Evaluation of finite rate homogenous mixture model in cavitation bubble collapse

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Abstract. The objective of this study is to evaluate the performance of finite rate homogeneous mixture models in three key aspects: (a) the ability of the model to predict the dynamics of resolved small scale vapor regions, (b) the importance of finite rate mass transfer and (c) the impact of assuming $2D$ flow as is done in RANS simulations with a statistically homogeneous direction. We consider a bubble collapse problem to evaluate all these effects.

1. Introduction

Cavitation often causes erosion due to the high impact loading caused by vapor cavity collapse. Sheet to cloud cavitation is a classic phenomenon where the cloud cavity often collapses and causes severe noise, vibration and surface erosion. Homogeneous mixture modelling is the most commonly used physical model to study turbulent sheet to cloud cavitation and these models employ both finite rate (e.g. Singhal *et al.* [1]) and equilibrium mass transfer terms (e.g. Schnerr *et al.* [2]) to model mass transfer between water and vapor. Using the finite rate mass transfer model of Saito *et al.* [4] as a representative example, we make use of the numerical method of Gnanaskandan and Mahesh [3] to study three key effects: the performance of the model in predicting the dynamics of resolved small scale vapor regions, the effect of finite rate mass transfer and the consequence of two dimensional assumptions as is commonly done in RANS simulations.

2. Spherical bubble collapse

While it is well known that homogeneous mixture models are reasonably accurate in predicting large scale cavitation dynamics, the final stages of a cloud collapse involve vapor regions of smaller length scales. The performance of homogeneous models in predicting the dynamics of these smaller scale vapor regions is of interest. In this section, the finite rate homogeneous mixture model is applied to study a 0.4 mm spherical bubble collapse problem. Evolution of a spherical cavity in the absence of surface tension and fluid viscosity with the assumption of liquid incompressibilty is given by [5] ,

$$R\ddot{R} + \frac{3}{2}\ddot{R} = \frac{p(R) - p_\infty}{\rho}, \tag{1}$$

where ρ is the liquid density, p_∞ is the ambient pressure of liquid and p is the pressure in the liquid at the interface. The collapse time ($t_{rayleigh}$) can be obtained by integrating the above

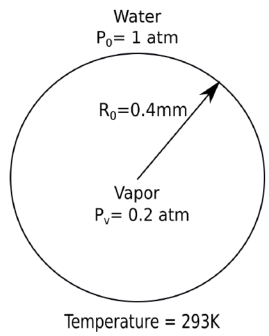

Figure 1. Schematic of the cylindrical bubble collapse.

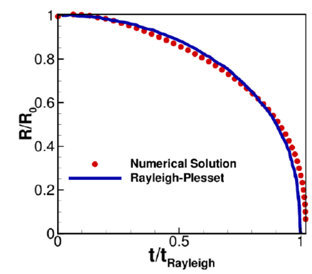

Figure 2. Evolution of radius with time.

equation. We consider a spherical vapor bubble of initial radius (R_0) in water as shown in Figure 1. The ambient pressure is P_0 and pressure inside the bubble is considered to be equal to vapor pressure ($P_v = 0.02$ atm at 293 K). Initial volume fraction of vapor (α_0) is 0.99 inside the bubble and 0.01 outside the bubble. The reference values considered are $L_r = 0.4mm$, $U_r = 10m/s$ and $\rho_r = 1000kg/m^3$. We consider 75 equidistant grid points spanning the initial bubble radius. Reflections are minimized by placing boundaries at $25R_0$ from the the bubble region with absorbing sponge layer starting from $20R_0$. The numerical result in Figure 2 shows good agreement with the analytical solution for evolution of bubble radius with time showing the suitability of the model in predicting the dynamics of small scale vapor regions as long as they are resolved on the computational grid.

3. Effect of finite rate mass transfer

Equilibrium models assume mass transfer between phases to occur at rates much faster than that of convection. In this section we evaluate this effect by first interpreting the condensation and convection time scales from the mass transfer terms. Then a series of numerical experiments are performed to corroborate our analysis and finally the interesting finite rate phenomenon of 'dynamic delay' is discussed.

3.1. Non-dimensional analysis

The collapse process generates very high pressure and the magnitude of this collapse pressure depends on the relative rate of condensation and convection. In a cavity collapse problem, the evaporation term is zero. The non-dimensional mass fraction transport equation is thus given by

$$\frac{\partial \tilde{\rho} Y}{\partial \tilde{t}} = -\frac{\partial \tilde{\rho} Y \tilde{u_j}}{\partial \tilde{x_j}} - \tilde{S_c}, \tag{2}$$

where $\tilde{S_c}$ is the non-dimensional condensation source term given by

$$\tilde{S_c} = \frac{C_c \alpha^2 (1-\alpha)^2 \max{(P - P_v, 0)}}{\sqrt{2\Pi R_g T_s}} * (\frac{L_r}{\rho_r * U_r}), \tag{3}$$

where α is the void fraction, C_c is an empirical constant, P is the pressure, P_v is the vapor pressure, R_g is the characteristic gas constant of vapor and T_s is the saturation temperature. Here L_r, ρ_r and U_r are the reference quantities. Defining a_s as speed of sound in vapor at saturation temperature ($\sqrt{\gamma R_g T_s}$), $\tilde{S_c}$ can be rearranged as a product of three non–dimensional parameters,

$$\tilde{S_c} = (C_c L_r) * (\frac{P - P_v}{\rho_r U_r^2}) * (\frac{U_r}{a_s}) * [\alpha^2 (1 - \alpha^2)]. \tag{4}$$

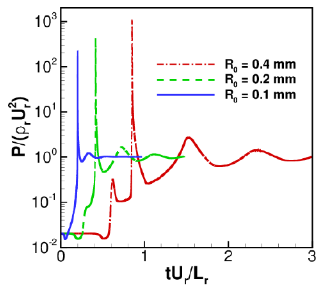

Figure 3. Temporal evolution of pressure at center for different initial bubble radius.

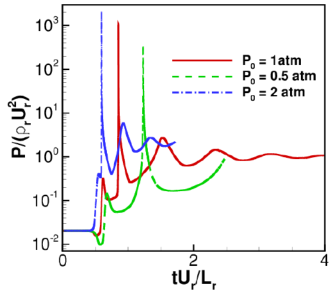

Figure 4. Temporal evolution of pressure at center for different ambient pressures.

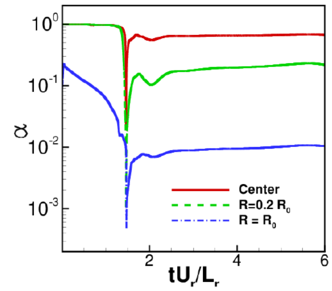

Figure 5. Volume fraction evolution at several locations showing residual vapor.

For the bubble collapse problem, the reference length is taken as the initial radius (R_0), and the reference velocity is the initial material front speed $\sqrt{(P_0 - P_v)/\rho_r}$. The non-dimensional source term can be considered as the relative strength of condensation with respect to convection. It can also be construed as the ratio of the convection and condensation time scales. Consequently, these non-dimensional parameters allow us to comment on the rate and strength of collapse under various flow conditions. As the bubble radius is increased, the non-dimensional parameter $C_c L_r$ increases, which increases the condensation strength. Thus collapse of larger bubbles should result in larger collapse pressure. Similarly larger driving pressure $P - P_v$ should also result in larger collapse pressures. Further, if we interpret this behavior in terms of time scales, increasing the driving pressure decreases the condensation time scale which allows rapid collapse of the bubble. These results are verified using numerical simulations in Section 3.2.

3.2. Effect of initial radius R_0 and liquid pressure P_0

Parametric studies are conducted for $R_0 = 0.4, 0.2$ and 0.1mm, and $P_0 = 0.5, 1$ and 2 atm. Figure 3 shows the pressure evolution at the center of the bubble for $R_0 = 0.4, 0.2$ and 0.1 mm. It is observed that both the collapse pressure magnitude and the time taken for collapse (τ_c) reduce as the initial radius is reduced. The larger bubble has more mass to be compressed and hence leads to larger collapse pressure. Figure 4 shows pressure evolution at the center for $P_0 = 0.5, 1$ and 2 atm. The main consequence of changing P_0 is in changing the initial velocity with which the material front moves (U_0). For the highest P_0, the difference in speed between material front and the compression wave is negligible and hence the initial pressure peak due to compression is not visible. Further maximum collapse pressure and minimum collapse time are obtained for $P_0 = 2$ atm. It is evident that higher the pressure gradient ($P_0 - P_v$), the faster the collapse will be and higher the collapse pressure.

3.3. Dynamic delay

For $R_0 = 0.4$ mm and $P_0 = 1$ atm, an interesting phenomenon is observed. Once the material front reaches the center, all the vapor is expected to be converted to water since the pressure reaches values above vapor pressure. However, we observe that even long after collapse not all vapor gets destroyed. This is evident from the void fraction evolution at two locations close to the center in Figure 5. It is observed that the void fraction does not relax back to 0.01 and remains at a value as high as 0.7 at the center. The void fraction is significant even at $r = 0.2R_0$. This “dynamic delay” [6] phenomenon occurs because of the large difference in the values of time scales associated with condensation and convection. As the size of the vapor becomes very small, the condensation time scale becomes very large when compared to convection time scale and hence it takes a long amount of time to observe the effect of condensation. This phenomenon

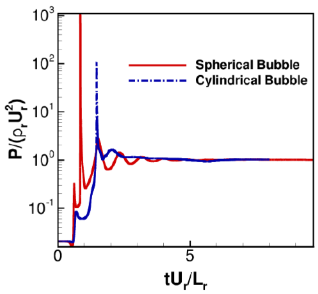

Figure 6. Comparison of temporal evolution of pressure at center for spherical and cylindrical bubbles.

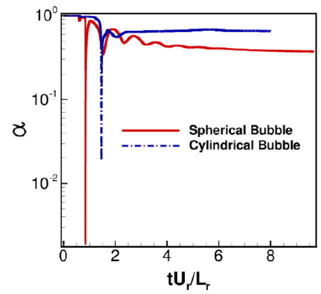

Figure 7. Comparison of temporal evolution of volume fraction at center for spherical and cylindrical bubbles.

is of practical significance since the undestroyed vapor can get convected by the flow and grow or collapse in other regions depending on the conditions. Equilibrium mass transfer models are not expected to predict this phenomenon.

4. Comparison of cylindrical and spherical bubble collapse
Here, we consider a case of collapsing cavitation bubble in 2D (i.e. a cylindrical shape bubble with infinite length in span wise direction) and compare it with spherical bubble collapse. All the parameters are same as those used for spherical bubble collapse in Section 3.2. As seen from Figure 6, a spherical bubble of same size under same flow parameters generates an order of magnitude higher collapse pressure compared to a cylindrical bubble. Further, from Figure 7, we can see that volume fraction of vapor remaining at the center is much lower for spherical bubble collapse compared to cylindrical bubble collapse, which shows that condensation strength is higher in the case of spherical bubble collapse. The practical significance of this result is that, 2D RANS simulations will under predict the condensation strength and collapse pressure.

5. Summary
A finite rate mass transfer homogeneous mixture model is used to study bubble collapse. It is observed that as long as the vapor region is sufficiently resolved, homogeneous models are accurate in predicting the dynamics of small vapor regions, making them a suitable approach to study sheet to cloud cavitation transition. The effect of finite rate mass transfer is analyzed using a non–dimensional analysis and a parametric study corroborates the non–dimensional analysis. A finite rate phenomenon - 'Dynamic delay' is also observed in the simulations. Finally the effect of considering $2D$ collapse as is commonly assumed in RANS is illustrated by comparing cylindrical and spherical bubble collapse. It is shown that cylindrical bubble collapse produces much smaller collapse pressure than a spherical bubble collapse which is an important design parameter.

6. Acknowledgments
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