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# Performance Limitations of Observer-Based Feedback for Transient Energy Growth Suppression

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**Transient energy growth suppression is a common control objective for feedback flow control aimed at delaying transition to turbulence. A prevailing control approach in this context is observer-based feedback, in which a full-state feedback controller is applied to state estimates from an observer. The present study identifies a fundamental performance limitation of observer-based feedback control: whenever the uncontrolled system exhibits transient energy growth in response to optimal disturbances, control by observer-based feedback will necessarily lead to transient energy growth in response to optimal disturbances for the closed-loop system as well. Indeed, this result establishes that observer-based feedback can be a poor candidate for controller synthesis in the context of transient energy growth suppression and transition delay; the performance objective of transient energy growth suppression can never be achieved by means of observer-based feedback. Further, an illustrative example is used to show that alternative forms of output feedback are not necessarily subject to these same performance limitations and should also be considered in the context of transient energy growth suppression and transition control.**



## Nomenclature

| | | |
|---|---|---|
| $(A, B, C)$ | = | linear time-invariant state-space realization of plant |
| $\tilde{A}$ | = | closed-loop observer–plant operator |
| $E$ | = | open-loop energy |
| $\tilde{E}$ | = | closed-loop observer–plant energy |
| $G$ | = | open-loop plant maximum transient energy growth |
| $\tilde{G}_x$ | = | closed-loop plant maximum transient energy growth |
| $\tilde{G}_{\hat{x}}$ | = | closed-loop observer maximum transient energy growth |
| $\tilde{G}$ | = | closed-loop observer–plant maximum transient energy growth |
| $K$ | = | controller feedback gain |
| $L$ | = | observer gain |
| $u, x, y$ | = | plant input, state, and output vectors, respectively |
| $\hat{x}$ | = | observer (estimated) state |
| $\tilde{x}$ | = | closed-loop observer–plant state, $(x, \hat{x})$ |

## I. Introduction

An ability to delay or fully suppress transition to turbulence has the potential to benefit a variety of technological systems, including air and maritime transportation systems, by enabling improvements to efficiency and performance. Transition to turbulence in many shear flows arises at a Reynolds number *Re* well below the critical *Re* predicted by a linear stability analysis of the Navier–Stokes equations about a laminar solution [1]. The onset of this so-called subcritical transition can often be explained by a linear mechanism for transient energy growth: a nonmodal stability analysis of the linearized dynamics reveals that, for linearly stable flows, small disturbances will often grow before they decay [2,3]. Indeed, this linear mechanism for transient energy growth can cause the fluid state to depart from the basin of attraction for the laminar solution, triggering transition and ultimately giving rise to turbulence.

Numerous investigations have sought to delay transition by aiming to reduce transient energy growth through various forms of linear feedback control [4–14]. Many of these studies have relied upon the well-established separation principle at some stage in the synthesis of a dynamic output feedback compensator (i.e., observer-based feedback) [10–14]. The separation principle is commonly invoked in this design process because it considerably simplifies controller synthesis when a dynamic output feedback law is desired; a stabilizing full-state feedback controller can be synthesized independently of a stable state estimator, then combined together to yield a stabilizing dynamic output feedback compensator [15]. Despite guarantees on linear stability of the closed-loop system, invoking the separation principle can dramatically degrade closed-loop performance. Yet, the adverse consequences of controller synthesis by means of the separation principle are not fully appreciated in the context of transition delay and transient energy growth control, in which closed-loop performance is paramount. For instance, the separation principle is central to linear quadratic Gaussian (LQG) control, which remains a common controller synthesis approach for transient energy growth reduction and transition delay [10–14]. Although degraded closed-loop performance of observer-based feedback controllers has been reported in the literature [5], no previous studies have explicitly identified the separation principle as the source of these performance limitations. The state estimation problem is sometimes identified as "the primary pacing item" for realizing acceptable closed-loop performance [12]; however, as we will show here, observer peaking and associated performance issues are more deeply rooted with the separation principle itself.

In this Paper, we will show that any linear system that exhibits transient energy growth to disturbances in open-loop will also exhibit transient energy growth to disturbances in closed-loop whenever the separation principle is invoked for controller synthesis. Indeed, even when a full-state feedback controller can fully suppress transient energy growth, a dynamic output feedback compensator designed via the separation principle will invariably lead to nontrivial transient energy growth in response to some disturbances. Further, the separation principle will guarantee that the estimator dynamics (and estimation error) will exhibit peaking in closed-loop from some initial conditions whenever the uncontrolled plant exhibits transient energy growth. The results in this Paper highlight the inherent performance limitations that arise by invoking the separation principle for observer-based feedback control in the context of transition control and transient energy growth suppression. As we will show via example, not all output feedback control approaches are

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restricted to the same performance limitations as observer-based feedback; indeed, alternative output feedback approaches should be considered as candidates for controller synthesis when closed-loop performance is a primary design objective.

## II. Observer-Based Feedback and Maximum Transient Energy Growth

Consider the linear time-invariant system,

$$\dot{x}(t) = Ax(t) + Bu(t)$$
$$y(t) = Cx(t) \tag{1}$$

where $x \in \mathbb{R}^n$ is the state vector, $u \in \mathbb{R}^m$ is the input vector, $y \in \mathbb{R}^p$ is the output vector, and $t \in \mathbb{R}$ is time. In considering transient energy growth in this study, we focus on the free response of the system to an initial perturbation $x(t_o) = x_o$ away from an equilibrium solution (e.g., a laminar base flow). The perturbation dynamics are given in terms of the matrix exponential, $x(t) = e^{A(t-t_o)}x_o$ and the associated perturbation energy will have a response given by

$$E(t) = x^T(t)Qx(t) \tag{2}$$

where $Q = Q^T > 0$. Without loss of generality, we take $Q = I$, since the state can always be transformed as $\bar{x} = Q^{1/2}x$ to satisfy this definition of energy. Further, define the maximum transient energy growth $G$ as

$$G = \max_{t \ge t_o} \max_{E(t_o) \ne 0} \frac{E(t) - E(t_o)}{E(t_o)} = \max_{t \ge t_o} \max_{\|x_o\|=1} E(t) - 1 \ge 0 \tag{3}$$

When the system in Eq. (1) is unstable, $E(t)$ will be unbounded, and $G$ will be infinite; when the system in Eq. (1) is stable, $G$ is simply the peak value in transient energy growth due to a so-called optimal disturbance [16]. The lower-bound $G = 0$ corresponds to the case of suppressed transient energy growth (i.e., monotonic stability [1]), which is the ultimate aim of controllers designed for transient energy growth suppression.

In light of these definitions, we introduce a Lemma [17] (herein referred to as the MTEG Lemma), which will be central to the analysis here.

*MTEG Lemma:* $G = 0$ if and only if $A^T + A \le 0$.

*Proof*: To show sufficiency, consider that if $\dot{E}(t) \le 0$ for all $t \ge t_o$ and all initial conditions then $G = 0$. Since $\dot{E}(t) = x^T(t)(A^T + A)x(t)$, it follows that $A^T + A \le 0$ is a sufficient condition for $G = 0$. Necessity can be shown by noting that $(A^T + A) \nleq 0$ implies the existence of an initial perturbation $x(t_o) = x_o$ that yields $\dot{E}(t_o) > 0$; thus, $E(t) > E(t_o)$ for some time $t > t_o$, and so $G > 0$. Therefore, $(A^T + A) \le 0$ is a necessary condition for $G = 0$.

The maximum transient energy growth can be reduced or suppressed by altering the system response characteristics via appropriate actuation $u(t)$. In principle, a full-state feedback control law $u(t) = -Kx(t)$ can be used to achieve various control objectives, including optimal regulation by means of a linear quadratic regulator (LQR) [15]. Here, $K \in \mathbb{R}^{m \times n}$ is the controller feedback gain. The closed-loop dynamics of the associated stable full-state feedback system will then be

$$\dot{x}(t) = (A - BK)x(t) \tag{4}$$

However, in practice, the full-state $x(t)$ is typically not directly available for feedback. Instead, measurements of the system outputs $y(t)$ can be used to estimate the full state by means of a stable state estimator of the form

$$\dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t) + L[y(t) - C\hat{x}(t)] \tag{5}$$

where $\hat{x}(t)$ is the state estimate. The observer gain $L \in \mathbb{R}^{n \times p}$ is chosen to yield desirable estimator performance, including adequate convergence rates and reliability in the face of both process and measurement uncertainties, as in the case of the optimal state estimator (i.e., the Kalman–Bucy filter).

The well-known separation principle establishes that a stabilizing full-state feedback controller and a stable state estimator can be designed independently of one another, then combined to yield a stabilizing dynamic compensator by means of the observer-based feedback law $u(t) = -K\hat{x}(t)$. To see this, consider the dynamics of the closed-loop observer–plant system with state $\tilde{x}(t) = [x(t), \hat{x}(t)] \in \mathbb{R}^{2n}$,

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{bmatrix} x(t) \\ \hat{x}(t) \end{bmatrix} = \underbrace{\begin{bmatrix} A & -BK \\ LC & A - BK - LC \end{bmatrix}}_{\tilde{A}} \begin{bmatrix} x(t) \\ \hat{x}(t) \end{bmatrix} \tag{6}$$

The observer–plant system in Eq. (6) can be brought into a form that replaces the estimated state $\hat{x}(t)$ by the estimation error $e(t) = x(t) - \hat{x}(t)$ via similarity transformation,

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{bmatrix} x(t) \\ e(t) \end{bmatrix} = \begin{bmatrix} A - BK & BK \\ 0 & A - LC \end{bmatrix} \begin{bmatrix} x(t) \\ e(t) \end{bmatrix} \tag{7}$$

Since the closed-loop operator in Eq. (7) appears in block-triangular form, the eigenvalues of the observer-based feedback system are simply the union of the eigenvalues of the full-state feedback system $(A - BK)$ and the estimator $(A - LC)$; indeed, since Eqs. (6) and (7) are related by similarity transformation, this establishes the well-known separation principle. In the remainder of the Paper, the term "observer-based feedback" will be used to refer to a closed-loop system as in Eq. (6), formed by means of the separation principle.

Although the separation principle provides guarantees on closed-loop stability, it does not provide any guarantees on closed-loop performance—an important point that is often overlooked in the context of transition control and transient energy growth suppression. Consider the closed-loop energy $\tilde{E}(t) = \tilde{x}^T(t)\tilde{Q}\,\tilde{x}(t)$ of the observer–plant system, with maximum transient energy growth $\tilde{G}$. Here, the first $n \times n$ subblock of $\tilde{Q}$ will be $Q$, to be consistent with the original state energy defined in Eq. (2). However, as with Eq. (2) and without loss of generality, we let $\tilde{Q} = I$ in the remainder of the Paper. From the MTEG Lemma, the closed-loop operator $\tilde{A}$ will exhibit $\tilde{G} = 0$ if and only if $\tilde{A}^T + \tilde{A} \le 0$. Considering this more closely,

$$\tilde{A}^T + \tilde{A} = \begin{bmatrix} A^T + A & (LC)^T - BK \\ LC - (BK)^T & (A - BK - LC)^T + (A - BK - LC) \end{bmatrix} \tag{8}$$

reveals that $\tilde{G}$ will depend on the plant $(A, B, C)$ as well as the compensator gains $K$ and $L$. Further, since $(A^T + A)$ is a principal submatrix of $(\tilde{A}^T + \tilde{A})$, it follows that $A^T + A \le 0$ is a necessary condition for $\tilde{A}^T + \tilde{A} \le 0$ [18]; thus, again by the MTEG Lemma, $G = 0$ is a necessary condition for $\tilde{G} = 0$. Hence, in the context of transient energy growth suppression and transition control, for which the open-loop dynamics exhibit $G > 0$, using the separation principle will limit closed-loop performance by guaranteeing that $\tilde{G} > 0$ as well. Note that $G = 0$ (or, equivalently, $A^T + A \le 0$) is a necessary condition for $\tilde{G} = 0$, but not a sufficient condition; all principal submatrices must be considered to establish $\tilde{A}^T + \tilde{A} \le 0$ [18]. As such, even if $G = 0$, $\tilde{G}$ will also depend on the particular system $(A, B, C)$ and the choices of $K$ and $L$, in general.

Since the maximum transient energy growth $\tilde{G}$ corresponds to the cyber-physical state $\tilde{x} = (x, \hat{x})$ of the observer–plant system, it appears possible that only one of either $x(t)$ or $\hat{x}(t)$ is contributing to the transient energy growth. Thus, we now establish the influence of the separation principle on the maximum transient energy growth $\tilde{G}_x$ of the physical plant and $\tilde{G}_{\hat{x}}$ of the observer. The maximum transient energy growth $\tilde{G}_x$ for the physical plant under control via the separation principle can be determined by first considering a modified energy $\tilde{E}_\epsilon(t) = \tilde{x}_\epsilon^T(t)\tilde{x}_\epsilon(t)$, where $\tilde{x}_\epsilon = W_\epsilon \tilde{x}$, $W_\epsilon = \mathrm{diag}(I_n, \epsilon I_n)$, $I_n$ is the $n \times n$ identity matrix, and $\epsilon > 0$ is a scalar. Then, noting that $\lim_{\epsilon\to 0}\tilde{E}_\epsilon(t) = E(t)$ and that $\tilde{A}$ and $W_\epsilon = W_\epsilon^T$ commute, it follows from the MTEG Lemma that $\tilde{G}_x = 0$ if and only if $\lim_{\epsilon\to 0} W_\epsilon(\tilde{A}^T + \tilde{A})W_\epsilon \le 0$. As before, $A^T + A \le 0$ is a necessary condition for $W_\epsilon(\tilde{A}^T + \tilde{A})W_\epsilon \le 0$, so we conclude that $A^T + A \le 0$ (and therefore $G = 0$) is a necessary condition for $\tilde{G}_x = 0$. That is, if $G > 0$, then it is guaranteed that $\tilde{G}_x > 0$ as well. The same can be said about the maximum transient energy growth $\tilde{G}_{\hat{x}}$ for the estimator, which corresponds to a case of observer peaking. To show this, consider the modified energy $\tilde{E}_\epsilon(t)$ defined using the coordinate transformation $W_\epsilon = \mathrm{diag}(\epsilon I_n, I_n)$; then, proceed as before.

In summary, $A^T + A \le 0$ (and therefore $G = 0$) is a necessary condition for $\tilde{G} = 0$, $\tilde{G}_x = 0$, and $\tilde{G}_{\hat{x}} = 0$. Thus, if the uncontrolled plant exhibits nontrivial maximum transient energy growth (i.e., $G > 0$), then it is guaranteed that the separation principle will result in nontrivial maximum transient energy growth in closed-loop for the physical plant (i.e., $\tilde{G}_x > 0$), the estimator (i.e., $\tilde{G}_{\hat{x}} > 0$), and the cyber-physical observer–plant system (i.e., $\tilde{G} > 0$). The performance limitations identified here apply only to dynamic output feedback laws synthesized using the separation principle, i.e., observer-based feedback as in Eq. (6). Not all output feedback control laws are necessarily subject to these same limitations. As we will see in the illustrative example that follows, even when a static output feedback law can be determined to achieve zero maximum transient energy growth a necessary condition for guaranteeing the existence of a dynamic output feedback compensator that can achieve the same [17], the separation principle will not (and cannot) yield a closed-loop system that achieves the same if the uncontrolled plant exhibits $G > 0$.

## III. Illustrative Example: Simple Nonnormal System

Consider the system

$$\dot{x}(t) = \begin{bmatrix} -1/R & 0 \\ 1 & -2/R \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u(t)$$
$$y(t) = [0 \quad 1]x(t) \tag{9}$$

where $R > 0$ is a scalar parameter. Variants of this system are often used to demonstrate the role of nonnormality in giving rise to transient energy growth [1,2,17,19]. The origin is asymptotically stable, but the system exhibits $G > 0$ for $R > R^* = 2\sqrt{2}$. In Fig. 1, we compare the worst-case transient energy response $E(t)$ with $R = 2 < R^*$ and $R = 3 > R^*$ for the uncontrolled plant with the corresponding worst-case closed-loop system response from each of three different controller synthesis techniques: LQR, LQG, and static output feedback (SOF). In all cases, the optimal disturbance for the worst-case response is computed by means of the iterative Algorithm 3.1 of Whidborne and Amar [20]. In the case of LQG control, the worst-case response corresponds to an optimal disturbance on the full observer–plant state $\tilde{x} = (x, \hat{x})$. In instances for which the maximum transient energy growth is zero, there is no optimal disturbance, and so the initial condition is set to either $x(t_o) = (1, 0)$ or $\tilde{x}(t_o) = (1, 0, 0, 0)$. All energy responses in Fig. 1 correspond to the energy of the physical plant $E(t)$ normalized by the initial energy $E(t_o) = E_o$.

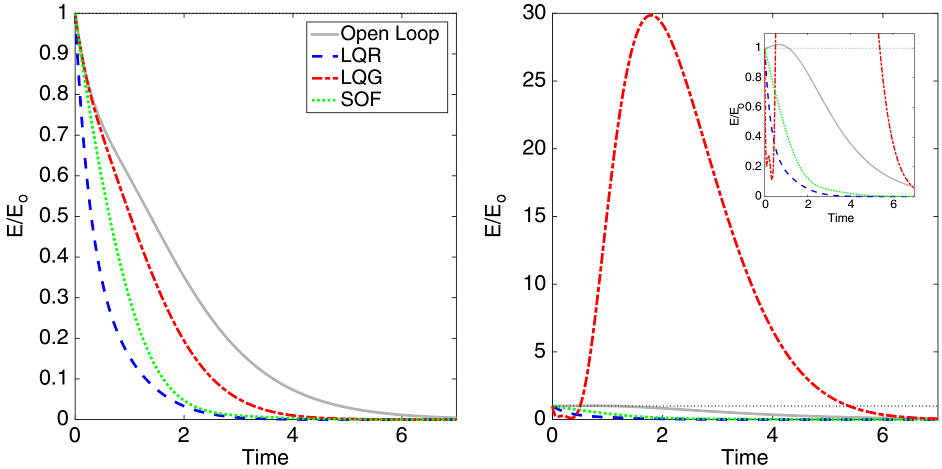
The LQR design here yields a full-state feedback law that minimizes the performance index,

$$J = \int_0^\infty x^T(t)x(t) + u^T(t)u(t)\,\mathrm{d}t \tag{10}$$

subject to the linear dynamic constraint $\dot{x}(t) = Ax(t) + Bu(t)$. In the present study, the LQR control gain is computed directly via the MATLAB® command lqr. The response under the LQR full-state feedback control yields $G = 0$ for both $R = 2$ and $R = 3$, as seen in Fig. 1. Note that LQR control is not guaranteed to yield zero transient energy growth in general; rather, for this example, the objective function in Eq. (10) was specifically selected after tuning to achieve zero maximum transient energy growth for the closed-loop response for both $R = 2$ and $R = 3$.

The separation principle is invoked for LQG optimal controller synthesis. The same LQR control gains determined by Eq. (10) are used in an observer-based feedback capacity [i.e., $u(t) = -K\hat{x}(t)$]. The observer gain $L$ is computed via the MATLAB® command lqe, which computes a solution to the optimal estimation problem, which is dual to the optimal control problem. Here, the observer gain $L$ was selected by tuning the estimator objective function to reduce $\tilde{G}_x$ associated with the closed-loop energy response. Recall, that $\tilde{G}_x$—and the associated optimal disturbance—can be computed by considering $\lim_{\epsilon\to 0}\tilde{E}_\epsilon(t) = E(t)$. As expected, in the case of $R = 3 > R^*$, the LQG controller yields $\tilde{G}_x > 0$, whereas the same LQG controller synthesis approach applied to the case of $R = 2 < R^*$ yields $\tilde{G}_x = 0$. Further, despite tuning to reduce $\tilde{G}_x$, the physical energy $E(t)$ grows to approximately 30 times its initial value. In fact, $\tilde{G}_x \approx 30G$, meaning the LQG controller degrades the transient energy growth performance relative to the uncontrolled (open-loop) response.

Lastly, we consider the SOF control law $u(t) = -y(t)$. By the MTEG Lemma, the resulting closed-loop operator $(A - BC)$ is guaranteed to exhibit zero maximum transient energy growth for all values of $R > 0$. Note that the SOF controller considered here is one




a) $R = 2 < R^*$ $(G = 0)$ b) $R = 3 > R^*$ $(G > 0)$

**Fig. 1 Comparison of worst-case responses for the controlled system in Eq. (9), for which $R^* = 2\sqrt{2}$.**

of a family of SOF controllers that can achieve zero transient energy growth [17].

The results of this simple example illustrate the limitations of the separation principle that were proven in Sec. II. These results further suggest that controllers synthesized by the separation principle (LQG or otherwise) can be unreliable for transient energy growth suppression and transition control. Although the separation principle can greatly simplify controller synthesis in many instances, the performance consequences of the separation principle must also be taken into consideration. As shown in this example, alternative output feedback approaches are not necessarily subject to the same performance limitations and should be considered as well as if not instead of observer-based feedback.

## IV. Conclusions

This Paper has proven that if the maximum transient energy growth for an uncontrolled plant is nontrivial (i.e., $G > 0$), then controller synthesis by the separation principle will necessarily result in nontrivial maximum transient energy growth in closed-loop (i.e., $\tilde{G} > 0$, $\tilde{G}_x > 0$, and $\tilde{G}_{\hat{x}} > 0$). These results were established by invoking the MTEG Lemma and properties of negative semidefinite operators to show that $A^T + A \leq 0$ (and thus $G = 0$) is a necessary condition for $\tilde{G} = 0$, $\tilde{G}_x = 0$, and $\tilde{G}_{\hat{x}} = 0$. This result highlights a fundamental performance limitation of observer-based feedback control in the context of transient energy growth suppression and transition control. As illustrated in the example of Sec. III, performance under full-state feedback is not necessarily a reliable indicator for closed-loop performance under observer-based feedback. Indeed, no amount of controller and estimator tuning can overcome the performance limitations of observer-based feedback. If closed-loop transient energy growth is of primary importance, then alternative approaches for output feedback control may yield better performance in this regard and should also be considered as candidate control approaches. Further, the result for $\tilde{G}_{\hat{x}}$ establishes that the separation principle will lead to observer peaking from some initial conditions in closed-loop whenever $A^T + A \nleq 0$ for the open-loop system. Thus, although observer peaking is commonly rooted out as a limiting factor on performance in observer-based feedback control, this result shows that observer peaking and the associated transient energy growth of the physical plant can actually be seen as direct consequences of using the separation principle to control a system for which $G > 0$, or equivalently $A^T + A \nleq 0$.

The results presented here can be generalized further. Note that all stabilizing controllers can be constructed by means of a $Q$ parameterization, in which a separation-principle-based controller/observer structure is combined with a free parameter $Q(s)$ [21]. Then, since the separation principle yields a dynamic compensator that is strictly proper, it follows that when $Q(s)$ is strictly proper the resulting parameterized dynamic compensator will also be strictly proper; in contrast, when $Q(s)$ is semiproper, the resulting parameterized dynamic compensator will also be semiproper. With this in mind, performing an analysis similar to that of Sec. II shows that all strictly proper stabilizing controllers will result in nontrivial maximum transient energy growth whenever $G > 0$. In contrast, it may be possible to achieve zero maximum transient energy growth in closed-loop when the dynamic compensator is semiproper; indeed, this corresponds to the analogous necessary condition for the generalized result. For example, static output feedback control constitutes a semiproper control structure, which was seen to fully suppress transient energy growth in the illustrative example of this Paper. Interestingly, the existence of a static output feedback controller that achieves zero maximum transient energy growth is a necessary condition for the existence of a dynamic compensator that can achieve the same [17]. Further, as shown in [17], a $Q$ parameterization can be used to design controllers that minimize the maximum transient energy growth.

Retrospectively, the performance limitations of observer-based feedback for transient energy growth suppression are not entirely surprising. Consider that the closed-loop operator in Eq. (7) is nonnormal. That nonnormality is a necessary condition for transient energy growth is well established in the flow control community; it is the high degree of nonnormality of the linearized Navier–Stokes operator that is commonly attributed to transient energy growth in the context of subcritical transition in shear flows [1]. And, yet, the nonnormality of the closed-loop operator in Eq. (7) seems to have evaded attention in many flow control studies. An important ramification of this nonnormality is that, even if a full-state feedback law can fully remove nonnormality of the physical plant (as in [19]), invoking the separation principle to synthesize an observer-based feedback law will inevitably yield a nonnormal cyber-physical observer–plant system. In this case, although the modes of the physical system $(A - BK)$ will still be orthogonal, the modes of the coupled observer–plant system will be oblique, since the associated operator is nonnormal. Further, the degree of nonnormality will depend on the specific plant $(A, B, C)$ and the particular choice of feedback and observer gains, $K$ and $L$, respectively. Thus, $K$ and $L$ can be tuned to improve performance for a given flow control configuration; yet, if $A^T + A \nleq 0$, then $\tilde{G} = 0$, $\tilde{G}_x = 0$, and $\tilde{G}_{\hat{x}} = 0$ will never be achieved. Further, the performance limitations of the separation principle presented here hold true regardless of the system terms $(B, C)$. Thus, although efforts at optimal actuator/sensor placement and selection can be useful at reducing the maximum transient energy growth, such efforts will never yield $\tilde{G} = 0$, $\tilde{G}_x = 0$, nor $\tilde{G}_{\hat{x}} = 0$ when $A^T + A \nleq 0$, if observer-based feedback is used for controller synthesis. Thus, to overcome these performance limitations, alternatives to observer-based feedback control structures need to be considered as candidates for control.

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