40)$$

where, since $\text{Tr}\mathbf{A} = \text{div } \mathbf{u} = 0$ and $\text{Tr}\mathbf{B} = \text{Tr}\mathbf{A}^2$,

$$\tilde{p} = -\frac{1}{3} \text{Tr}\mathbf{T} = p - \frac{1}{3}(\alpha_1 + \alpha_2) \text{Tr}\mathbf{A}^2 \quad (41)$$

and

$$\mathbf{S} = \alpha_1\mathbf{B} + \alpha_2\mathbf{A}^2 - \frac{1}{3}(\alpha_1 + \alpha_2) \text{Tr}\mathbf{A}^2, \text{Tr}\mathbf{S} = 0 \quad (42)$$

The reader has already been alerted several times to the fact that the use of p in cavitation studies has no rational foundation.

Joseph [1992] has shown that equations of motion for a second order fluid admit all potential flow solutions; that is, velocity fields of the form

$$\mathbf{u} = \nabla\phi, \nabla^2\phi = 0 \quad (43)$$

give rise to solutions of

$$\rho \frac{d\mathbf{u}}{dt} = \text{div}\mathbf{T}, \text{div}\mathbf{u} = 0 \quad (44)$$

where \mathbf{T} is given by (38) and p has a Bernoulli equation with an extra term:

$$p + \rho \frac{|\mathbf{u}|^2}{2} - \frac{\hat{\beta}}{4} \text{Tr}\mathbf{A}^2 + \rho \frac{\partial\phi}{\partial t} = 0 \quad (45)$$

The extra term

$$-\frac{\hat{\beta}}{4} \text{Trace } \mathbf{A}^2 = \hat{\beta} \nabla\mathbf{u} : \nabla\mathbf{u} < 0 \quad (46)$$

is positive whenever the climbing constant $\hat{\beta} = 3\alpha_1 + 2\alpha_2$ is positive and it acts in opposition to inertia $\rho|\mathbf{u}|^2/2$. Using the estimate $\Psi_2 = -\Psi_1/10$ we get

$$\hat{\beta} = 2\Psi_2 + \Psi_1/2 = \frac{3}{10}\Psi_1 > 0.$$

Since nearly all polymeric solutions climb rotating rods we may assume that $\hat{\beta} > 0$. Looking at p in (45) we might conclude that the compression due to the extra

term in the Bernoulli equation is increased, suppressing cavitation as was supposed by Ellis and Ting [1970]. However, the mean normal stress \tilde{p} is another (more relevant) pressure given by

$$\tilde{p} + \rho \frac{|\mathbf{u}|^2}{2} - \frac{1}{3}(5\alpha_1 + 2\alpha_2)\nabla \mathbf{u} : \nabla \mathbf{u} + \rho \frac{\partial \varphi}{\partial t} = 0 \quad (47)$$

gives rise to a different result. Since

$$5\alpha_1 + 2\alpha_2 = 2\Psi_2 - \Psi_1/2 < 0 \quad (48)$$

the extra term in the mean normal stress promotes rather than suppresses cavitation.

I have argued that cavitation at any point in a flowing liquid depends on the state of stress and breaking stress there. For potential flow

$$\mathbf{B} = \left(\frac{\partial}{\partial t} + \mathbf{u} \cdot \nabla \right) \mathbf{A} + \mathbf{A}^2 \quad (49)$$

and the deviatoric part \mathbf{S} of the stress \mathbf{T} is given by

$$\mathbf{S} = (\eta + \alpha_1 \frac{d}{dt}) \mathbf{A} + (\alpha_1 + \alpha_2) [\mathbf{A}^2 - \frac{1}{3} \text{Tr } \mathbf{A}^2] \quad (50)$$

and the mean normal stress $-\tilde{p}$ is given by (47) as

$$-\tilde{p} = +\rho \frac{\partial \varphi}{\partial t} + \rho \frac{|\mathbf{u}|^2}{2} - \frac{1}{12}(2\Psi_2 - \Psi_1/2) \text{Tr } \mathbf{A}^2 \quad (51)$$

since $-2\Psi_2 + \Psi_1/2 > 0$ the term with $\text{Tr } \mathbf{A}^2$ is positive, tensile promoting cavitation. Therefore the suppression of cavitation must arise from deviatoric part of the stress. However, the deviator has both positive and negative eigenvalues (see Joseph [1992]) with the positive ones corresponding to tension, promoting cavitation.

The progressive reduction of the breaking strength with increasing concentration which was observed by Sedgewick and Trevena [1978] is consistent with the analysis just given, but the use of a second order fluid model to study cavitation in potential flows is not justified, too many effects are left out of the model.

12 Flow cavitation on solids

We can study flow cavitation for nearly steady flow of a viscoelastic fluid over bodies on which the no-slip condition applies as a viscoelastic perturbation of Stokes flow. The second order fluid arises universally as the asymptotic form taken by all

the special models in common use. Though the motions to which such an analysis can rigorously apply are restricted, the rheology for those motions is completely general; every special model has η, α_1, α_2 as limiting values of model parameters in nearly steady slow motion.

The perturbation of Stokes flow with a second order fluid gives rise to a pressure function for motions in two-dimensions (Tanner [1966]) and for motions in three dimensions when $\alpha_1 = -\alpha_2$ (Giesekus [1963]). The implications for forces on solid particles of the two-dimensional reduction were studied by Joseph [1996] and Joseph & Feng [1966] who showed that the normal stress on each and every point on the boundary of a rigid solid is given by

$$T_{nn} = -p_s + \frac{\Psi_1}{2\eta} \left(\frac{\partial p_s}{\partial t} + \mathbf{u} \cdot \nabla p_s \right) - \frac{\Psi_1}{4} \dot{\gamma}^2 \quad (52)$$

where n is a coordinate along the outward \mathbf{n} on the boundary $\Psi_1 = -2\alpha_1$, $\dot{\gamma}$ is the shear rate of the Stokes flow at that point and p_s is the Stokes flow pressure obtained from solving $\nabla p_s = \eta \nabla^2 \mathbf{u}$ for no-slip boundary condition. For steady flows over stationary bodies like those used in cavitation studies we get

$$T_{nn} = -p_s - \frac{\Psi_1}{4} \dot{\gamma}^2 \quad (53)$$

which is positive in tension; the viscoelastic contribution $-\Psi_1 \dot{\gamma}^2 / 4$ is a compression. In the three dimensional case with $\alpha_1 + \alpha_2 = 0$, the Stokes flow problem can be solved and $p_s = p_\infty - \frac{3}{2} \eta \frac{U}{a} \cos\theta$ and $\dot{\gamma} = \frac{5}{2} \frac{U}{a} \sin\theta$ where θ is the polar angle.

The shear rate $\dot{\gamma}$ is greatest where the streamlines of the Stokes flow are most crowded; these are the “high” velocity points on the body. Since compressive stresses are generated at “high” velocity points, we get additional “pressure” due to viscoelasticity precisely at the “cavitating” points where the pressure is lowest in potential flow. The compressive contribution $-\Psi_1 \dot{\gamma}^2 / 4$ is largest, for example, near the equator of a sphere where cavitation bubbles would appear if no polymers were present.

Van der Meulen [1973] has shown that cavitation inception on a hemispherical-nosed stainless steel body in a water tunnel is greatly reduced by the presence of polyethylene oxide, while a teflon coating showed a much smaller effect. If gas is nucleated at the surface of the hydrophilic stainless steel body it will be replaced by water. The additional pressure due to (52) is effective in suppressing this nucleation of gas. The teflon coated sphere is hydrophobic. Since it likes gas, the liberation of cavitation bubbles is not so evident and not readily replaced by

water. The additional pressure should also work here, but its overall effect could be less evident due to the adhesion of nucleated gas on the teflon.

The formula (53) should not be expected to apply strictly to the high speed flows in which cavitation is suppressed in experiments. Moreover, Ψ_1 is not known in the dilute solutions used in these experiments. It is nevertheless of interest to use (53) for an estimate, to see if it gives rise to a pressure level large enough to effect cavitation.

To construct such an estimate we first use data from figure 3 of Arndt et al. [1976] for flow at $U = 18.3\text{m/s}$ of a 20ppm solution in water of Polyox WSR-301 over a $2R=7.62\text{cm}$ diameter hemispherical nosed body. Estimating $\dot{\gamma} = O(U/R)$ we get $\dot{\gamma} \approx 500\text{sec}^{-1}$, $\dot{\gamma}^2 \approx 2.5 \times 10^5\text{sec}^{-2}$. To get pressure increases of the order of the vapor pressure; i.e., one hundredth of an atmosphere we would need to satisfy

$$\frac{\Psi_1}{4}(2.5 \times 10^5\text{sec}^{-2}) = 10^4 \frac{\text{dynes}}{\text{cm}^2} \quad (54)$$

or

$$10\Psi_1 = O(1) \frac{\text{gm}}{\text{cm}} \quad (55)$$

This is a value of Ψ_1 smaller than those which can be measured on rheometers but it is possibly larger than the value of Ψ_1 which could be expected from solutions so dilute as 20ppm.

A second estimate taken from data in Table 2 in the paper by Ellis, Waugh and Ting [1970] for flow of different fluids over a $1/4$ in radius hemispherical nose body; typical shear rates are of the order of

$$\frac{U}{r} \approx \frac{500}{1/4} = 2000\text{sec}^{-1}$$

For fluids, like aqueous 50 ppm polyox, for which cavitation suppression was observed we need Ψ_1 larger than

$$100\Psi_1 = O(1) \frac{\text{gm}}{\text{cm}}. \quad (56)$$

Ting [1978] studied cavitation on flat top posts protruding from a disk rotating in aqueous polyox FRA solutions with concentrations of 100, 250 and 500 ppm. The 0.29cm diameter posts were 0.29cm high and cavitation suppression was observed for values of $Re = \rho Ud/\eta > 10^5$ where, for water $\rho/\eta = 100^{-1}$ and $d = 0.29$. Hence

$$\dot{\gamma} \cong \frac{U}{d/2} \cong 5000\text{sec}^{-1}$$

corresponding to estimates like (54) giving rise to

$$\Psi_1 \approx 1.6 \times 10^{-3} \frac{\text{gm}}{\text{cm}}. \quad (57)$$

Dilute polymer solutions might have normal stress coefficients larger than (57) but nothing is known. In section 12 we will argue that these viscoelastic effects, with very small $\Psi_1 = 2\eta\lambda_1$ where λ_1 is the relaxation time of the fluid, may be seen on small but not large bodies.

The estimates following from (53) are perhaps not so outrageous as to eliminate consideration of the mechanism of inhibition of cavitation inception by shear-induced normal stress. This mechanism has not much in common with mechanisms involving extensional effects due to the irrotational stretching of polymer coils which are popular in theory of drag reduction. Some of the problems encountered in estimating the viscoelastic effects in an irrotational strain field were discussed by Arndt et al. [1976] and they appear to support an alternate explanation presented first by Arakeri and Acosta [1973] which is discussed next.

Arakeri and Acosta [1973] have shown that polymer additives influence the point of laminar separation and inhibit cavitation inception. Van der Meulen [1976] did holographic studies in which he shows that when polymers are added, separation of the boundary layer does not occur and the boundary layer exhibits a turbulent character. This may be compatible with the inhibition of cavitation inception by shear induced normal stresses. In a Newtonian fluid, the position of separation is controlled by the pressure variation on the solid surface. In a viscoelastic fluid the normal stress T_{nn} is not given by pressure alone and terms contributed by normal stresses due to shearing do not vanish; obviously the fluid at the boundary of a solid can sense T_{nn} only and various decompositions of the stress giving rise to different expressions for the pressure are not relevant. To their credit, Ellis et al. [1970] recognize this when they say (p. 463) that "... cavitation bubble inception is inhibited by an actual change of principal stress (or less precisely, pressure)...". The effects of viscoelastic contributions to the variation of T_{nn} on the boundary of a solid which determines the position of points of separation are not clear. Arndt [1981] notes that "... All that can be said at the moment is that available experimental and theoretical information do not provide any clear cut criteria for the determination of early transition in the flow of polymer solutions."

13 Size effects and the elasticity number

A very interesting and robust effect in cavitation inception inhibition is that the inhibition is large for small bodies and is negligible for large bodies. Arndt et al.

[1976] studied cavitation in two recirculating water tunnels using four hemispherical nosed bodies ranging in size from 6.3mm to 203mm. They found that the desinent value of the cavitation index was reduced as much as 50% on the smaller bodies but the effect on the 203mm was negligible. The effect persisted even in well degraded polymer solution. Huang [1971] noted that the cavitation inception reduction was much smaller when a larger model was used in a water tunnel. A small propellor was observed by White [1971] to exhibit greatly reduced cavitation, in agreement with the diameter effect hypothesis.

A size effect scaling with the square of the radius is consistent with the equations governing the motion of viscoelastic fluids over bodies. For example, in the flow of a second order fluid the effects of inertia proportional to ρu^2 and viscoelastic normal stresses proportional to $\Psi \dot{\gamma}^2$ are in opposition and estimating $\dot{\gamma} \approx u/a$ where a is the particle size, we find that viscoelasticity dominates near the body when $\Psi_1/\rho a^2$ is large.

The same scaling follows from analysis of Oldroyd B fluids made dimensionless by $[a, U, a/U, \eta U/a] = [\text{length}, \text{velocity}, \text{time}, \text{stress}]$. The dimensionless equations for momentum and stress then take the form

$$\begin{aligned} Re \left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} \right) &= -\nabla p + \nabla \cdot \mathbf{T}, \\ \mathbf{T} + De \overset{\nabla}{\mathbf{T}} &= \mathbf{A} + \frac{\lambda_2}{\lambda_1} De \overset{\nabla}{\mathbf{A}} \end{aligned}$$

where $\nabla \cdot \mathbf{u} = 0$ and

$$\overset{\nabla}{\mathbf{T}} = \frac{\partial \mathbf{T}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{T} - \nabla \mathbf{u} \cdot \mathbf{T} - \mathbf{T} \cdot \nabla \mathbf{u}^T.$$

The parameters are:

$$\begin{aligned} Re &= U a \rho / \eta \quad (\text{Reynolds number}), \\ De &= U \lambda_1 / a \quad (\text{Deborah number}), \\ \lambda_2 / \lambda_1 &\quad (\text{retardation / relaxation time ratio}). \end{aligned}$$

The maximum elastic effects are for $\lambda_2 = 0$ (Maxwell model). The fluid is Newtonian when $\lambda_2 = \lambda_1$ and $0 \leq \lambda_2 / \lambda_1 \leq 1$.

Huang, Hu and Joseph [1997] used these equations to study the sedimentation of an ellipse with semi-major axis a in an Oldroyd B fluid. They found that the results correlated better with

$$E = De/Re = \lambda_1 \eta / \rho a^2 \quad (\text{elasticity number})$$

and

$$M = \sqrt{Re De} = U/c \quad (\text{viscoelastic Mach number})$$

where $c = \sqrt{\eta/\lambda_1\rho}$ is the speed of shear waves. They found that when the elasticity number is smaller than critical the fluid is essentially Newtonian with broadside-on falling at the centerline of the channel. For larger elasticity numbers the settling turns the long side of the particle along the stream in the channel center for all velocities below a critical one; identified with a critical Mach number of order one.

The elasticity number depends on the fluid and the particle size and not on the velocity. It says that small particles can experience elastic response in circumstances in which larger particles have essentially Newtonian response, as in experiments on cavitating flows over bodies. On the other hand, the Mach number does not depend on the particle size. The elastic response of small particles is thus rather firmly implied by direct and elementary study of the equations governing the motion of Oldroyd B fluids.

14 Discussion

It is perhaps useful to think of cavitation as a topic in a more general theory of breaking and mixing of incompressible materials in which cavitation and fracture of amorphous solids are different, but perhaps related phenomena. Such a theory might lead to better understandings of the dynamics of mixing and mixers which is an important but as yet undeveloped subject.

The breaking of an incompressible continuum is obviously controlled by breaking stresses and cavitation is not different. The fluid knows the state of stress at a point but it cannot decompose this stress into a pressure determined by the flow dynamics and a constitutively determined extra stress. It is however useful, and is always possible, to define the pressure as the mean normal stress, even when, as in many viscoelastic fluids, it is not originally so defined; in this case the extra stress is deviatoric, with plus and minus entries on its leading diagonal.

It may be assumed that in each liquid or solid there is a field of breaking strengths defined at each and every point of the continuum; this strength field could be discontinuous with singular values at nucleation sites. The breaking of a liquid requires that we compare some function of the principal stresses with the strength criterion; for instance it would be consistent with the literature on cavitation to look for the tensile strength of liquids. In this case we look at the magnitude of the maximum value in the diagonalized extra stress, which must be positive and compare it to the breaking strength (cavitation inception) of the liquid. Our comparison requires that we distinguish also the direction of the maximum tension in the moving continuum, which is determined by diagonalization of the stress tensor; we could look at equi-tension lines and seek the locus of maximum values. It is not completely clear that the maximum tension is the right criterion, though it

is in some sense conventional and does appear to agree with experiments on the tensile strength of liquids. One interesting consequence is that we could test the criterion and the directionality of this criterion by setting the pressure level slightly above the breaking strength in a pure shear flow between parallel plates in which the maximum tension is 45° from the flow direction.

The state of stress in a flowing polymeric liquid could in principle have strong effects on cavitation. The evidence from experiments with dilute solutions used for drag reduction weighs strongly for cavitation inhibition, though neutral inhibition results for cavitation on large bodies and a reduction in the breaking strength of liquids with polymer concentration have been reported. It is argued that potential flow approximations are not appropriate for cavitation studies except possibly in the case of acoustic cavitation. An analysis of cavitation inhibition of polymeric liquid based on the second order fluid model, in which the original pressure in the model is embedded in a more appropriate decomposition based on the mean normal stress, shows that inhibition will not arise in potential flows, increased cavitation is predicted. It was stressed that the definition of pressure in non-Newtonian fluids is not unique and differs from one constitutive equation to another, but the mean normal stress is uniquely defined and sets an appropriate level against which one can measure the differences between principal compressive and tensile stresses.

Another positive feature of the second order fluid model for studies of cavitation is that a pressure function can be derived a-priori for flows which perturb Stokes flow as well as for potential flows. In the Stokes flow perturbation the no-slip condition is applied so that the analysis is appropriate for discussions of cavitation which originate from nucleation on solid boundaries. An additional normal stress T_{nn} equal to $-\Psi_1 \dot{\gamma}^2 / 4$ which is always compressive and proportional to the square of the shear rate at each point on the boundary of a solid body is equivalent to a viscoelastic pressure which is large at places where the flow is fast, the opposite of inertia. Though the speeds of cavitating flows are larger than those for which analysis based on second order models is valid, estimates of the size effects do give rise to additional pressures which could suppress nucleation at solids. An important additional effect is that the variation of “backpressure” which controls the position of points of separation would be changed by the presence and variation of this additional viscoelastic pressure.

All evidence from experiments indicates that cavitation inhibition is much more pronounced on small than on large bodies. This observation is consistent with the analysis of response Oldroyd B fluids. The controlling parameters of these fluids are the Reynolds number Ua/ν and the Deborah number $U\lambda/a$. The Deborah number shows a size dependence of the required kind, but the number depends on U which also depends on a . The elasticity number E which is the ratio $De/Re =$

$\lambda\nu/a^2$ is independent of U and it indicates enhanced elasticity for small bodies in a given fluid and the size effect is proportional to a^2 rather than a . The predictions of viscoelastic response based on the elasticity number should be robust since it arises from direct analysis of dimensions but it has not yet been tested in cavitation studies.

15 Conclusions

- The pressure in incompressible Newtonian fluids is the mean normal stress. The stress is decomposed into a pressure and stress deviator with a zero trace. The pressure in incompressible Non-Newtonian liquids is given by the constitutive equation and has no intrinsic significance. Cavitation criteria for liquids in motion must be based on the stress and not on the pressure. The liquid cannot average its stresses or recognize the non-unique quantity called pressure in non-Newtonian fluids.
- It is convenient for the study of cavitation of flowing liquids to decompose the stress into a deviator and mean normal stress. The deviator has positive and negative normal stresses, deviating from the average. The most positive value of principal stresses is the maximum tension. The stress in non-Newtonian liquids should also decompose the stress into average and deviator.
- A cavitation bubble will open in the direction of maximum tension in principal coordinates. The angles defining the principal axis determine how a cavity will open; angles are important.
- A liquid can cavitate in shear. However, it is pulled open by tension in the direction defined by principal stresses; Newtonian liquids in pure plane shear will open 45° from the direction.
- Cavitation in a flowing liquid will occur at a nucleation site when the maximum tensile stress in principal axes coordinates is such as to make the total stress smaller than the cavitation pressure.
- Cavitation is a fast, non-equilibrium event resembling fracture in which the cavity first opens and then fills with gas.
- Outgassing is cavitation of liquid gas in solution.
- Analyses of the second order fluid shows that explanations based on irrotational strain fields are not consistent with inhibition but cavitation on solid

bodies can be inhibited by compressive normal stresses due to shear which give rise to increased pressures preventing the nucleation of gas on bodies. Boundary layer separation is controlled by the variation of the total normal stress which has a substantial viscoelastic component. The observed size effects on cavitation in the presence of polymer additives in which inhibition is seen on small but not large bodies is implied by the elasticity number which is independent of velocity and increases with the reciprocal of the square of particle radius.

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Acknowledgement. This work was supported NSF/CTS - 9523579 and ARO grant DA/DAAH04. Helpful discussions with Roger Arndt are gratefully acknowledged.