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ADAPTIVE PASSIVITY-BASED NONLINEAR CONTROL FOR STRICT FEEDBACK FORM SYSTEMS

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ABSTRACT

This paper presents a tracking algorithm for the adaptive control of nonlinear dynamic systems represented in Strict Feedback Form with parametric uncertainty. The construction of the stabilizing algorithm is given using Passivity-based arguments that result in an Adaptive Passivity-Based Controller (APBC). This paper also shows a comparison with a controller designed via Adaptive Backstepping with tuning functions. The Adaptive Backstepping Controller (ABC) has many additional coupling terms that make its design and implementation more complex. These coupling terms may give the ABC better robustness; however they may also result in unwanted transients. On the other hand, the APBC has a convenient decoupling property that provides a diagnostic tool for detection of non-parametric model error.

1. INTRODUCTION

In this paper we will study I/O feedback linearizable systems that can be transformed into Strict Feedback Form (Krstic et. al., 1995). Further, these systems will have parametric uncertainty of the following form:

$$\dot{\xi} = f(\xi) + g(\xi)x_1 \qquad (1.1\text{ a})$$

$$\begin{aligned} \dot{x}_1 &= \theta_1 f_1(\xi, x_1) + g_1(\xi, x_1)x_2 \\ &\vdots \\ \dot{x}_{r-1} &= \theta_{r-1} f_{r-1}(\xi, x_1, \ldots, x_{r-1}) + g_{r-1}(\xi, x_1, \ldots, x_{r-1})x_r \\ \dot{x}_r &= \theta_r f_r(\xi, x_1, \ldots, x_r) + g_r(\xi, x_1, \ldots, x_r)u \end{aligned} \qquad (\text{b})$$

$$y = h(x) \qquad (\text{c})$$

Here, r denotes the relative degree of the system and $x \in \Re^r$ denotes the portion of the state which is visible through the output equation, (c). Equation (a) describes the evoloution of the state, $\xi \in \Re^{n-r}$, which is not visible at the output. In the following, the system is assumed to be nonlinear minimum phase (Isidori, 1995) and therefore the zero dynamics, ξ, are stable. We assume that f_i, g_i, and h are smooth vector fields and $g_i(x) \neq 0\ \forall x$. 'Smooth' means that the functions are differentiable to any order necessary, potentially C^∞. The parameters $\theta_i \in \Re$ are unknown constants and we will use an adaptation scheme to estimate them. The following can be generalized to include the case where θ_i are vectors in $\Re^{n_i}$ and f_i correspondingly maps to $\Re^{n_i}$.

Control of systems in the form of Equation (1.1) has attracted a great deal of interest. Adaptive Backstepping techniques presented by Kanellakopoulos, et. al. (1991) and Kristic and Kokotovic (1996) apply an iterative method to develop 'synthetic inputs' and estimators for a stabilizing controller. The controller presented in this paper differs significantly from the Backstepping approach by reducing the problem to r-simpler problems. The objective is to perform output tracking by creating multiple errors between the individual states and the desired value of each state. The desired state values are then used as synthetic inputs to control each state error. Notice that the assumption, $g_i(x) \neq 0\ \forall x$, ensures that x_{i+1} can always be used to affect x_i. There are two key benefits to separating the dynamics into multiple errors: (1) the effect of model uncertainty can be localized and (2) differentiation of the system model in the controller can be avoided. The first benefit provides a useful diagnostic tool to locate model uncertainty while the second benefit reduces controller complexity. Both of these benefits are important in the presence of model uncertainty. Previous stability results for this type of control strategy have been obtained for systems in parametric strict feedback form under the assumption of known bounds for the parametric uncertainty (Yip, 1997).

The remainder of this paper has the following structure: Section 2 describes the Adaptive Passivity-Based Controller design. Section 3 gives a comparison of APBC and an Adaptive Backstepping controller on a simple model. Then Section 4 furthers this comparison by examining the effect of non-parametric model uncertainty. This will clarify the statement that the APBC controller results in decoupled error dynamics. A conclusion will then summarize the results of this work.

2. ADAPTIVE PASSIVITY-BASED CONTROLLER (APBC) DESIGN

Suppose the goal is to choose the control, u, such that the output of the system, y, tracks some desired value. Define the tracking error as:

$$e = y - y_{desired} \tag{2.1}$$

For simplicity, assume that $y = h(x) = x_1$. Create r separate error dynamics as follows:

$$e_i = x_i - x_{(i)desired} \qquad i = 1,\ldots,r \tag{2.2}$$

where $x_{(i)desired}$ will be defined shortly. Differentiating each error in Equation (2.2) gives:

$$\begin{aligned} \dot{e}_i &= \dot{x}_i - \dot{x}_{(i)desired} = \theta_i f_i + g_i x_{i+1} - \dot{x}_{(i)desired} \quad i = 1,\ldots,r\text{-}1 \\ \dot{e}_r &= \dot{x}_r - \dot{x}_{(r)desired} = \theta_r f_r + g_r u - \dot{x}_{(r)desired} \end{aligned} \tag{2.3}$$

We have transformed the r-dimensional system given by Equation (1.1 b) into 'r' 1-dimensional error systems. Similar to Green and Hedrick (1990), the desired state values, $x_{(i+1)desired}$, are 'synthetic inputs' used to the control the i^th state for $i = 1,\ldots,r-1$. For the r^th system, no synthetic input is needed because u enters the e_r dynamics directly. Equation (2.3) can be rewritten to explicitly show the dependence on the synthetic inputs:

$$\begin{aligned} \dot{e}_i &= \theta_i f_i + g_i e_{i+1} + g_i x_{(i+1)desired} - \dot{x}_{(i)desired} \quad i = 1,\ldots,r\text{-}1 \\ \dot{e}_r &= \theta_r f_r + g_r u - \dot{x}_{(r)desired} \end{aligned} \tag{2.4}$$

The synthetic inputs, $x_{(i+1)desired}$, are chosen to force their respective error dynamics to decay to zero:

$$\begin{aligned} \overbrace{x_{(i+1)desired}}^{\text{synthetic inputs}} &\equiv \frac{1}{g_i}\left(-\hat{\theta}_i f_i + \dot{x}_{(i)desired} - k_i e_i\right) \quad i = 1,\ldots,r\text{-}1 \\ u &\equiv \frac{1}{g_r}\left(-\hat{\theta}_r f_r + \dot{x}_{(r)desired} - k_r e_r\right) \end{aligned} \tag{2.5}$$

where $\hat{\theta}_i$ is our best estimate of the actual parameter θ_i. Note again that the assumption $g_i(x) \neq 0\,\forall x$ ensures that (2.5) is well defined. The controller is augmented with the following set of estimators to obtain this best estimate:

$$\begin{aligned} \dot{\hat{\theta}}_i &= \begin{cases} e_i \gamma_i f_i & \text{if } |e_i| \geq |e_{i+1}| \\ 0 & \text{if } |e_i| < |e_{i+1}| \end{cases} \quad i = 1,\ldots,r\text{-}1 \\ \dot{\hat{\theta}}_r &= e_r \gamma_r f_r \end{aligned} \tag{2.6}$$

where $\gamma_i>0$ is used to tune the parameter converence rate. The justification for turning off the estimators when $e_i<e_{i+1}$ will be given in the proof. A superficial justification is that we will be trying to force the errors to converge sequentially. Therefore, $e_i<e_{i+1}$ means that the i^th error is converging faster than necessary and so the estimation is not required.

Define the parameter error as $\tilde{\theta}_i = \theta_i - \hat{\theta}_i$. Since we have assumed that θ_i is constant, $\dot{\tilde{\theta}}_i = -\dot{\hat{\theta}}_i$. Combining equations (2.4)-(2.6) leads to a chain of interconnected (state and estimator) error dynamics:

$$\begin{aligned} \dot{e}_i &= -k_i e_i + \tilde{\theta}_i f_i + g_i e_{i+1} \quad i = 1,\ldots,r\text{-}1 \\ \dot{\tilde{\theta}}_i &= \begin{cases} -e_i \gamma_i f_i & \text{if } |e_i| \geq |e_{i+1}| \\ 0 & \text{if } |e_i| < |e_{i+1}| \end{cases} \\ \dot{e}_r &= -k_r e_r + \tilde{\theta}_r f_r \\ \dot{\tilde{\theta}}_r &= -e_r \gamma_r f_r \end{aligned} \tag{2.7}$$

The stability properties of this controller / estimator structure are summarized by the following theorem:

<u>*Theorem:*</u>

Given the system in Equation (1.1) and using the synthetic and actual inputs in Equations (2.5) and the estimators in Equation (2.6) with controller gains chosen to be $\{k_i = |g_i|\,\forall i = 1,\ldots,r\text{-}1 \text{ and } k_r > 0\}$ *and estimator gains given by positive contants γ_i, the output tracking error $e_1 = y - y_{desired}$ is globally asymptotically stable.*

Proof:

Initially we will assume that $|e_i| \geq |e_{i+1}|$. We will deal with the alternative subsequently. By assumption, g_i is smooth and $g_i \neq 0\,\forall(x,t)$ so we only have to consider two cases:

<u>Case 1</u>: $g_i>0$ and $i = 1,\ldots,r-1$

In this case the i-th error dynamics can be written as:

$$\begin{aligned} \dot{e}_i &= -|g_i|(e_i - e_{i+1}) + \tilde{\theta}_i^T f_i \\ \dot{\tilde{\theta}}_i &= -\Gamma_i f_i e_i \end{aligned} \tag{2.8}$$

This system is output strictly passive (Khalil, 1996), which can be verified with the following positive definite storage function:

$$\Phi_i(e_i, \tilde{\theta}_i) = \frac{e_i^2}{2} + \frac{1}{2}\tilde{\theta}_i^T \Gamma_i^{-1} \tilde{\theta}_i \tag{2.9}$$

Consider $|g_i|e_{i+1}$ as the input and e_i as the only output of the error dynamics given in Equation (2.8). Differentiating the storage function gives:

$$\begin{aligned} \frac{d}{dt}\Phi_i(e_i, \tilde{\theta}_i) = \dot{\Phi}_i &= e_i \dot{e}_i + \tilde{\theta}_i^T \Gamma_i^{-1} \dot{\tilde{\theta}}_i \\ &= e_i\left[-|g_i|(e_i - e_{i+1}) + \tilde{\theta}_i^T f_i\right] + \tilde{\theta}_i^T \Gamma_i^{-1}\left[-\Gamma_i f_i e_i\right] \\ &= -|g_i| e_i^2 + \underbrace{e_i}_{\text{output}}\,\underbrace{|g_i| e_{i+1}}_{\text{input}} \end{aligned} \tag{2.10}$$

Rearranging this equation gives:

$$\underbrace{e_i}_{\text{output}}\,\underbrace{|g_i| e_{i+1}}_{\text{input}} = \dot{\Phi}_i + \underbrace{|g_i| e_i^2}_{\geq 0} \tag{2.11}$$

This shows that the error dynamics are output strictly passive from $|g_i|e_{i+1} \to e_i$. Since $|g_i|$ is strictly positive, the mapping from $e_{i+1} \to e_i$ is also output strictly passive.

Case 2: $g_i<0$ and $i=1,\ldots,r-1$

In this case choosing $k_i=|g_i|=-g_i$ gives the error dynamics:

$$\begin{aligned}\dot{e}_i &= -|g_i|(e_i-(-e_{i+1}))+\tilde{\theta}_i^T f_i \\ \dot{\tilde{\theta}}_i &= -\Gamma_i f_i e_i\end{aligned} \tag{2.12}$$

Use the same storage function as in Equation (2.9) to get:

$$\begin{aligned}\frac{d}{dt}\Phi_i(e_i,\tilde{\theta}_i) = \dot{\Phi}_i &= e_i\dot{e}_i+\tilde{\theta}_i^T\Gamma_i^{-1}\dot{\tilde{\theta}}_i \\ &= e_i\left[-|g_i|(e_i-(-e_{i+1}))+\tilde{\theta}_i^T f_i\right]+\tilde{\theta}_i^T\Gamma_i^{-1}\left[-\Gamma_i f_i e_i\right] \\ &= -|g_i|e_i^2+\underbrace{e_i}_{\text{output}}\underbrace{(-|g_i|e_{i+1})}_{\text{input}}\end{aligned} \tag{2.13}$$

This is similar to Case 1 except the input function is now $(-|g_i|e_{i+1})$. Rearranging the equation above gives:

$$\underbrace{e_i}_{\text{output}}\underbrace{(-|g_i|e_{i+1})}_{\text{input}}=\dot{\Phi}_i+\underbrace{|g_i|e_i^2}_{\geq 0} \tag{2.14}$$

Thus the error dynamics are output strictly passive from $(-|g_i|e_{i+1})\to e_i$. Again since $|g_i|$ is strictly positive, the mapping from $-e_{i+1}\to e_i$ is also output strictly passive.

From Case 1 and Case 2, we conclude that $|e_{i+1}|\to|e_i|$ is output strictly passive and hence finite gain L_2 stable for all $i\in[1,r-1]$.

Next, examine the r-th error dynamics:

$$\begin{aligned}\dot{e}_r &= -k_r e_r+\tilde{\theta}_r^T f_r+u^* \\ \dot{\tilde{\theta}}_r &= -\Gamma_r f_r e_r\end{aligned} \tag{2.15}$$

where u^* is a fictitious input which will be used here only for the analysis. Use the same storage function given in Equation (2.9)

$$\Phi_r(e_r,\tilde{\theta}_r)=\frac{e_r^2}{2}+\frac{1}{2}\tilde{\theta}_r^T\Gamma_r^{-1}\tilde{\theta}_r \tag{2.16}$$

and differentiate with respect to time:

$$\begin{aligned}&\dot{\Phi}_r=-k_r e_r^2+e_r u^* \\ &\Rightarrow e_r u^*=\dot{\Phi}_r+k_r e_r^2\end{aligned} \tag{2.17}$$

This shows that, similar to the other r-1 error systems, the r-th error dynamics are also output strictly passive and hence finite gain L_2 stable. However, the input to this system is the fictitious input $u^*\equiv 0\ \forall t$. Clearly, u^* is an L_2 bounded signal and therefore e_r is also L_2 bounded. Since serial interconnections of finite gain L_2 stable systems are also finite gain L_2 stable, all the e_i signals are also L_2 bounded for $i=1,\ldots,r-1$.

We can also prove that all the errors, $(e_i,\tilde{\theta}_i)$ are uniformly bounded in time for $i=1,\ldots,r$. Notice that the storage function Φ_r is also a Lyapunov function for the r-th error dynamics. Further, Equation (2.17) shows that its derivative is negative semidefinite when the fictitious input is zero, $u^*\equiv 0$. Therefore, $(e_r,\tilde{\theta}_r)$ are bounded in time.

For the i-th error dynamics we will make the induction assumption that e_{i+1} is uniformly bounded in time. Let M_{i+1} be this bound, i.e. $|e_{i+1}|\leq M_{i+1}\ \forall t$. When $|e_i|\geq|e_{i+1}|$ we can again use the storage function Φ_i as a Lyapunov function. Equations (2.10) and (2.13) show that the time derivative is:

$$\frac{d}{dt}\Phi_i(e_i,\tilde{\theta}_i)=\dot{\Phi}_i=-|g_i|e_i^2\pm e_i|g_i|e_{i+1}\leq 0 \tag{2.18}$$

The derivative is negative semidefinite because the first term overpowers the second term when $|e_i|\geq|e_{i+1}|$. Therefore the error states $(e_i,\tilde{\theta}_i)$ stay bounded when $|e_i|\geq|e_{i+1}|$.

Alternatively, when $|e_i|<|e_{i+1}|$ we can see in Equation (2.18) that the storage function is sign indefinite. Therefore, if we left the estimator on, it is possible that $\tilde{\theta}_i\to\infty$ while $|e_i|<|e_{i+1}|$. To prevent this from happening, the estimator is turned off. As a consequence, $\tilde{\theta}_i$ is constant when $|e_i|<|e_{i+1}|$ and e_i must be bounded by M_{i+1}, which is the bound for e_{i+1}. For example, suppose e_{i+1} is a constant. If the estimator is turned off at τ_1 and then turned back again at τ_2 then $\Phi_i|_{t=\tau 1}=\Phi_i|_{t=\tau 2}$ since the parameter errors remain unchanged and $|e_i|\,|_{t=\tau 1}=|e_i|\,|_{t=\tau 2}$.

By the two scenarios, $(e_i,\tilde{\theta}_i)$ must stay bounded for all time. By induction, this statement is true for all $i=1,\ldots,r$. The boundedness of all error states and the smoothness of f_i, g_i implies that all synthetic inputs, $x_{(i)desired}$, and system states x_i are bounded.

We can now apply Barbalat's Lemma (Khalil, 1996) to conclude that e_r asymptotically converges to zero. First note that f_r is bounded since it is a continuous function of bounded arguments. Therefore, $\dot{e}_r$ is bounded. We conclude that $\ddot{\Phi}_r$ is bounded and hence $\dot{\Phi}_r$ is uniformly continous. By Barbalat's Lemma, $\dot{\Phi}_r\to 0$ and thus $e_r\to 0$.

Since $e_r\to 0$ we can show that $e_{r-1}\to 0$. Either $|e_{r-1}|<|e_r|$, in which case the conclusion is obvious or $|e_{r-1}|\geq|e_r|$. In the latter case we have proved that the mapping from $|e_r|\to|e_{r-1}|$ is output strictly passive. Since the input to this output strictly passive system is decaying to zero, the output, e_{r-1}, must also decay to zero. By induction all intermediate errors and hence the output error must converge to zero.

■

Remarks:

(1) The Passivity-Based structure of the algorithm dictates a sequential convergence of the tracking errors. The e_r error will converge which causes the e_{r-1} error to converge and so on. All intermediate errors converge to zero which eventually causes the output error to converge. If the i^{th} error (e_i) were non-zero and bounded (e.g. due to model error or disturbances), then we cannot conclude convergence of all subsequent intermediate errors, including the output. We can only conclude (as shown in the proof) that all intermediate errors will stay bounded. This provides a useful tool for detection of non-parametric uncertainty.

(2) The proof only guarantees that the output and intermediate tracking errors will converge to zero. The parameter errors will remain bounded but may not necessarily converge to zero. For parameter convergence, we need to impose some persistency of excitation constraints. For example, assume the i^{th} error system is zero-state observable (Khalil 1996), i.e. the input and output identically zero implies that the state is identically zero. Then the i^{th} parameter vector, θ_i, will converge along with the output, e_i. If θ_i is a scalar, then zero-state observability of Equation (2.8) means:

$$e_i, e_{i+1} \equiv 0 \Rightarrow \dot{\tilde{\theta}}_i \equiv 0, \tilde{\theta}_i f_i \equiv 0 \tag{2.19}$$

To guarantee that $\tilde{\theta}_i \equiv 0$ as required for zero-state observability, we need $f_i \neq 0$.

(3) The main benefit of this approach is that the controller design problem can be decoupled into r simple problems. This reduction offers two key advantages when compared with the Adaptive Backstepping with tuning functions (Krstic et al, 1996). First, the design of each of the decoupled problems is very simple while the Adaptive Backstepping approach can be quite tedious and quickly leads to many terms. Second, the decoupled nature of the APBC is useful for identifying sources of model error. For example, suppose there exists an error in the i^{th} state equation of the model but the rest of the model is accurate. By the analysis in the proof all errors from e_{i+1} to e_r will converge to zero, but the errors from e_1 to e_i may not converge to zero. It will quickly be apparent where the model error exists. In the Adaptive Backstepping approach, all the errors and estimators are coupled which make this type of modeling uncertainty localization and identification impossible. Furthermore, the decoupled nature of the APBC potentially reduces the size of transients. Since the Adaptive Backstepping controller has many coupling terms, model error in one area of the system may lead to large transients in any of the error dynamics.

(4) In practice, parameter estimates are projected onto a compact set to ensure reasonable parameter estimates and the safety of physical components. The boundedness of all error signals in the proof of the Theorem could also be achieved via parameter projection with some slight modifications to the control law. Consider the case where θ_i is a scalar (see Sastry and Bodson, 1989, for the general case) and project $\hat{\theta}_i$ onto $[\theta_{LOW}, \theta_{HIGH}]$. When the parameter hits the boundary and tries to move out of the box, $\dot{\tilde{\theta}}_i = 0$ and Equation (2.7) reduces to:

$$\dot{e}_i = -k_i e_i + \tilde{\theta}_i f_i + g_i e_{i+1} \tag{2.20}$$

This is the type of error dynamics obtained when the PBC described in (Alleyne, 1999) is used and model uncertainty is present. The model uncertainty in this case is bounded in magnitude by $(\theta_{HIGH} - \theta_{LOW}) \cdot f_i$. Alleyne has considered the PBC robustness problem (Alleyne and Liu, 1999) and strengthened the control gains, k_i, to ensure that $|e_i|$ reamins bounded and converges to some tracking error bound $\phi_i > 0$ in finite time.

3. COMPARISON OF ADAPTIVE TECHNIQUES

Consider the following nonlinear plant:

$$\begin{aligned} \dot{x}_1 &= a_1 x_1^3 + x_2 \\ \dot{x}_2 &= a_2 x_2^3 + x_3 \\ \dot{x}_3 &= a_3 x_3 + u \\ y &= x_1 \end{aligned} \tag{3.1}$$

The scalars, a_i, are the unknown plant parameters which have nominal values of $a_1=a_2=a_3=1.0$. As a result, this three state system is unstable without control and feedback is required. This plant is a simple system that is in Strict Feedback form with parametric uncertainty as required by Equation (1.1). For simplicity, the plant we consider has no zero dynamics and hence it trivially satisfies the nonlinear minimum phase condition. This plant looks quite simple, but we will use it to compare the APBC control with an Adaptive Backstepping controller. Even this simple system will result in quite an explosion of terms when the Adaptive Backstepping controller is designed.

The APBC design of the previous section gives the following synthetic/actual inputs:

$$\begin{aligned} x_{(2)desired} &= -\hat{a}_1 x_1^3 - k_1 e_1 \\ x_{(3)desired} &= -\hat{a}_2 x_2^3 + \dot{x}_{(2)desired} - k_2 e_2 \\ u &= -\hat{a}_3 x_3 + \dot{x}_{(3)desired} - k_3 e_3 \end{aligned} \tag{3.2}$$

Furthermore, the following set of estimators is used:

$$\begin{aligned} \dot{\hat{a}}_1 &= \begin{cases} \gamma_1 e_1 x_1^3 & \text{if } |e_1| \geq |e_2| \\ 0 & \text{if } |e_1| < |e_2| \end{cases} \\ \dot{\hat{a}}_2 &= \begin{cases} \gamma_2 e_2 x_2^3 & \text{if } |e_2| \geq |e_3| \\ 0 & \text{if } |e_2| < |e_3| \end{cases} \\ \dot{\hat{a}}_3 &= \gamma_3 e_3 x_3 \end{aligned} \tag{3.3}$$

Since $g_1=g_2=1$, the Theorem says that $k_1=k_2=1.0$ and $k_3>0$ will guarantee convergence of all intermediate errors. For comparison with the Adaptive Backstepping controller, the following gains are used in the simulations: $k_1=k_2=k_3=2.0$. The estimator gains are chosen as $\gamma_1=\gamma_2=\gamma_3=1.0$. Since our simulation is devoid of noise, we obtain the synthetic input derivatives via finite differences.

We assume that the objective is just setpoint control, i.e. $y_{desired}$=1.0. The system given by (3.1) with controller / estimator given by (3.2-3.3) is simulated using Simulink/Matlab at a sample time of .001 seconds. The initial condition vector for the system is chosen as $[0.5, 0.8, 0.2]^T$ and the parameter estimates are started at the initial conditions: $[\hat{a}_1(0), \hat{a}_2(0), \hat{a}_3(0)]^T = [1.1, 0.9, 0.9]^T$. Figures 1 and 2 show the tracking/estimator performance of the APBC approach. The upper subplot of Figure 1 shows that the output is converging to the desired setpoint. The middle subplot shows all individual errors are also converging to zero. If we zoom in on the intermediate errors (lower subplot of Figure 1) we notice that e_3 converges first followed by e_2 and then e_1. This is the sequential convergence dictated by the Passivity-based design.

Figure 2 shows the parameter estimates are also converging to the true values. In this case, all three error systems satisfy the zero-state observability condition given in Equation (2.19). Specifically, when $y_{desired}$=1.0 then $f_i \neq 0$ for i=1,2,3 at the steady state. Also note that the estimators, $\hat{a}_1$ and $\hat{a}_2$, are each turned off at some point in the simulation as dictated by Equation (3.3). Most notably, $\hat{a}_1$ is turned off from ~1 second to ~8 seconds (Figure 2). During this time, e_1 is "ahead" of e_2 and the estimation is not needed. When $|e_2|$ drops below $|e_1|$, this estimator is turned on again to ensure that the first error system is output strictly passive.

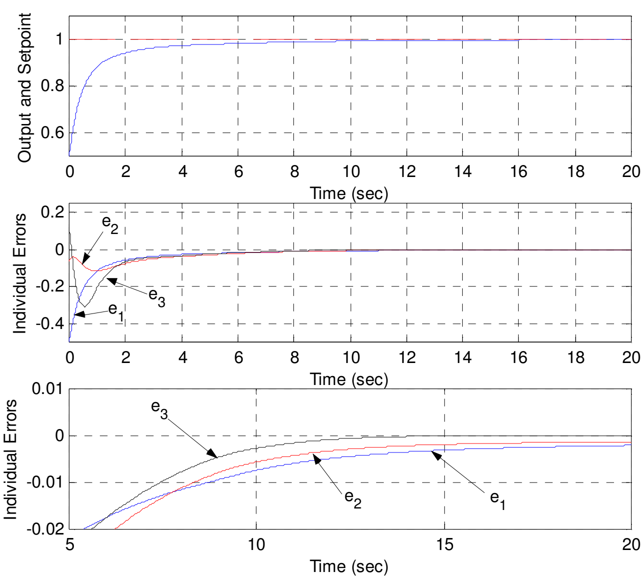

Figure 1: APBC with Perfect Model (Tracking Errors)

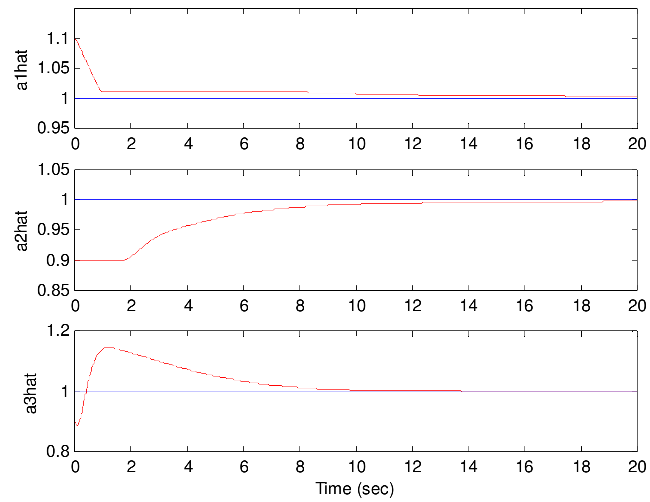

Figure 2: APBC with Perfect Model (Parameter Estimates and True Values)

For comparison we design a controller using Adaptive Backstepping with tuning functions (ABC) as developed in (Krstic and Kokotovic, 1996). For the same system, this design gives the following synthetic and actual inputs:

$$\begin{aligned} x_{(2)desired} &= -\hat{a}_1 x_1^3 - k_1 e_1 \\ x_{(3)desired} &= -\hat{a}_2 x_2^3 - k_2 e_2 - e_1 + \frac{\partial x_{(2)desired}}{\partial x_1}\left(x_2 + \hat{a}_1 x_1^3\right) \\ &\quad + \frac{\partial x_{(2)desired}}{\partial \hat{a}_1}\gamma_1 x_1^3\left(e_1 - e_2\frac{\partial x_{(2)desired}}{\partial x_1}\right) \\ u &= -\hat{a}_3 x_3 - k_3 e_3 - e_2 + \frac{\partial x_{(3)desired}}{\partial x_1}\left(x_2 + \hat{a}_1 x_1^3\right) \\ &\quad + \frac{\partial x_{(3)desired}}{\partial x_2}\left(x_3 + \hat{a}_2 x_2^3\right) + v_3 \end{aligned} \tag{3.4}$$

where v_3 is given by:

$$\begin{aligned} v_3 &= \gamma_1 \frac{\partial x_{(3)desired}}{\partial \hat{a}_1} x_1^3\left(e_1 - e_2\frac{\partial x_{(2)desired}}{\partial x_1} - e_3\frac{\partial x_{(3)desired}}{\partial x_1}\right) \\ &\quad + \gamma_2 \frac{\partial x_{(3)desired}}{\partial \hat{a}_2} x_2^3\left(e_2 - e_3\frac{\partial x_{(3)desired}}{\partial x_2}\right) \\ &\quad + \gamma_1 x_1^3 e_2 \frac{\partial x_{(2)desired}}{\partial \hat{a}_1}\frac{\partial x_{(3)desired}}{\partial x_1} \end{aligned} \tag{3.5}$$

The gains for this controller are any k_i>0 and the γ_i>0 are the estimator gains. The controller is augmented with the following estimators:

$$\dot{\hat{a}}_1 = \gamma_1 x_1^3 \left(e_1 - e_2 \frac{\partial x_{(2)desired}}{\partial x_1} - e_3 \frac{\partial x_{(3)desired}}{\partial x_1} \right)$$

$$\dot{\hat{a}}_2 = \gamma_2 x_2^3 \left(e_2 - e_3 \frac{\partial x_{(3)desired}}{\partial x_2} \right) \tag{3.6}$$

$$\dot{\hat{a}}_3 = \gamma_3 x_3 e_3$$

Equations (3.5-3.6) are constructed iteratively from Lyapunov's Direct Method. Specifically, the following Lyapunov function is used:

$$V = \frac{1}{2}\tilde{\theta}^T \Gamma^{-1} \tilde{\theta} + \sum_{i=1}^{3} \frac{1}{2} e_i^2 \tag{3.7}$$

where $\tilde{\theta}^T = [\hat{a}_1\ \hat{a}_2\ \hat{a}_3]$ and $\Gamma = \mathrm{diag}(\gamma_1, \gamma_2, \gamma_3)$. If Equation (3.7) is differentiated with respect to time along the trajectories of Equation (3.1) with the ABC given by Equations (3.4-3.6), then the Lyapunov function derivative is negative semidefinite:

$$\dot{V} = -\sum_{i=1}^{3} k_i e_i^2 \le 0 \tag{3.8}$$

Barbalat's lemma can then be applied to conclude that all individual errors converge to zero.

Notice that the ABC (3.4-3.6) looks similar to the APBC (3.2-3.3) except that it has additional coupling terms along with terms to approximate $\dot{x}_{(2)desired}$ and $\dot{x}_{(3)desired}$. We can see that the ABC uses analytically computed derivatives which results in an explosion of terms. As a result, the ABC is much more complex than the APBC, a fact which is exacerbated by the partial derivatives (e.g. $\frac{\partial x_{(3)desired}}{\partial x_1}$ has 14 terms). The coupling terms result in another drawback. Notice that $x_{(3)desired}$ and u depend on multiple intermediate errors and parameter estimates (Equation 3.4-3.5). Moreover, the first and second estimators are coupled to multiple errors (Equation 3.6). Therefore, if an error occurs in a specific synthetic input or estimator, the error will leak to other synthetic inputs or estimators. This may cause undesired transients and make it difficult to pinpoint which section of the system is causing the error. This idea will be explored further in the comparisons below.

For this comparison, the ABC gains, k_i, are all set equal to 6.0 and the estimator gains are chosen as $\gamma_1=\gamma_2=\gamma_3=0.1$. Figures 3 and 4 show the performance of the ABC with the same setup as above. Specifically, we can see that all intermediate and output errors converge to zero (Figure 3). Figure 4 shows the parameter estimates along with the nominal values. It appears that $\hat{a}_3$ is not converging to the true value. The Adaptive Backstepping design only guarantees the boundedness of parameter errors, so the nonconvergence of $\hat{a}_3$ is not uncommon. The fact that all parameters converged for the APBC is not a general result, it just happens that in this example the zero-state observability conditions were satisfied for this example. In summary, the APBC and ABC controllers behave similarly for this case of well-known system dynamics. Any differences in performance on the known plant can probably be eliminated with further tuning of the appropriate controller.

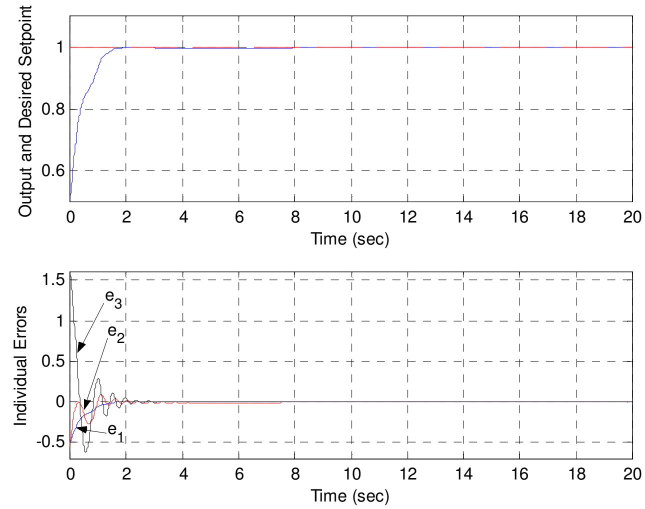

Figure 3: ABC with Perfect Model (Tracking Errors)

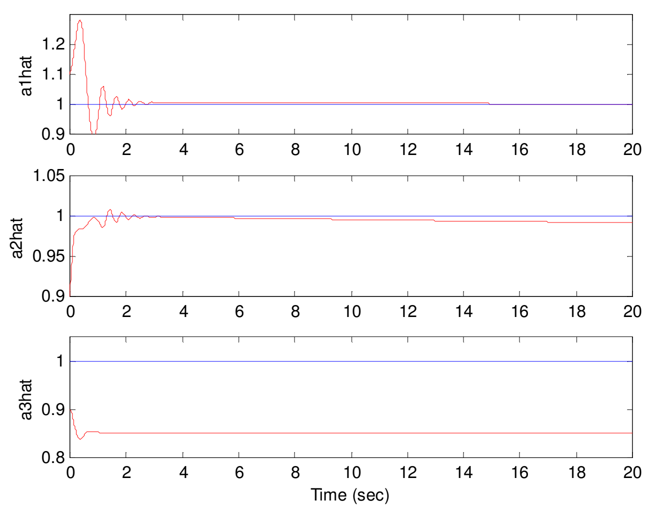

Figure 4: ABC with Perfect Model (Parameter Estimates and True Values)

4. EFFECT OF NON-PARAMETRIC UNCERTAINTY

As shown in Section 3, both the APBC and ABC can handle the case of parametric model uncertainty. In reality, a control designer does not have access to the "true" plant dynamics and must deal with model uncertainty. The adaptive control laws are well-suited to handle parametric uncertainty, so the actual concern lies in the case of unmodeled dynamics. A controls engineer may wonder if a specific set of dynamical equations models the plant accurately or if neglected dynamics truly were irrelevant. The real benefit of the APBC approach arises in this case of non-parametric model uncertainty. To simulate the effect of non-parametric model error, the previous plant (3.1) is perturbed by adding a slowly varying, sinusoidal disturbance to the first state equation:

$$\begin{aligned}
\dot{x}_1 &= a_1 x_1^3 + x_2 + \overbrace{.25 * \sin\left(\frac{2\pi t}{20}\right)}^{\text{non-parametric uncertainty}} \\
\dot{x}_2 &= a_2 x_2^3 + x_3 \\
\dot{x}_3 &= a_3 x_3 + u \\
y &= x_1
\end{aligned} \tag{3.9}$$

The time-varying model error will display the error localization property which is the key benefit of APBC design. It is assumed that neither controller has knowledge of this additional term. Figure 5 shows the individual errors for both the APBC (upper subplot) and the ABC (lower subplot). Figure 6 shows the parameter estimates for both controller designs. For the APBC, Figure 5 shows the trajectories for e_2 and e_3 converge to zero, while the trajectory for e_1 retains the artifacts of the sinusoidal disturbance. Furthermore, Figure 6 shows that $\hat{a}_2$ and $\hat{a}_3$ converge to their true values, but $\hat{a}_1$ oscillates to compensate for the disturbance. In summary the disturbance has been localized in the first estimator and error!

This simulation shows the decoupling property of the APBC design method: only states associated with the uncertain part of the model along with any states further up the passivity chain show an error. This allows the controls engineer to focus their attention on improving a specific portion of the system model and then improve the controller performance.

The lower subplot of Figure 5 shows the individual errors for the ABC design. The output error, e_1, is smaller than the APBC design, but the magnitude is simply a function of the Backstepping gains. The important property of this plot is that all three intermediate errors oscillate similarly about 0. Furthermore, Figure 6 shows that $\hat{a}_1$ and $\hat{a}_2$ for the ABC design oscillate at the disturbance frequency. The third estimator, $\hat{a}_3$ has a response similar to the perfect model case (Figure 4). This result is explained by Equation (3.6) which shows that the first two estimators have coupling terms while the third estimator only depends on e_3 and x_3. The key result of these two figures is that the disturbance is not localized. In fact it has leaked to all the errors and the first two estimators making it impossible to determine the source of error in the system.

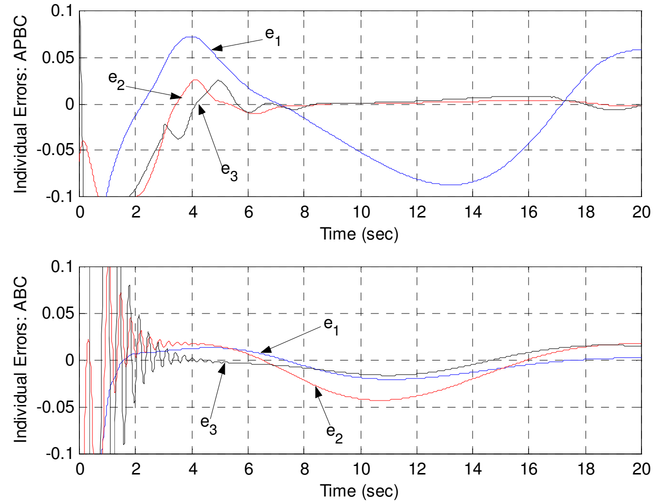

Figure 5: APBC and ABC Individual Errors

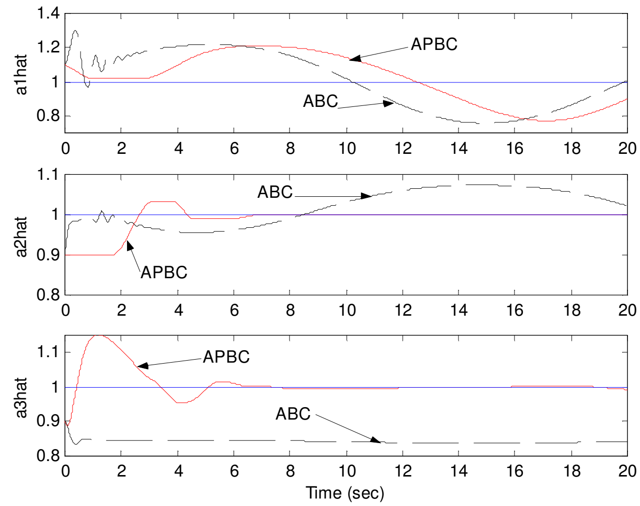

Figure 6: APBC and ABC Parameter Errors

Finally, we examine the effect of the Adaptive Backstepping coupling terms on the system transients. The plant given by Equation (3.1) is perturbed by adding a constant to the first state equation:

$$\begin{aligned}
\dot{x}_1 &= a_1 x_1^3 + x_2 + \overbrace{.25}^{\text{non-parametric uncertainty}} \\
\dot{x}_2 &= a_2 x_2^3 + x_3 \\
\dot{x}_3 &= a_3 x_3 + u \\
y &= x_1
\end{aligned} \tag{3.10}$$

Again, this uncertainty is unknown to both the APBC and ABC. It turns out that all intermediate errors converge to zero for both the APBC and ABC designs. The estimators for both controllers are able to compensate for this constant disturbance and therefore the outputs still converge to the desired setpoint. It is the transient behavior of the systems which we would like to investigate for this case. Figure 7 shows the synthetic and actual inputs for the system with no model error (3.1) and the perturbed system (3.9) when the APBC design is used. These plots show that the synthetic/actual inputs have comparable transients for the perfect and perturbed models (although the steady state values are different to compensate for the model error). For comparison, the ABC was also simulated on the perturbed plant. Figure 8 shows the synthetic/actual inputs for the ABC on the model with no error and the perturbed model. The lowest subplot shows that the control effort, u, has very large initial transients on the perturbed plant. Recall from Equations (3.4-3.5) that this input had the most coupling terms and we conjecture that it is these coupling terms which is resulting in the large transients.

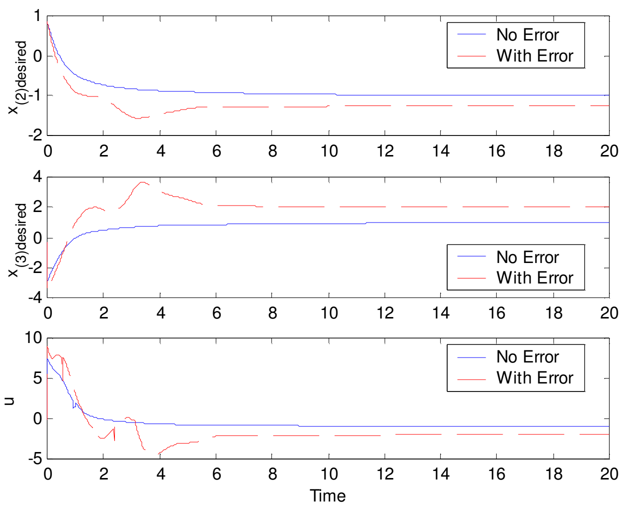

Figure 7: APBC with/without Model Error (Comparison of Synthetic/Actual Inputs)

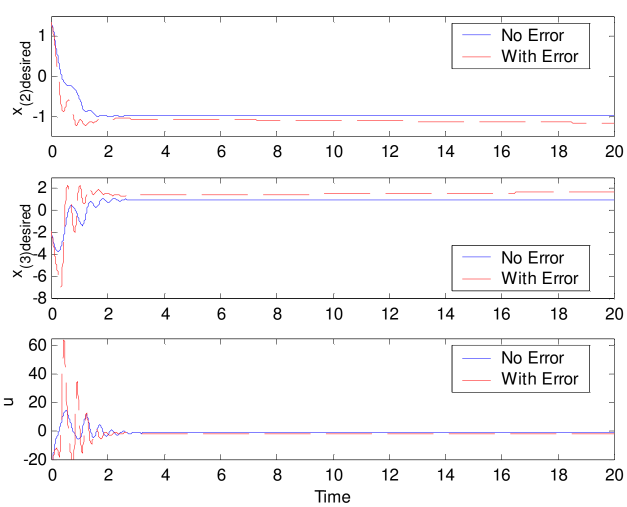

Figure 8: ABC with/without Model Error (Comparison of Synthetic/Actual Inputs)

CONCLUSIONS

In this paper we developed an Adaptive Passivity Based Controller for nonlinear systems in Strict Feedback Form with parametric uncertainty. We then compared this design method against Adaptive Backstepping with tuning functions on a simple 3-state nonlinear model. While the APBC and ABC performance are comparable when the model is perfectly known (other than the parametric error), the APBC offers an easier design procedure. The APBC controller is far less complex than the Adaptive Backstepping controller, a fact which may lead to shorter implementation/debugging time and reduced real-time processing constraints. This simpler controller may lead to reduced transients, particularly in the case where model error is present. The real benefit of the APBC approach lies when the model contains non-parametric uncertainty. This design method has a decoupling property which can be used as a diagnostic tool to determine where uncertainty lies in the system.

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