

Aircraft Navigation Aided by Vehicle Equations of Motion

Elan M. Graupe, Maziar S. Hemati, Demoz Gebre-Egziabher *University of Minnesota*

BIOGRAPHY

Elan M. Graupe is a PhD Candidate at the University of Minnesota whose research interests are in guidance, navigation, and controls of aerospace vehicles.

Maziar S. Hemati is an Associate Professor at the University of Minnesota. Professor Hemati's research aims to develop efficient and reliable strategies for the modeling, analysis, and control of fluid flows.

Demoz Gebre-Egziabher is a Professor in the Department of Aerospace Engineering and Mechanics at the University of Minnesota Twin Cities and Director of NASA's Minnesota Space Grant Consortium. His research focuses on multi-sensor positioning, navigation, and timing (PNT) for aerospace and autonomous vehicle applications. He is a Fellow of the ION.

ABSTRACT

In this work, we present an approach that fuses measurements from inertial sensors, a GNSS receiver, and the aircraft equations of motion. The results show that during GNSS-stressed or denied operations, fusing inertial sensor measurements with the equations of motion provides a positioning and navigation solution that can be substantially more accurate than that obtained from an unaided inertial navigation system. The study further demonstrates that accurate modeling of aerodynamic forces, moments, and wind, together with proper characterization of their associated uncertainties, is critical to achieving this performance. Aerodynamic model uncertainty, along with other system uncertainties, are modeled as additive biases on the thrust force and aerodynamic coefficients. A comprehensive sensitivity analysis is presented to quantify the effect of errors in the estimated aerodynamic coefficients on navigation performance. This analysis identifies and prioritizes the aerodynamic model parameters that most strongly influence navigation accuracy. The proposed approach is evaluated using a longitudinal model of a Ryan Navion general aviation aircraft flying a final approach trajectory. The results show that fusing commercial- or industrial-grade inertial sensors with the aircraft equations of motion yields navigation performance comparable to that of a low-end tactical-grade unaided inertial navigation system.

I. INTRODUCTION

Accurate knowledge of kinematic states (position, velocity, attitude) and air data parameters (airspeed, mach number, angle of attack, air density, etc.) is essential for the safe operation of aircraft. Traditionally, important air data parameters are estimated using measurements from a collection of pressure and temperature sensors collectively referred to as an air data system. Kinematic states are estimated using an integrated navigation system which consists of an inertial measurement unit (IMU) coupled with an aiding system such as GNSS or other radio-navigation systems.

IMUs are central to integrated navigation systems and are often categorized into *broad* performance groups based on the accuracy of the resulting position solution generated using them in a standalone (or unaided) fashion. Two such categories that will be the focus of this paper are the so-called consumer-grade sensors, which can produce position errors on the order of thousands of kilometers after an hour of standalone operation, and tactical-grade sensors, which can be expected to produce position errors on the order of tens of kilometers after an hour of standalone operation [Groves (2013)].

By far, the most commonly used IMUs today fall into the category of consumer-grade sensors owing to their affordability. They are prevalent in everyday consumer devices (phones, automobiles, VR headsets, etc.) and are also used in cost-sensitive aviation applications such as general aviation attitude and heading reference systems [Federal Aviation Administration (2011)]. When used as part of positioning, navigation, and timing (PNT) systems to generate position, velocity, and attitude estimates, they are almost always coupled with aiding systems such as GNSS receivers or optical cameras.

The air data sensors that are used to track the air data states of an aircraft are also subject to size, weight, and power constraints on a vehicle. Additionally, traditional air data sensors must be placed beyond the surface of a vehicle, making them vulnerable to the environment outside of the vehicle and therefore more prone to failure. Air data sensors have a history of failing in unfavorable weather conditions [Freeman et al. (2013)]. In extreme environments such as those relevant for hypersonic vehicles, traditional air data sensors are infeasible due to the high temperatures and pressures experienced. To deal with this vulnerability, IMU-based solutions such as synthetic air data systems have been proposed for use in general aviation and autonomous aerial

systems [Lie and Gebre-Egziabher (2013)]. However, it should be noted that synthetic systems which rely on low cost IMUs must be coupled with an aiding system such GNSS.

The continued vulnerability of GNSS to malicious or inadvertent jamming and interference makes it crucial that applications relying on low-cost (i.e., consumer-grade) inertial sensors, such as those found in general aviation, have a means of coping with GNSS-denied scenarios. In this paper, we present a method that can potentially enable inertial navigation systems using low-cost inertial sensors to tolerate limited periods of GNSS outage. As made clear by a recent general aviation mishap [National Transportation Safety Board (2026)], if not properly mitigated, these jamming and interference events can lead to catastrophic outcomes.

The proposed approach combines aircraft equations of motion with inertial sensor measurements to constrain the inertial sensor outputs, thereby limiting the growth in positioning and navigation errors during GNSS outages that are typical of standalone inertial navigation. We demonstrate that this approach allows systems employing low-cost, consumer-grade inertial sensors to achieve navigation performance approaching that of low-end tactical-grade sensors. Consequently, the duration over which a navigation system using this approach can maintain acceptable performance in the absence of GNSS aiding can be extended.

A byproduct of the proposed method is the estimation of air data parameters, such as angle of attack and atmospheric conditions such as wind speed, without having to rely on traditional air data sensors. This capability offers a redundant means of determining these air data quantities and, in the future, could potentially provide a means for analytical (i.e., software-based as opposed to hardware-based) redundancy for air data systems.

Accordingly, this paper attempts to answer the following questions: How can we optimally and effectively incorporate an airplane's equations of motion into the vehicle's navigation system to aid inertial navigation? How well must the dynamic model be known to effectively estimate kinematic states and air data parameters?

1. Prior Work

The idea of using vehicle equations of motion to aid in navigation is not new and has been used extensively in spacecraft guidance, navigation, and control applications [Markley and Crassidis (2016)]. However, applying a similar approach to ground vehicle and aircraft PNT applications is considerably more challenging because the equations of motion governing terrestrial and aerial vehicles are substantially more complex. An earlier exploration of this idea for aircraft PNT is presented in Koifman and Bar-Itzhack (2002). This work presents a feasibility study into the use of vehicle equations of motion to aid in an aircraft's inertial navigation system. In this feasibility study, they considered an estimation architecture where both the kinematic inertial navigation equations and the vehicle equations of motion are effectively placed into a single Extended Kalman Filter (EKF) where each dynamics model contains its own set of states that are driven by their corresponding dynamics equations. The feasibility study indicated that aircraft dynamics can effectively be used to aid inertial navigation when the inertial navigation system is using low-grade inertial navigation sensors, however a complete, practical estimation architecture design was not presented. The work in Lie and Gebre-Egziabher (2013) uses concepts related to synthetic air data estimation to estimate the air data states of a vehicle such as the angle of attack, sideslip, and airspeed. Synthetic air data estimation is the idea of using synthetic sensors to estimate the air data parameters of a vehicle. The synthetic *sensors* that would typically be used in these use cases are the vehicle's equations of motion. While the vehicle's equations of motion are not sensors in the traditional sense, they provide additional information about the motion of the vehicle that can be used to improve any dynamical state estimates.

Crocoll et al. (2013) and Crocoll et al. (2014) present algorithms for aiding in the navigation of small unmanned aerial vehicles using a vehicle dynamics model. These algorithms, in effect, run two parallel Kalman filters with separate dynamics and kinematics models, similar to what is described in Koifman and Bar-Itzhack (2002). However, these works employ model reduction techniques to remove correlated states and improve the efficiency of the filter. Most recently, Khaghani and Skaloud (2016) describes an approach where the vehicle equations of motion are used to drive a single EKF, while the inertial measurement unit is used in the measurement update. Our approach is closely related to the approach proposed in this work. We extend this line of work in several ways, including improvements to the wind and turbulence model, a sensitivity analysis, and improvements to the computational efficiency of the algorithm.

2. Paper Contribution & Organization

In the existing literature discussed above, effects from wind are either ignored, or are assumed to be a constant bias on the vehicle velocity [Koifman and Bar-Itzhack (2002)]. Because of this, existing methods tend to be best suited for estimating air data states of the vehicle and important kinematic states are often excluded. Another gap in the existing literature is methods to properly account for model uncertainty in the aerodynamics model. A full nonlinear model of aircraft forces and moments is rarely available, and available linearized models can include large amounts of uncertainty, especially when the vehicle state deviates from the model linearization state.

As such, the remainder of this paper is organized as follows: In section II, we present an overview of the proposed approach. In

section IV, we provide the details of the filtering architecture. In section V, we describe the covariance analysis used to evaluate the filtering architecture. In section VI, we present results from a set of simulation experiments. Finally, in section VII, we draw conclusions and present avenues for future work.

II. OVERVIEW OF PROPOSED APPROACH

We will use Figure 1 to provide an overview of the proposed concept. Without loss of generality, we limit our discussion to the longitudinal motion of an airplane [Etkin and Reid (1995)] operating over a sufficiently small geographical area such that a flat-Earth model may be assumed. This assumption means that we will not sense Earth's rotation rate and transport rate, which is consistent with the use of consumer-grade inertial sensors whose output errors are generally too large and therefore mask these two very small angular rates.

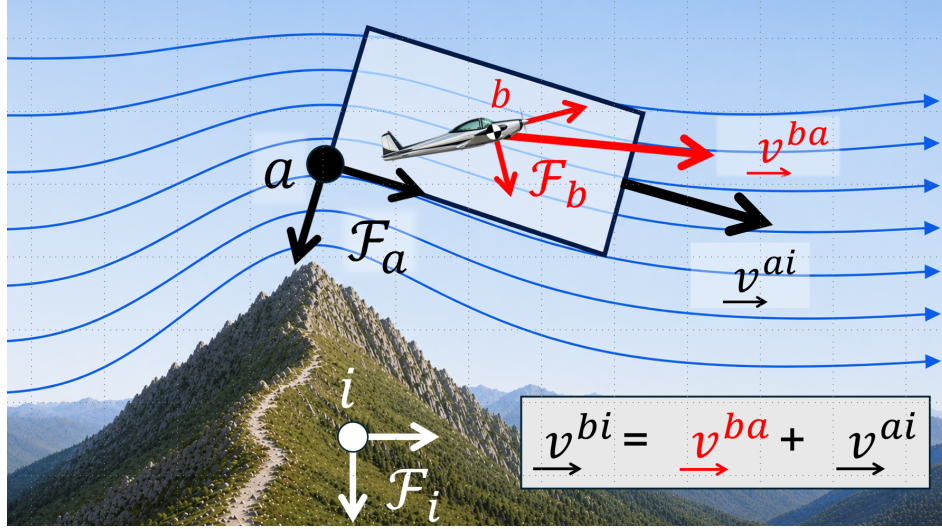


Figure 1: Proposed concept and reference frames. The algebraic sum of the airspeed vector $\vec{v}^{ba}$ (predicted by the aircraft equations of motion) and the wind velocity vector $\vec{v}^{ai}$ (measured, estimated, or modeled) equals the aircraft velocity with respect to the inertial frame $\mathcal{F}_i$, which can be estimated using an IMU. $\mathcal{F}_w$ is not shown.

To understand the kinematics and kinetics that underpin the filtering equations derived later, we consider four reference frames: an inertial frame denoted by $\mathcal{F}_i$; an air-mass frame, $\mathcal{F}_a$, whose translational and rotational motion are identical to those of the air mass in which the airplane is flying; a wind frame, $\mathcal{F}_w$, which is used extensively in aircraft dynamics and whose translational and rotational motion are defined by the velocity vector of the aircraft's center of mass relative to the air mass; and the aircraft body frame, $\mathcal{F}_b$, whose translational and rotational motion are identical to those of the aircraft. Points i , a , and b are points whose positions are fixed in the frames $\mathcal{F}_i$, $\mathcal{F}_a$, and $\mathcal{F}_b$, respectively. We will assume that point b is coincident with the aircraft's center of mass.

The air mass translates relative to the inertial frame with velocity $\vec{v}^{ai}$ and we will refer to this as the *wind speed*. That is, it is the velocity of point a relative to an observer at point i . What we will call the aircraft's *airspeed vector* or $\vec{v}^{ba}$ is the velocity of point b as seen by an observer at point a . Consequently, the aircraft's velocity as seen by an inertial observer will be

$$\vec{v}^{bi} = \vec{v}^{ba} + \vec{v}^{ai} \quad (1)$$

Note that, as written above in the equation above, these vectors may be resolved in any of the four reference frames. For filtering convenience later, we resolve $\vec{v}^{ba}$ in the wind frame, $\mathcal{F}_w$, and denote its magnitude by V_∞ to indicate that it represents the freestream speed and not the local flow velocity in the vicinity of the aircraft surfaces. The wind speed will be resolved in the inertial frame and its components in that frame will be denoted w_x and w_y , respectively. Note that the wind speed includes overall non-zero translation and small variation that, as discussed later, will be model by the Dryden Turbulence Model [Madden (2018)].

The accelerations and rotations sensed by the IMU consist of two contributions: the acceleration and rotation of the aircraft with

respect to the air mass (i.e., $\mathcal{F}_a$) and the acceleration and rotation of the air mass with respect to the inertial frame. The aircraft equations of motion describe the former because the aerodynamic forces and moments acting on an airplane are functions of the relative motion between the airplane and the surrounding air mass [Nelson et al. (1998); Etkin and Reid (1995); Cook (2007)]. Therefore, to predict the total accelerations and rotation rates sensed by the IMU, the aircraft equations of motion must be augmented with measurements or models of the wind dynamics. As we will see later, uncertainty in the wind dynamics is a major contributor to the accuracy of the proposed approach.

Viewed in this manner, the aircraft equations of motion and the wind model together act as a virtual inertial sensor that generates estimates of the accelerations and angular rates that would be observed by an IMU. By fusing these model-based predictions with measurements from low-cost inertial sensors, the resulting filter equations constrain inertial drift to remain consistent with the physical capabilities of the aircraft. The standalone solution from a consumer-grade inertial sensor is known to drift by hundreds of meters in a few seconds. However, such behavior is not consistent with the dynamics of the airplane as described by its equations of motion. Thus, the information gleaned from the equations of motion can be used to very effectively constrain inertial sensor drift, as we will demonstrate later in this paper.

III. AIRCRAFT EQUATIONS OF MOTION

Since the aircraft equations of motion are central to the concept discussed in this paper, we will take a brief detour to describe them briefly. More detail on these equations can be found in any standard flight dynamics text such as Etkin and Reid (1995), Cook (2007), Nelson et al. (1998), or Morelli and Klein (2016). In the context of Figure 1, the equations of motion that describe the evolution of $\vec{v}^{ba}$ can be written in terms of the following state vector

$$\mathbf{x}_a = [V_\infty \quad \gamma \quad \theta \quad p_x \quad p_z \quad q]^T \quad (2)$$

where V_∞ is the magnitude of the airspeed vector $\vec{v}^{ba}$, γ is the flight path angle, θ is the pitch angle, p_x is the down range distance traveled, p_z is the altitude and $q = \dot{\theta}$ is the pitch rate. The angle of attack α is related to γ and θ by $\alpha = \theta - \gamma$. In terms of $\mathbf{x}_a$ equations of motion are written as

$$\dot{\mathbf{x}}_a = \begin{bmatrix} -\frac{F_D}{m} - g \sin \gamma + \frac{T}{m} \cos(\theta + \theta_T - \gamma) - \dot{w}_x \cos \gamma - \dot{w}_z \sin \gamma \\ \frac{1}{V_\infty} \left(\frac{F_L}{m} - g \cos \gamma + \frac{T}{m} \sin(\theta + \theta_T - \gamma) + \dot{w}_x \sin \gamma - \dot{w}_z \cos \gamma \right) \\ q \\ V_\infty \cos \gamma + w_x \\ V_\infty \sin \gamma + w_z \\ \frac{M}{I} \end{bmatrix}, \quad (3)$$

where the lift force F_L , the drag force F_D , and the pitching moment M are given by

$$F_L = \frac{1}{2} \rho V_\infty^2 S C_L, \quad F_D = \frac{1}{2} \rho V_\infty^2 S C_D, \quad M = \frac{1}{2} \rho V_\infty^2 S \bar{c} C_M. \quad (4)$$

Table 1: Aircraft Force and Moment Model.

Quantity	Definition	Description
C_L	$C_{L0} + C_{L\alpha}\alpha$	Lift coefficient
C_D	$C_{D0} + \frac{C_L^2}{\pi a e_0}$	Drag coefficient
C_M	$C_{M0} + C_{M\alpha}\alpha + C_{Mq}\frac{q\bar{c}}{2V_\infty}$	Pitching moment coefficient
F_L	$\frac{1}{2}\rho V_\infty^2 S C_L$	Lift force
F_D	$\frac{1}{2}\rho V_\infty^2 S C_D$	Drag force
M	$\frac{1}{2}\rho V_\infty^2 S \bar{c} C_M$	Pitching moment
g	$\frac{\mu}{p_z^2}$	Gravitational acceleration

In turn, C_L , C_D , and C_M are the lift, drag, and pitching moment coefficients, respectively, and are given by the relationships listed in Table 1. The parameters in the models for C_L , C_D , and C_M (e.g., C_{L0} , C_{Mq} , etc.), as well as the aircraft model parameters listed in Table 2, are determined through modeling, simulation, and testing (e.g., wind tunnel tests and flight tests) and are therefore their exact values are uncertain. Part of the innovation in this paper is that we consider these uncertainties explicitly and assess their impact on the resulting positioning and navigation solution.

Table 2: Aircraft Model Parameters.

Parameter	Description	Parameter	Description
ρ	Air density	m	Aircraft mass
S	Wing planform area	I	Pitch moment of inertia
$\bar{c}$	Mean aerodynamic chord	μ	Earth's gravitational parameter
a	Wing aspect ratio	T	Engine thrust
e_0	Oswald efficiency factor	θ_T	Thrust line inclination angle

As we will show later, air density is an important parameter that must be known with reasonable accuracy. For the work presented here, air density is modeled using an eighth-degree polynomial parameterized by the coefficient vector $\mathbf{a} \in \mathbb{R}^{8 \times 1}$. The coefficients are obtained by performing a polynomial regression of the standard atmosphere density profile as a function of altitude.

$$\begin{aligned}\rho &= a_0 + a_1 p_z + a_2 p_z^2 + \cdots + a_8 p_z^8 \\ \dot{\rho} &= a_1 \dot{p}_z + 2a_2 p_z^1 \dot{p}_z + \cdots + 8a_8 p_z^7 \dot{p}_z\end{aligned}\tag{5}$$

IV. FILTERING ARCHITECTURE

In this section we will show how the concept described above can be translated into the equations for a positioning and navigation filter. A centralized filtering approach is pursued in this work whereby we use the vehicle equations of motion in an Extended Kalman Filter (EKF) time update and the accelerometer and gyroscope measurements from the IMU into the EKF measurement update. This proposed approach is summarized in the block diagram presented in Figure 2.

All of the information available from the aircraft equations of motion, the inertial sensors, and, when available, GNSS measurements can be cast into the standard dynamic and measurement models, which have the following generic form:

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{u}, \mathbf{w}) \quad \text{where } \mathbf{w} \sim N(\mathbf{0}, \mathbf{Q}), \tag{6}$$

$$\mathbf{y} = \mathbf{h}(\mathbf{x}, \mathbf{v}) \quad \text{where } \mathbf{v} \sim N(\mathbf{0}, \mathbf{R}). \tag{7}$$

The first equation is the dynamic model, while the second is the measurement model. Here, $\mathbf{w}$ denotes the process noise with covariance $\mathbf{Q}$, and $\mathbf{v}$ denotes the measurement noise with covariance $\mathbf{R}$. The input vector $\mathbf{u}$ for an aircraft in longitudinal flight

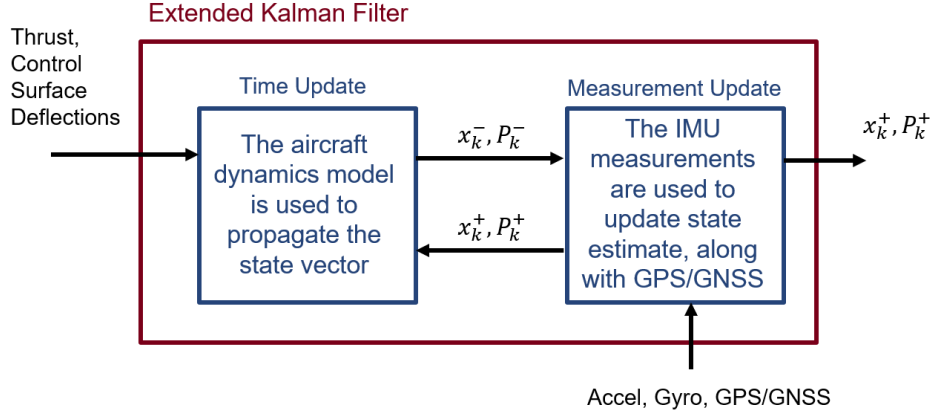


Figure 2: Block diagram of proposed EOM-aided inertial navigation algorithm for state estimation algorithm

is given by

$$\mathbf{u} = [\delta_e \quad \delta_t]^T. \quad (8)$$

where δ_e and δ_t are the commanded elevator deflection and throttle input, respectively. As noted in the equations of motion presented earlier, the lift and pitching moment coefficients depend on the elevator deflection, i.e., $C_L = C_L(\delta_e)$ and $C_M = C_M(\delta_e)$, while the thrust depends on the throttle setting, i.e., $T = T(\delta_t)$. For the exploratory work presented in this paper, we limit our attention, without loss of generality, to a single control input, namely the throttle input δ_t .

The filter state vector $\mathbf{x}$ is defined as follows:

$$\mathbf{x} = [\mathbf{x}_a^T \quad \mathbf{x}_w^T \quad \mathbf{x}_{IMU}^T \quad \mathbf{x}_c^T]^T. \quad (9)$$

It is seen that it is composed of four sub-vectors, where sub-vector $\mathbf{x}_a$ is the state vector defined earlier, $\mathbf{x}_w$ is the wind state vector, $\mathbf{x}_{IMU}$ is the IMU error state vector, and $\mathbf{x}_c$ is the aircraft model uncertainty state vector. This last sub-vector $\mathbf{x}_c$ represents uncertainty in the parameters that define the aircraft aerodynamic model.

The measurement vector $\mathbf{y}$ is defined as follows:

$$\mathbf{y} = [\mathbf{y}_{GNSS}^T \quad \mathbf{y}_{IMU}^T]^T. \quad (10)$$

When GNSS is available, the measurement sub-vector $\mathbf{y}_{GNSS}$ consists of GNSS position and velocity measurements corresponding to a loosely coupled INS/GNSS integration architecture. When GNSS is unavailable due to jamming or other signal disruptions, the sub-vector $\mathbf{y}_{GNSS}$ is omitted from the measurement update. The sub-vector $\mathbf{y}_{IMU}$ consists of the accelerometer and gyroscope measurements provided by the IMU.

In what follows, we will discuss both the dynamic and measurement models in some detail.

1. Dynamic Model

In this section, we present the dynamics, process noise, and physical interpretation of each state sub-vector, with the exception of $\mathbf{x}_a$, whose dynamic model was presented in the previous section. The dynamic model is developed by considering each of the sub-vectors that comprise $\mathbf{x}$ and the differential equations that govern their time evolution.

a) The Wind State Vector

The system state vector also includes the wind. The wind is modeled as the sum of constant horizontal and vertical components and a turbulence component described by a simplified Dryden Turbulence Model. The wind state vector is defined as

$$\mathbf{x}_w = [d_x \quad d_{z1} \quad d_{z2} \quad w_x \quad w_z]^T, \quad (11)$$

where $w_x = \bar{w}_x + w_x^t$ and $w_z = \bar{w}_z + w_z^t$ are the components of $\vec{v}^{ai}$. Here, $\bar{w}_x$ and $\bar{w}_z$ denote the constant horizontal and vertical wind components, respectively, while w_x^t and w_z^t represent the corresponding turbulence components. The constant wind components are modeled as random constants, i.e., $\dot{\bar{w}}_x = 0$ and $\dot{\bar{w}}_z = 0$, while the turbulence components are modeled using the Dryden Turbulence Model [Madden (2018)].

The Dryden Turbulence Model is a mathematical model used to represent continuous atmospheric gusts encountered by an aircraft flying through an air mass. It defines the power spectral densities of the turbulence in a wind-relative coordinate system. In this work, a simplified Dryden model is adopted by assuming that the aircraft angle of attack is zero for the purpose of modeling turbulence and by neglecting rotational turbulence. Under these assumptions, the longitudinal and vertical turbulence are modeled by the Dryden transfer functions

$$G_x(S) = \sigma_x \sqrt{\frac{2L_x}{\pi V_\infty}} \frac{1}{1 + \frac{L_x}{V_\infty} S} \quad \text{and} \quad G_z(S) = \sigma_z \sqrt{\frac{2L_z}{\pi V_\infty}} \frac{1 + \sqrt{3} \left(\frac{L_z}{V_\infty} S \right)}{1 + \left(\frac{L_z}{V_\infty} S \right)^2}. \quad (12)$$

The inputs to these transfer functions are unit spectral density white-noise processes, while the outputs are the longitudinal and vertical turbulence velocities in the inertial frame. The parameters L_x and L_z denote the turbulence length scales, while σ_x and σ_z are the corresponding RMS turbulence velocities. Using standard state-space realizations, the Dryden model is written as

$$\dot{\mathbf{d}}_x = \mathbf{A}_x \mathbf{d}_x + \mathbf{B}_x \eta_{dx}, \quad w_x^t = \mathbf{C}_x \mathbf{d}_x, \quad (13)$$

$$\dot{\mathbf{d}}_z = \mathbf{A}_z \mathbf{d}_z + \mathbf{B}_z \eta_{dz}, \quad w_z^t = \mathbf{C}_z \mathbf{d}_z, \quad (14)$$

where w_x^t and w_z^t are the turbulence-induced components of the horizontal and vertical wind velocities, respectively, while $\mathbf{d}_x$ and $\mathbf{d}_z$ are the internal states of the state-space realization of the Dryden model, defined as

$$\mathbf{d}_x = d_x, \quad \mathbf{d}_z = [d_{z1} \quad d_{z2}]^T. \quad (15)$$

Thus, the final form of the wind dynamics are:

$$\dot{\mathbf{x}}_w = \mathbf{A}_w \mathbf{x}_w + \mathbf{B}_w \boldsymbol{\eta}, \quad \mathbf{A}_w = \begin{bmatrix} \mathbf{A}_x & \mathbf{0}_{1 \times 2} & 0 & 0 \\ \mathbf{0}_{2 \times 1} & \mathbf{A}_z & \mathbf{0}_{2 \times 1} & \mathbf{0}_{2 \times 1} \\ \mathbf{C}_x \mathbf{A}_x & \mathbf{0}_{1 \times 2} & 0 & 0 \\ 0 & \mathbf{C}_z \mathbf{A}_z & 0 & 0 \end{bmatrix} \quad \text{and} \quad \mathbf{B}_w = \begin{bmatrix} \mathbf{B}_x & 0 \\ \mathbf{0}_{2 \times 1} & \mathbf{B}_z \\ \mathbf{C}_x \mathbf{B}_x & 0 \\ 0 & \mathbf{C}_z \mathbf{B}_z \end{bmatrix} \quad (16)$$

where $\boldsymbol{\eta}_w = [\eta_{dx} \quad \eta_{dz}]^T$ is the process noise vector for the turbulence. This will be used later in forming the overall system process noise matrix used the covariance analysis discussed below.

b) Sensor Error Vector

There are three states in the state vector to account for correlated biases in the inertial measurement unit. This includes a correlated bias in the forward-facing accelerometer δf_u , a correlated bias in the downward-facing accelerometer δf_w , and a correlated bias in the gyroscope $\delta \omega$.

$$\mathbf{x}_{IMU} = [\delta f_u \quad \delta f_w \quad \delta \omega]^T \quad (17)$$

These biases are modeled as Gauss-Markov processes and, thus, the dynamics of $\mathbf{x}_{IMU}$ becomes

$$\dot{\mathbf{x}}_{IMU} = \begin{bmatrix} -\frac{1}{\tau_{accel}} & 0 & 0 \\ 0 & -\frac{1}{\tau_{accel}} & 0 \\ 0 & 0 & -\frac{1}{\tau_{gyro}} \end{bmatrix} \mathbf{x}_{IMU} + \begin{bmatrix} \eta_{\delta f_u} \\ \eta_{\delta f_w} \\ \eta_{\delta \omega} \end{bmatrix} \quad (18)$$

where τ_{accel} and τ_{gyro} are the accelerometer and gyroscope bias time constants, respectively, and $\eta_{\delta f_u}$, $\eta_{\delta f_w}$, and $\eta_{\delta \omega}$ are the process noise terms driving the bias states. Their spectral densities are determined from the stochastic error models of the inertial sensors. These quantities are subsequently used to construct the overall system process noise covariance matrix $\mathbf{Q}$ employed in the covariance analysis presented the subsequent sections of the paper.

c) Equation of Motion Uncertainty Vector

The aircraft equations of motion are not known perfectly, and this lack of perfect knowledge of the parameters that define the aircraft dynamic model is captured by the uncertainty state vector $\mathbf{x}_c$. The uncertainty state vector is given explicitly by

$$\mathbf{x}_c = [\delta C_{d0} \quad \delta C_{l0} \quad \delta C_{l\alpha} \quad \delta C_{m0} \quad \delta C_{m\alpha} \quad \delta C_{mq} \quad \delta T \quad \delta e_0 \quad \delta \rho]^T \quad (19)$$

These uncertainties include aerodynamic coefficients and other parameters that describe the forces and moments acting on the aircraft and will be described later in the paper. In the work described here, we do not attempt to estimate these parameters. Instead, we examine the impact of uncertainty in the aircraft model on the resulting positioning and navigation solution. This allows us to obtain a realistic assessment of the performance of the proposed approach under real-world conditions while also providing insight into which aircraft model parameters are most important to model accurately. To this end, we treat these uncertainties as *consider* states (hence, the subscript “c”) and will use a Kalman-Schmidt filter to account for them. In the Kalman-Schmidt filter, consider states are included in the Kalman gain calculation, but they have no dynamics and are not updated. Thus,

$$\dot{\mathbf{x}}_c = \mathbf{0} \quad (20)$$

In closing our discussion of the dynamic model we note that the process noise matrix we use in the covariance analysis discussed later in the paper, we concatenate all the process noise vectors describe above into one large process noise matrix $\mathbf{w}$ given by:

$$\mathbf{w} = [\eta_{dx} \quad \eta_{dz} \quad \eta_{\delta f_u} \quad \eta_{\delta f_w} \quad \eta_{\omega}]^T \quad (21)$$

All the elements in this process noise vector are assumed to be uncorrelated and zero-mean Gaussian. The parameters η_{dx} and η_{dz} have a noise density with a standard deviation of 1, as prescribed by the Dryden Turbulence Model. The elements $\eta_{\delta f_u}$, $\eta_{\delta f_w}$, and η_{ω} , which drive the Gauss-Markov processes used to model the IMU sensor biases, have zero-mean Gaussian white noise with a standard deviation equal to the standard deviation of the bias in the accelerometer and gyroscope sensors.

2. Measurement Model

As discussed earlier, the measurement vector for the equations of motion-aided inertial navigation filter includes measurements from the accelerometer and the gyroscope. In the longitudinal model of an aircraft, the three elements of the measurement model are the specific force measurement in the body-frame forward direction f_u , the specific force measurement in the body-frame downward direction f_w , and the pitch rate measurement ω from the gyroscope. Accordingly, we define the measurement vector $\mathbf{y}$ as follows

$$\mathbf{y}_{\text{IMU}} = [f_u \quad f_w \quad \omega]^T \quad (22)$$

The measurement equations $\mathbf{y} = \mathbf{h}(\mathbf{x}, \mathbf{v})$ that relate the state vector to the system measurements is derived as follows. We begin with equations for u and w , which are the vehicle inertial velocities in the north and down body frame or $\mathcal{F}_b$.

$$\begin{aligned} u &= V_{\infty} \cos(\alpha) + w_x \cos(\theta) + w_z \sin(\theta) \\ w &= V_{\infty} \sin(\alpha) + w_x \sin(\theta) - w_z \cos(\theta) \end{aligned} \quad (23)$$

We can relate these equations to the outputs of the inertial sensors by making the following observations. First, for an aircraft constrained to motion in the longitudinal plane (the simplifying assumption adopted in this paper), the angular velocity of the body frame with respect to the inertial frame, as measured by the rate gyro, is ω^{bi} . In this case, ω^{bi} is a scalar equal to the aircraft pitch rate, q . Second, differentiating the equations above yields the acceleration of the aircraft with respect to the inertial reference frame. Recalling that accelerometers measure specific force rather than acceleration (i.e., the term “accelerometer” is technically a misnomer), we obtain the corresponding accelerometer outputs by subtracting the local gravitational acceleration from these inertial accelerations. This results in the following measurement equation:

$$f_u = \dot{V}_{\infty} \cos(\alpha) - \dot{\alpha} V_{\infty} \sin(\alpha) + \dot{w}_x \cos(\theta) - w_x \dot{\theta} \sin(\theta) + \dot{w}_z \sin(\theta) + w_z \dot{\theta} \cos(\theta) + q w + g \sin(\theta) + \delta f_u + \nu_{f_u} \quad (24)$$

$$f_w = \dot{V}_{\infty} \sin(\alpha) + \dot{\alpha} V_{\infty} \cos(\alpha) + \dot{w}_x \sin(\theta) + w_x \dot{\theta} \cos(\theta) - \dot{w}_z \cos(\theta) + w_z \dot{\theta} \sin(\theta) - q u - g \cos(\theta) + \delta f_w + \nu_{f_w} \quad (25)$$

$$\omega = q + \delta \omega + \nu_{\omega} \quad (26)$$

where $\dot{\theta} = q$ and ν_{f_u} , ν_{f_w} and ν_{ω} are the zero-mean Gaussian noise on the sensor measurements. That is,

$$\mathbf{v} = [\nu_{f_u} \quad \nu_{f_w} \quad \nu_{\omega}]^T \quad \text{where } \mathbf{v} \sim N(\mathbf{0}, \mathbf{R}) \quad (27)$$

The measurement covariance matrix $\mathbf{R}$ is determined from the given sensor characteristic either by testing or from a trusted specification sheet.

V. COVARIANCE ANALYSIS

As noted in the introduction one of the stated goals of this paper is to explore the expected performance of the proposed sensor fusion scheme. To that end, the results presented later in the paper are the results of a covariance analysis using the equations above.

The covariance analysis is performed using Kalman-Schmidt filter [Simon (2006)]. The Kalman-Schmidt filter is a reduced order Kalman filtering framework where the system's state space is divided into a set of estimated states and a set of states that are not estimated. The filter's estimated states are treated as they would be by a traditional Kalman filtering framework and are estimated via the filter's time and measurement updates. The set of non-estimated states, known as *consider states*, are updated as usual during the filter's time update. However, these consider states are not actually estimated by the filter. This way, if a bias, or other nuisance parameter in the system is designated as a consider state, the covariance innovations will properly account for these nuisance parameters without needing to estimate the parameters and increase the dimensionality of the state estimate.

To construct the Kalman-Schmidt filter we note that the state vector can be partitioned as follows:

$$\mathbf{x} = [\mathbf{x}_a^T \quad \mathbf{x}_w^T \quad \mathbf{x}_{\text{IMU}}^T \quad | \quad \mathbf{x}_c^T]^T \quad (28)$$

$$\underbrace{\hspace{10em}}_{\mathbf{x}_{\text{PNT}}} \quad (29)$$

This partitioning allows us to write the estimate covariance matrix $\mathbf{P}$ as follows:

$$\mathbf{P} = E \{ \epsilon_x \epsilon_x^T \} = \begin{bmatrix} \mathbf{P}_{\text{PNT}} & \mathbf{P}_{\text{cross}} \\ \mathbf{P}_{\text{cross}}^T & \mathbf{P}_c \end{bmatrix} \quad (30)$$

where $\epsilon_x = \hat{\mathbf{x}} - \mathbf{x}$ is the state estimation error, $\mathbf{P}_{\text{PNT}}$ is the covariance matrix associated with the PNT state estimate $\mathbf{x}_{\text{PNT}}$, $\mathbf{P}_c$ is the covariance matrix associated with the equation-of-motion uncertainty state estimate $\mathbf{x}_c$, and $\mathbf{P}_{\text{cross}}$ is the cross-covariance matrix between $\mathbf{x}_{\text{PNT}}$ and $\mathbf{x}_c$. Given these definitions, the time and measurement updates proceed as follows, where $\mathbf{F}_k$ and $\mathbf{H}_k$ are the Jacobians of $\mathbf{f}(\mathbf{x}, \mathbf{u}, \mathbf{w})$ and $\mathbf{h}(\mathbf{x}, \mathbf{v})$, respectively. The state and covariance estimates are propagated through the Kalman-Schmidt filter as follows, where an *a priori* state and covariance estimates are denoted by $(\bullet)^-$ and a *posteriori* state and covariance estimates are denoted by $(\bullet)^+$.

Time Update

$$\mathbf{x}_k^+ \xrightarrow{\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{u}, \mathbf{w} = \mathbf{0})} \mathbf{x}_{k+1}^- \quad \text{and} \quad \mathbf{P}_{k+1}^- = \mathbf{F}_k \mathbf{P}_k^+ \mathbf{F}_k^T + \mathbf{Q}_d \quad (31)$$

Measurement Update

$$\lambda_k = [\mathbf{H} \quad \mathbf{H}_c] \begin{bmatrix} \mathbf{P}_k^- & \mathbf{P}_{\text{cross},k}^- \\ (\mathbf{P}_{\text{cross},k}^-)^T & \mathbf{P}_{c,k}^- \end{bmatrix} \begin{bmatrix} \mathbf{H}^T \\ \mathbf{H}_c^T \end{bmatrix} + \mathbf{R}, \quad (32)$$

$$\mathbf{K}_k = \left(\mathbf{P}_k^- \mathbf{H}^T + \mathbf{P}_{\text{cross},k}^- \mathbf{H}_c^T \right) \lambda_k^{-1}, \quad (33)$$

$$\mathbf{x}_k^+ = \mathbf{x}_k^- + \mathbf{K}_k (\mathbf{y}_k - \mathbf{h}(\mathbf{x}_k^-)), \quad (34)$$

$$\mathbf{P}_k^+ = (\mathbf{I} - \mathbf{K}_k \mathbf{H}) \mathbf{P}_k^- - \mathbf{K}_k \mathbf{H}_c (\mathbf{P}_{\text{cross},k}^-)^T, \quad (35)$$

$$\mathbf{P}_{\text{cross},k}^+ = (\mathbf{I} - \mathbf{K}_k \mathbf{H}) \mathbf{P}_{\text{cross},k}^- - \mathbf{K}_k \mathbf{H}_c \mathbf{P}_{c,k}^-, \quad (36)$$

$$\mathbf{P}_{c,k}^+ = \mathbf{P}_{c,k}^- \quad \text{and} \quad \mathbf{x}_{c,k}^+ = \mathbf{x}_{c,k}^-. \quad (37)$$

The matrices $\mathbf{Q}$ and $\mathbf{R}$ are the process noise and measurement noise covariance matrices. Since the measurements noise vector contains just the Gaussian white-noise component of the accelerometer noise, the measurement noise covariance matrix $\mathbf{R}$ simply contains the covariance of the measurement noise vector defined in 27. Note that a discrete version of $\mathbf{Q}$ must be used in

the filtering equations where discretization is carried out using standard methods described filtering texts such as Franklin et al. (1998).

VI. RESULTS

The proposed algorithm was evaluated using a series of simulation experiments. These simulation experiments were performed using a longitudinal model of the Ryan Navion general aviation aircraft. The Navion was chosen because it is a general aviation aircraft whose aerodynamics database and vehicle dynamics model are publicly available at Teper (1969). The Navion is a single-engine, fixed-wing, unpressurized general aviation aircraft that was manufactured by North American Aviation and introduced to the market in 1948 [Simpson (2005)]. The aircraft has a dry mass of around 875 kg and a wing span of around 10 meters.

The state estimation architecture was evaluated on a notional instrument-approach trajectory over mountainous terrain. The trajectory is a 300 second flight, where in the first 100 seconds, the vehicle is in level flight at an altitude of 1000 meters and an airspeed of 54 meters per second. At the 100-second mark (after crossing over mountainous terrain), we adjust the throttle input to the simulation such that the vehicle enters an -3 degree flight path angle. The vehicle remains in this descent at the -3 degree flight path angle for the remaining 200 seconds of the simulation. In the longitudinal vehicle simulation, we assume a non-rotating, flat-Earth model, with gravitational forces calculated using the WGS 84 gravitational model and the air density is modeled using the International Standard Atmosphere model [Atmosphere (1976)]. Figure 3 shows a time history of the position, the airspeed, and the flight path angle of the vehicle during its flight. These plots are of the nominal trajectory, assuming no wind. The two-dimensional position vector states are designated as *Altitude* and *Forward Position*, where the *Altitude* is the altitude of the vehicle above sea level and *Forward Position* is the lateral distance that the vehicle has traveled. In the scenarios where there is wind, both the *Altitude* and *Forward Position* states are assumed to be with respect to the ground, whereas the *Airspeed* and *Flight Path Angle* states are wind-relative air data states.

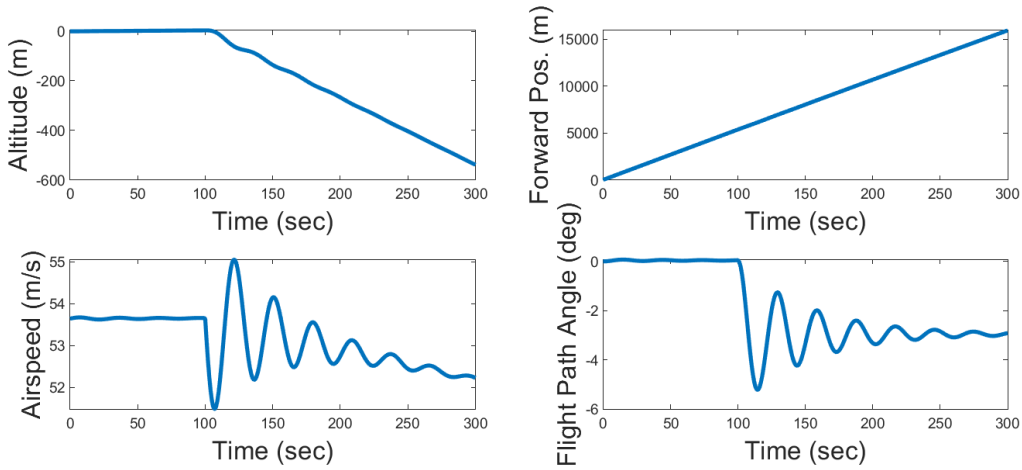


Figure 3: Navion ground truth trajectory

In these simulation experiments, the vehicle's initial wind-relative state is assumed to be perfectly known by the estimator. However, certain simulation experiments have wind and turbulence which contains uncertainty. Therefore, even though the initial condition of the vehicle's airspeed, angle of attack, and flight path angle are perfectly known, the vehicle's initial ground-relative velocity is not necessarily perfectly known. The vehicle's initial position, altitude, and pitch angle are assumed to be perfectly known.

We compare inertial navigation using consumer-grade inertial navigation sensors aided by different grades of vehicle dynamics models with inertial navigation using either tactical-grade or consumer-grade inertial navigation sensors alone. We grade the vehicle equations of motion as having either perfect knowledge of the vehicle equations of motion, or having 2% or 10% error in the vehicle aerodynamic coefficient parameters. When we say that the aerodynamic coefficient parameters have a given percent error, we mean that there is an unknown constant bias on each individual aerodynamic coefficient parameter that falls within a Gaussian distribution that has a standard deviation of some percentage of that aerodynamic coefficient parameter. For example, if the true value of the coefficient $C_{L\alpha}$ for the vehicle were equal to 4.44, a vehicle dynamics model with 10% uncertainty would mean that $C_{L\alpha}$ would be known by the filter to be constant and to lie within $4.44 + \mathcal{N}(0, 0.44^2)$.

Table 3: Consumer-Grade IMU Error Statistics.

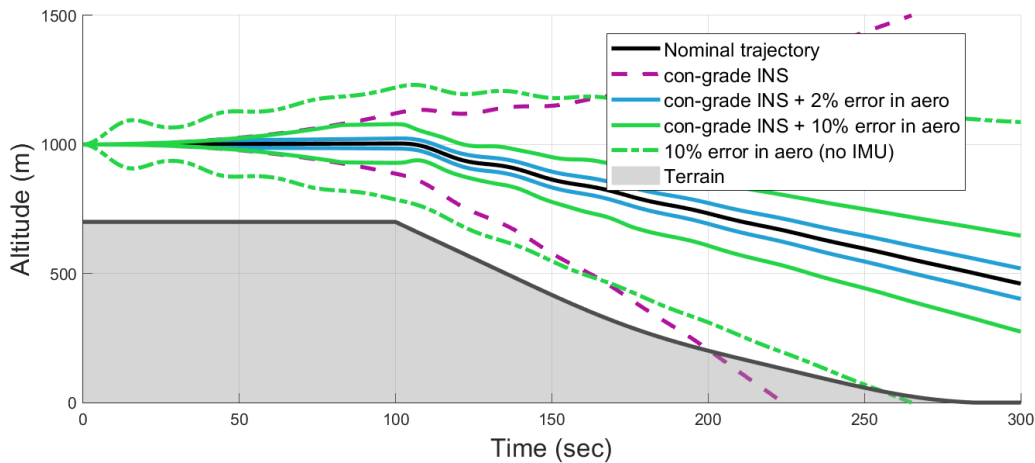
Parameter	Value	Parameter	Value
IMU Update Rate	100 Hz	Accelerometer White Noise Std. Dev.	$1 \times 10^{-3} g$
Accelerometer Bias Std. Dev.	$1.2 \times 10^{-3} g$	Accelerometer Correlation Time Constant	100 sec
Gyroscope White Noise Std. Dev.	$0.05^\circ/\text{sec}$	Gyroscope Bias Std. Dev.	$180^\circ/\text{hour}$
Gyroscope Correlation Time Constant	300 sec		

The analysis in this work considered three different scenarios, which each scenario making different assumptions about the wind and turbulence present in the environment. In Scenario 1, and there was no wind or turbulence present in the simulations. In Scenario 2, a moderate amount of turbulence was included in the simulation, with a turbulence length scale and RMS velocity of 300 seconds and 5 m/s respectively in the horizontal direction and 300 seconds and 0.5 m/s in the vertical direction. These turbulence properties reflect the ones associated with moderate turbulence in Moorhouse and Woodcock (1982). In Scenario 3, we consider turbulence as well as constant winds in the horizontal and vertical directions. The turbulence included in Scenario 3 is identical to the turbulence in Scenario 2. The constant wind has components in the horizontal and vertical direction. The exact magnitude of the wind is not known to the filter, but it is known to fall within a zero-mean Gaussian distribution, where the constant horizontal wind falls within a zero-mean Gaussian distribution that has a standard deviation of 3 m/s and the constant vertical wind falls within a zero-mean Gaussian distribution that has a standard deviation of 1 m/s.

Scenario 1: Without Wind and Turbulence

Scenario 1 presents a covariance analysis where no wind or turbulence is present in the simulation. In this scenario, all of the uncertainty in the system comes from inertial sensor noise and biases, and from a lack of knowledge of the vehicle aerodynamic coefficient parameters in the consider state vector.

Figure 4 shows a plot of the aircraft's descent trajectory, with $4 - \sigma$ error uncertainty bounds on state estimates from each of the estimator configurations. Some terrain is drawn in gray underneath the descent trajectory. The $4 - \sigma$ uncertainty bounds corresponds, roughly, to a 10^{-5} integrity for a fault-free condition. Figure 4 shows that for a descent trajectory like the one presented, when aiding from external GNSS measurements is not available, inertial navigation using solely the consumer-grade inertial navigation sensors typically found in general aviation aircraft would not be viable and would make this approach trajectory infeasible. Additionally, when navigating using the vehicle's equations of motion alone, the navigation solution also rapidly diverges and can not be relied upon for a successful approach and landing. However, these results show that aiding of the consumer-grade inertial navigation sensors by even a relatively low-fidelity vehicle dynamics model can be used to improve the continuity of the navigation solution by ensuring that the vehicle will not collide with the terrain.

**Figure 4:** Scenario 1 Navion descent trajectory with $4 - \sigma$ uncertainty bounds

Figures 5 and 6 show the $1 - \sigma$ bounds on the forward position, pitch angle, angle of attack, and airspeed respectively. Results are presented for aiding of the consumer-grade inertial sensors using several different grade vehicle dynamics models, as well

as navigation using consumer grade inertial sensors or tactical grade sensors alone. These results show that the inclusion of the vehicle equations of motion in the navigation solution has the potential to bring down the level of uncertainty that you see using consumer-grade inertial navigation sensors to something that is more comparable to navigation using tactical-grade inertial navigation sensors.

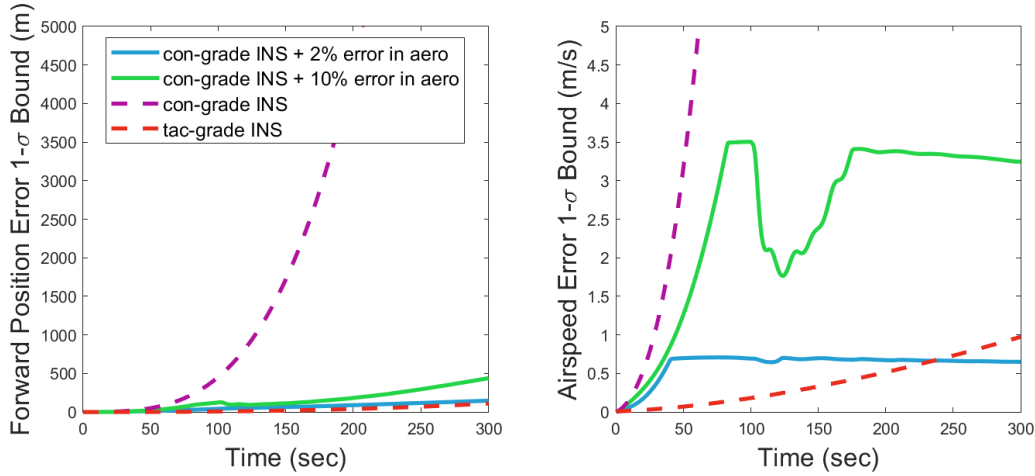


Figure 5: Scenario 1 covariance analysis of distance traveled and airspeed error

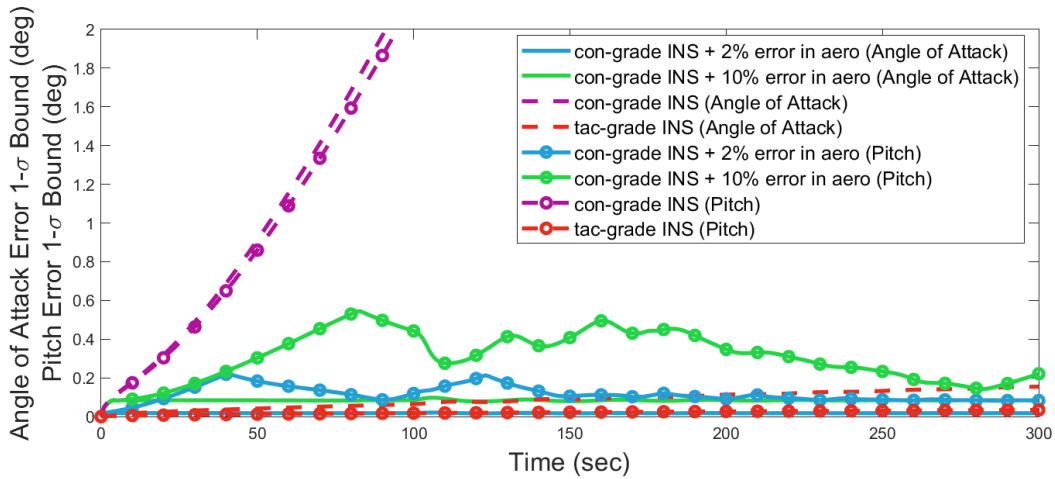


Figure 6: Scenario 1 covariance analysis of angle of attack and pitch angle error

Scenarios 2 and 3: With Turbulence and Mean Wind

Scenario 2 presents a covariance analysis where a moderate amount of turbulence is injected into the simulation. Scenario 3 presents a covariance analysis where in addition to the added turbulence, there is uncertainty in the mean wind acting on the aircraft. This mean wind is a constant movement of the air mass that offsets the inertial frame from the wind-relative frame. While the mean wind is modeled by states in the Kalman filter which the estimator can estimate, the turbulence states, modeled using the Dryden Turbulence Model, are configured as consider states and are not estimated by the filter. Figure 7 shows a comparison between Scenario 2 and Scenario 3 of the $4 - \sigma$ altitude estimation covariance bounds. These results show that if we let the state estimation filter estimate the mean wind, we can still see significant improvement in the navigation solution and see results similar to those seen in Scenario 1. However, if we assume that turbulence in the atmosphere, like the one included in Scenario 3, cannot be estimated, then we still see some improvement. However, the improvement is much less significant in the short term.

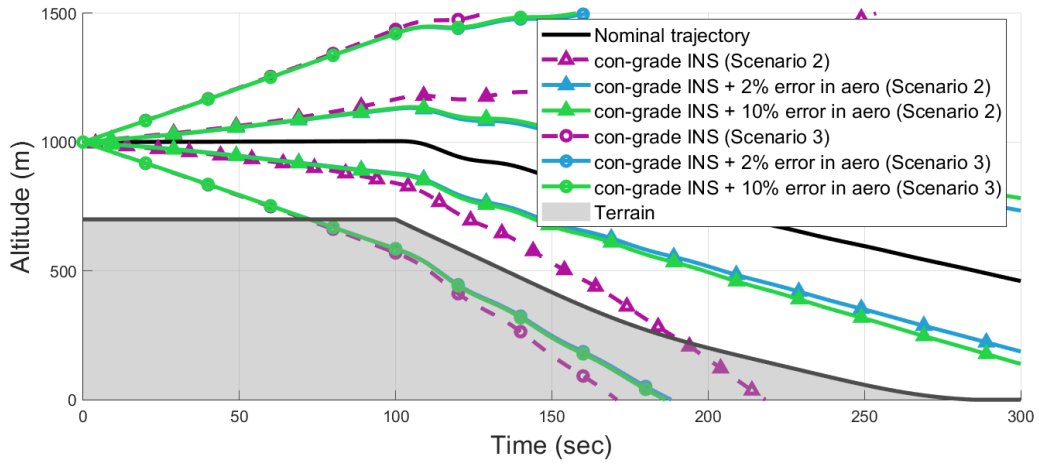


Figure 7: Scenarios 2 and 3 Navion descent trajectory with $4 - \sigma$ uncertainty bounds

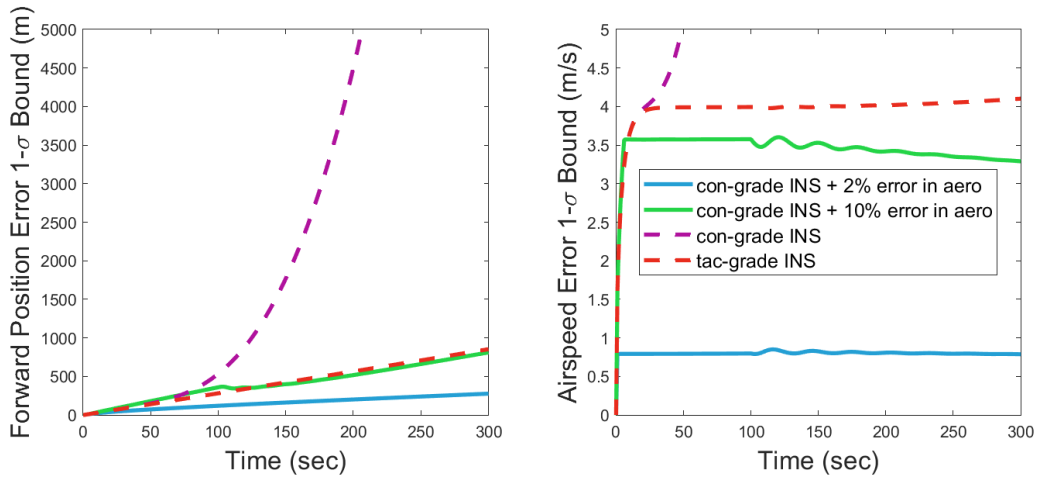


Figure 8: Scenario 2 covariance analysis of distance traveled and airspeed error

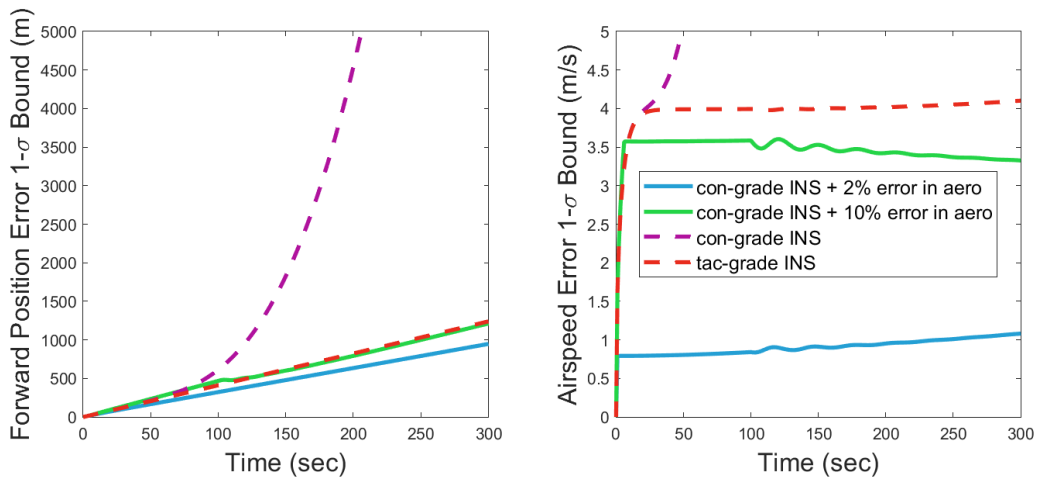


Figure 9: Scenario 3 covariance analysis of distance traveled and airspeed error

Figures 8, 9, 10, and 11 show more results from the covariance analysis of Scenarios 2 and 3. Again, in these covariance analyses, we present $1 - \sigma$ bounds on the estimate errors using consumer-grade inertial navigation sensors aided by vehicle equations of motion and consumer-grade or tactical-grade inertial navigation sensors alone. Since the filter is initialized with perfect knowledge of the state in the wind-relative reference frame, the initial airspeed and angle of attack are perfectly known. The initial position and altitude are also perfectly known. However, since there is uncertainty in the initial wind, there is also uncertainty in the initial velocity. This causes the estimates of the position and altitude from the inertial navigation sensors alone to diverge more quickly when there is wind in the simulation. For the analysis of the wind-relative states like angle of attack and airspeed where the initial conditions are perfectly known, the state estimate error bounds quickly jump up to near their steady state value. The performance of the filter in Scenario 2 is slightly better than the performance of the filter in Scenario 3, but the affects of including mean wind in the simulation are somewhat mitigated due to the fact that the state estimation filter is relatively good at estimating the mean wind states. In both of these scenarios, the inclusion of the vehicle equations of motion to aid the consumer-grade inertial navigation sensors brings the quality of the state estimates up to a level comparable to what would be expected from a tactical-grade inertial navigation system.

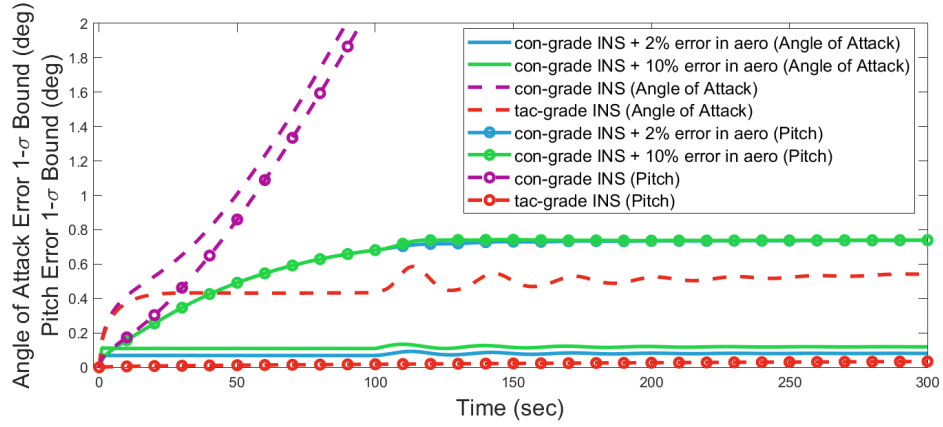


Figure 10: Scenario 2 covariance analysis of angle of attack and pitch angle error

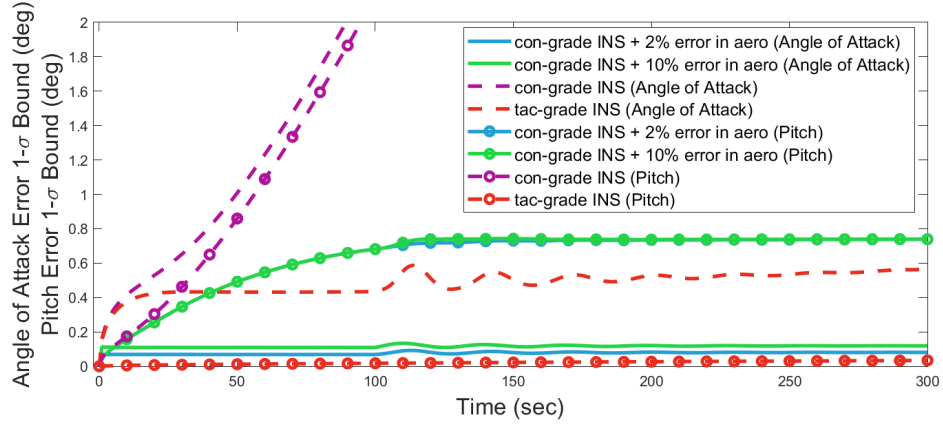


Figure 11: Scenario 3 covariance analysis of angle of attack and pitch angle error

Sensitivity Analysis

Finally, we present a sensitivity analysis. This sensitivity analysis quantifies the affect that errors in different aerodynamic coefficient parameters have on the estimation of the states. In this sensitivity analysis, we begin with perfect knowledge of the equations of motion of the vehicle and individually corrupt each of the aerodynamic coefficient parameters with 4% error. Next, we carry out the covariance analysis discussed at the beginning of this section to generate the estimate error characteristics due to that aerodynamic coefficient parameter alone. Figures 12 and 13 compare the responses in the horizontal position error variances and the angle of attack error variances due to corruption in each of the aerodynamic coefficient parameters.

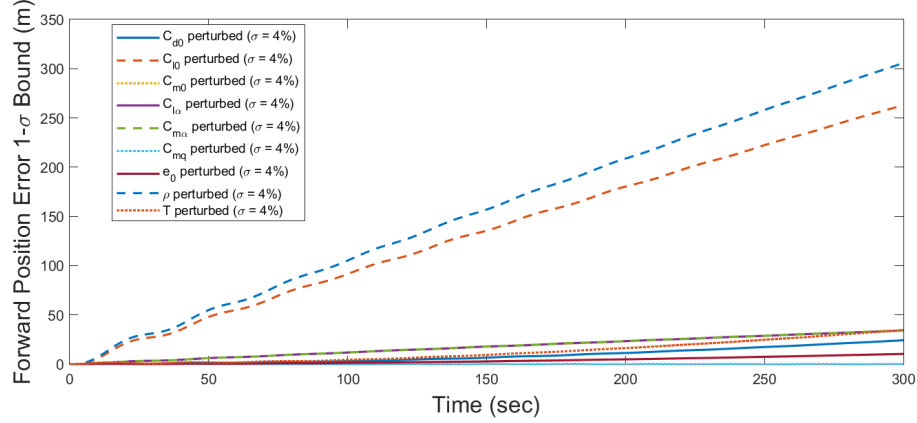


Figure 12: Analysis of sensitivity of estimation error in distance traveled to different aerodynamic coefficient parameters

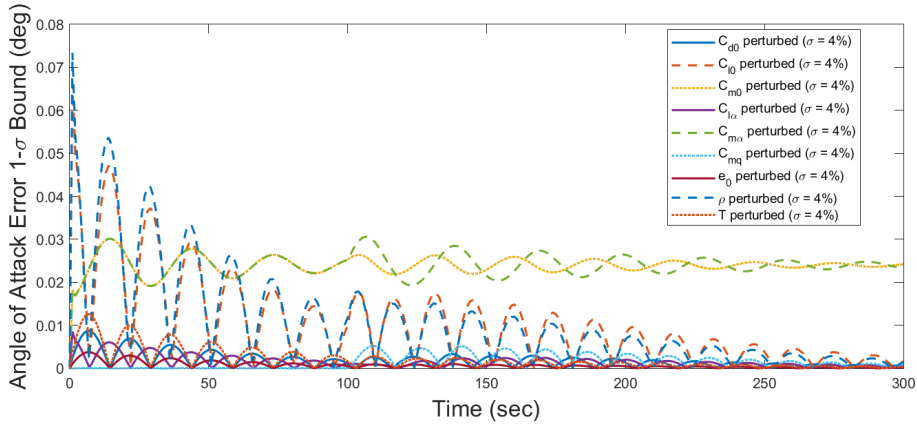


Figure 13: Analysis of sensitivity of estimation error in angle of attack to different aerodynamic coefficient parameters

VII. CONCLUSIONS

In this work, we presented a navigation architecture that fuses inertial sensor measurements with an aircraft dynamics model to improve positioning and navigation performance during GNSS-stressed or GNSS-denied operations. In the proposed approach, an Extended Kalman Filter (EKF) is used to fuse the aircraft equations of motion (used as the time update) and with the outputs of an inertial sensor measurements (measurement update). In effect what this fusion is doing is constraining the inertial solution to remain consistent with the physical dynamics of the aircraft and, thus, help to reduces inertial drift relative to a standalone inertial navigation system.

Simulation results demonstrate that this model-based aiding approach can substantially improve the performance of low-cost consumer- and industrial-grade inertial sensors, enabling navigation performance comparable to that obtained using lower-end tactical-grade IMUs in standalone operation. The results also show that the ultimate performance of the approach is limited by uncertainty in the aircraft dynamic model and the surrounding environment. In particular, the sensitivity analysis identified uncertainty in thrust and air density as two of the most significant factors affecting estimation accuracy, underscoring the importance of accurately modeling propulsion and atmospheric conditions.

The work presented here should be viewed as an exploratory investigation of the concept. The formulation considered only constrained longitudinal aircraft motion and therefore represents a first step toward a more comprehensive implementation. Future work should extend the approach to the full six-degree-of-freedom aircraft equations of motion and investigate the simultaneous estimation of selected aircraft model parameters, rather than treating them as fixed but uncertain quantities. Such an adaptive approach has the potential to further improve navigation performance and increase robustness to aircraft modeling errors and changing flight conditions. Another potential benefit of this approach that can be explored is that of using the aircraft equations of motion as a means for fault detection and isolation of navigation sensor failures.

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