3A02 Reducing Transient Energy Growth in Linearized Channel Flow with Output Feedback Control

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Abstract

Transient energy growth of small disturbances provides a mechanism for sub-critical transition in many shear flows. Here, we investigate sensor-based feedback strategies that aim to reduce transient energy growth in a linearized channel flow. Wall-normal blowing and suction is used as actuation at the upper and lower channel walls, while wall shear-stress measurements are used for feedback. We show that linear quadratic controller synthesis with a static output feedback structure outperforms the more conventional observer-based optimal feedback control strategies that are standard in flow control practice. It is found that the resulting static output feedback laws reduce the worst-case transient energy growth, while also being robust to parametric and modeling uncertainties, such as Reynolds number variations.

1. Introduction

Transient energy growth (TEG) of small perturbations about a linearly stable laminar equilibrium state provides a mechanism for bypass transition in channel flows and many other shear flows[1)]. Various feedback control strategies have been proposed to reduce the growth of perturbation by means of full-information linear quadratic synthesis[2)]. These controllers are then adapted for use in an output feedback capacity by estimating the flow state from a limited set of sensor measurements using an adequately designed state observer. Such approaches for observer-based feedback possess inherent TEG performance limitations due to adverse cyber-physical couplings that can arise between the control system dynamics and the fluid dynamics[3)]. As we will show here, observer-based feedback in linearized channel flow can result in higher levels of TEG than in the uncontrolled system. Rather than working with observer-based feedback, we demonstrate that linear quadratic controller synthesis with a static output feedback structure[3-5)] is able to mitigate these performance limitations and improve TEG performance. The resulting feedback flow control laws are also found to be robust to parametric and modeling uncertainties, such as Reynolds number variations.

2. Feedback Flow Control Synthesis

The linear quadratic regulator problem seeks to solve the following optimization:

$$\min_{u(t)} \int_0^\infty x(t)^T Q x(t) + u(t)^T R u(t) dt \qquad (1)$$

subject to the linear dynamic constraint

$$\dot{x}(t) = Ax(t) + Bu(t) \qquad (2)$$

where $Q \geq 0$ and $R > 0$. The standard solution to the linear quadratic regulator (LQR) problem is a full-state feedback control law $u(t) = -Kx(t)$, where the controller gain K can be determined from the solution of an algebraic Riccati equation[6)].

Here, we will consider a modified static output feedback (SOF) version of the LQR problem, i.e., SOF-LQR. We introduce an additional constraint that the controller must be in the form of SOF as $u(t) = -Ky(t)$, where $y(t) = Cx(t)$ denotes the sensor outputs. In doing so, the resulting solution can be found by solving the three sufficiency conditions for optimality:

$$(A - BKC)^T S + S(A - BKC) + Q + C^T K^T RKC = 0 \qquad (3)$$

$$(A - BKC)P + P(A - BKC)^T + I = 0 \qquad (4)$$

$$RKCPC^T - B^T SPC^T = 0 \qquad (5)$$

where I is the identity matrix and the initial state is assumed to be unknown.

Note that these sufficiency conditions constitute a system of coupled nonlinear matrix equations, which is significantly more challenging to solve compared to the algebraic Riccati equation required for the standard full-information LQR problem or the additional algebraic Riccati equations required in determining the optimal observer in linear quadratic Gaussian (LQG) synthesis for observer-based output feedback control. Nonetheless, several alternative numerical algorithms exist for solving these equations for the solution to the optimal SOF-LQR problem[5)].

In the present study, we make use of the iterative Anderson-Moore algorithm for SOF-LQR design. The Anderson-Moore algorithm is a gradient method that has guaranteed convergence properties to the unique optimal solution. In this study, we have further incorporated a tailored Armijo-type rule[7)] within the Anderson-Moore algorithm in order to increase the convergence rate of the method. This modification allows the optimal solution to be computed with fewer iterations than the standard approach.

3. Results

The alternative controller synthesis approaches described above will be applied to a linearized channel flow. Blowing and suction actuation is applied at the upper- and lower-walls of the channel, with the blowing and suction rate at each wall serving as a system input. Shear-stress measurements at the upper- and lower-walls are used for output feedback. The channel flow dynamics are modeled by means of a Fourier-Fourier-Chebyshev spectral collocation method. In the present study, all results correspond to the streamwise and spanwise wave number pair $(\alpha,\beta)=(0,1)$ for $Re=3000$. Full details of this model are outlined in McKernan et al.[8].

In this study, we consider the energy response to worst-case disturbance (i.e., a disturbance that causes the system to achieve the maximum TEG possible). Here, worst-case disturbances are computed using the method outlined in Whidborne and Amar[9]. Fig. 1 reports the worst-case response of the uncontrolled flow and of the flow controlled by means of LQR and SOF-LQR strategies. The response for LQG control here is not the worst-case; rather, it is determined by computing the worst-case, then setting the estimator states to zero. Even with this better behaved sort of disturbance, it is clear that LQG control leads to a significantly larger TEG than the worst-case TEG in the uncontrolled system. Both the full-information LQR and SOF-LQR strategies reduce the worst-case TEG relative to the uncontrolled case.

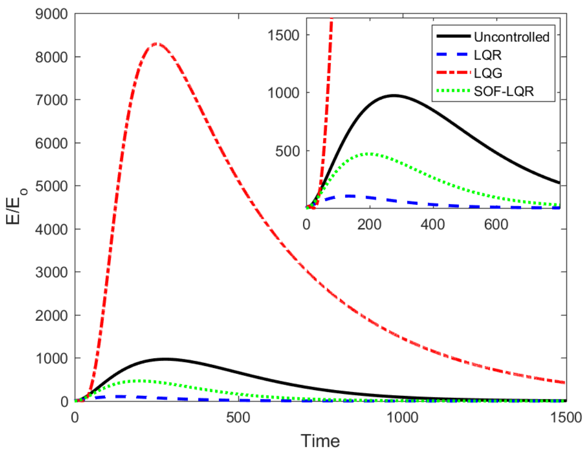

Fig. 1. LQG control increases TEG relative to the worst-case uncontrolled system. Full-information LQR and SOF-LQR both reduce the worst-case TEG. The LQG controller is initialized with the observer state set to zero, and so does not constitute the worst-case response here. All other responses are worst-case responses.

Beyond TEG performance, the full-information LQR and SOF-LQR control exhibit robustness to parametric uncertainties. Fig. 2 and 3 report the results of applying a controller designed for $Re=3000$ to flows with $Re=2500$ and $Re=3500$, respectively. The reported results correspond to worst-case responses for the associated closed-loop systems. The resulting performance for these "off-design" conditions suggest that SOF-LQR can achieve reduced worst-case TEG with acceptable robustness, a property that is already well-established for the full-information LQR strategy.

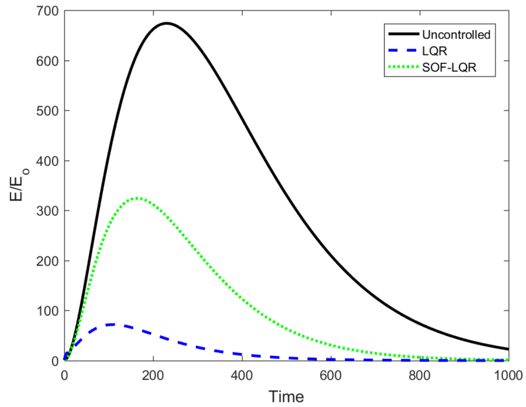

Fig. 2. Full-information LQR and SOF-LQR controllers are exhibit robust performance in the presence of parametric uncertainties. Here, controllers are designed for $Re=3000$, then applied to $Re=2500$. All responses correspond to the worst-case TEG.

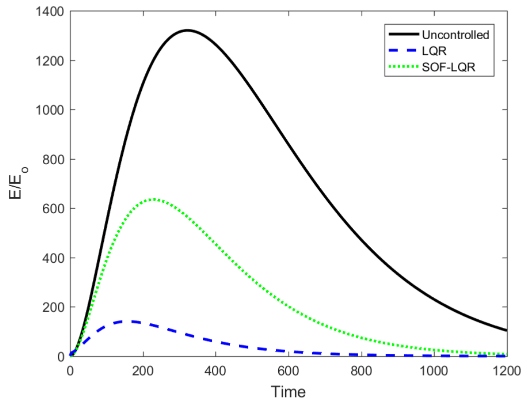

Fig. 3. Full-information LQR and SOF-LQR controllers are exhibit robust performance in the presence of parametric uncertainties. Here, controllers are designed for $Re=3000$, then applied to $Re=3500$. All responses correspond to the worst-case TEG.

5. Conclusions

The results of this study suggest that SOF-LQR synthesis can yield improved TEG performance over LQG synthesis when sensor-based output feedback is used in the context of linearized channel flow. Although the controller synthesis process associated with SOF-LQR design is slightly more challenging, the performance benefits in the context of output feedback control warrant the additional effort. Further, the calculation of SOF-LQR controllers can be expedited through the use of Armijo-type adaptations to the iterative Anderson-Moore algorithm. Indeed, SOF-LQR should be considered as an alternative to standard observer-based feedback strategies, such as LQG control, when sensor-based output feedback control is required for TEG reduction and transition delay.

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