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Model Reduction of Flexible Aircraft for Flutter Suppression using Smart Sensors

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ABSTRACT

This paper proposes a model reduction method for linear parameter-varying (LPV) systems based on traditional modal truncation and residualization techniques. The method is applied to a body freedom flutter vehicle with coupled short period and first bending mode and also with structural bending and torsion coupling. These models describe the dynamics with considerable accuracy, but result in high-order state space models which make the controller design extremely difficult. Hence, reduced order models for control synthesis are generated by retaining a common set of states across the flight envelope. As results, full order models of 148 states are reduced to 43 states. The reduced order models capture the dynamics of interest and can be used in the synthesis of active flutter suppression controllers.

INTRODUCTION

Modern aircraft designers are adopting light-weight, high aspect ratio wings to take advantage of wing flexibility for increased maneuverability. Those modifications can lead to improve performance and reduce operating costs. However, the high flexibility and significant deformation in flight exhibited by these aircraft increase the interaction between the rigid body and structural dynamics modes resulting in *Body Freedom Flutter*. This phenomenon occurs as the aircraft short period mode frequency increases with airspeed and comes close to a wing vibration mode, typically wing bending mode. This leads to poor handling qualities and may even generate dynamics instability. Hence, an integrated active approach to flight control, flutter suppression and structural mode attenuation will be required to meet the desired handling quality performance for modern flexible aircraft.

Several flutter suppression control strategies have been proposed to address the coupled rigid body and aeroelastic dynamics including optimal control [9][15], dynamic inversion control [17], robust multivariable control [22], predictive control [8] and gain scheduled control [1][3][6]. Nevertheless, almost all of these control strategies are model based and require accurate aerodynamic and structural dynamic models of the aircraft. Numerous investigations have addressed the aeroelastic modeling for highly flexible aircraft [9][16][17]. Modeling of a flexible aircraft requires a geometric structural model coupled with a consistent aerodynamic model. Linear aeroelastic models are based on structural finite elements and lifting-surface theory, both of which are available in general purpose commercial codes [18]. Unsteady aerodynamics are often modeled using the doublet lattice method, which results in a matrix of linear aerodynamic influence coefficients that relate the pressure change of an aerodynamic degree-of-freedom due to the change in pressure of the another aerodynamic degree-of-freedom. Then, the fully coupled, nonlinear aircraft model is a combination of the mass and stiffness matrices derived from the aeroelastic model and the unsteady aerodynamics.

Recently, researchers [12][13][14] have proposed using smart hot-film anemometry to directly measure aerodynamic coefficients at multiple locations along the wings. The conventional state-space system is augmented with an additional output associated to these measurements. Accurate aerodynamic coefficients are obtained measuring surface parameters on the boundary layer to characterize the circulation and, consequently, the aerodynamic forces. A real-time sensor-agnostic algorithm is used for estimating the aerodynamic coefficients with minimal calibration requirements and the measurement accuracy validation through wind tunnel test.

Unfortunately, the inclusion of structural dynamic and aeroelastic effects from the smart sensor models will still result in linear, dynamic models with a large number of degrees-of-freedom defined across the flight envelope. It is unrealistic to use these high order and complex models for control design since modern control methods will result in controllers with very high state order. Even more, practical implementation of high order controllers is usually avoided since numerical errors may increase and the resulting system may present undesired behavior. Hence, a reduced-order linear model of the flexible aircraft will allow linear parameter-varying controllers to be designed.

Several model reduction techniques for linear, parameter-varying (LPV) have been reported in the literature. Balanced truncation [10], LMIs [19], bounded parameter variation rates [23], coprime factorizations and singular perturbation [7] [24] are presented as an extension of the model reduction techniques for linear time invariant (LTI) systems. However, those transformations are time-dependent and include then additional derivative terms to the models. This paper describes the development of a low order, control-oriented aircraft model whose states are consistent across the flight envelope which is useful for intuition and also ensure easily schedule of the controllers. Traditional modal residualization techniques are extended to linear, parameter-varying (LPV) aeroservoelastic (ASE) models. These techniques are applied to an experimental body freedom flutter test vehicle model developed by the U.S. Air Force and described below.

BODY FREEDOM FLUTTER MODEL

The Air Force Research Laboratory (AFRL) have developed a flight test vehicle, denoted the Body Freedom Flutter (BFF) vehicle, to demonstrate active aeroelastic control technologies. The vehicle is a high aspect ratio flying wing with light weight airfoil. Details of the vehicle's design can be found in [4]. The aircraft configuration with the location of accelerometers and control surfaces for flutter suppression is presented in the Fig. 1. The aeroservoelastic (ASE) model of the BFF vehicle was assembled using MSC/NASTRAN. The initial structural model was created with 2556 degrees-of-freedom and then was reduced to 376 degrees-of-freedom via a Guyan reduction. A Ground Vibration Test was performed to validate the structural model and six critical modes were found. Table 1 lists the mode shapes and frequency values of the structural model [5].

Table 1. Ground Vibration Test Frequencies

Mode shape:	**Frequency (rad/s)**
Symmetric Wing 1st Bending	35.37
Anti-symmetric Wing 1st Bending	54.98
Symmetric Wing 1st Torsion	123.34
Anti-symmetric Wing 1st Torsion	132.76
Symmetric Wing 2nd Bending	147.28
Anti-symmetric Wing 2nd Bending	185.73

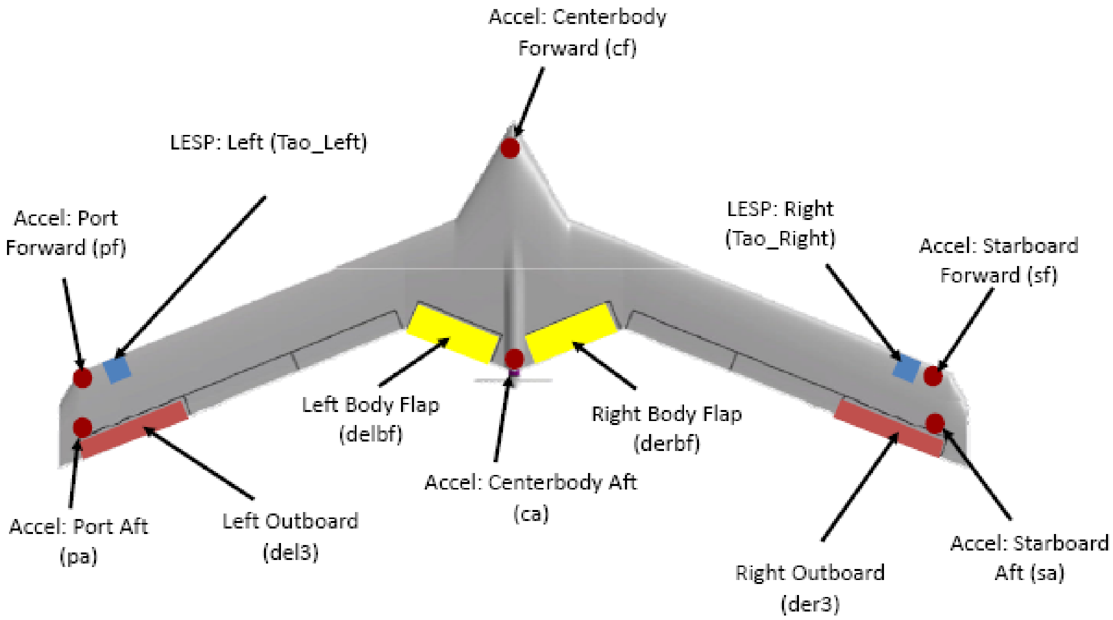

Fig. 1. Body Freedom Flutter Vehicle [5]

The aerodynamics were modeled using the doublet lattice method which is a technique used to model oscillating lifting surfaces. This model produces a matrix of linear aerodynamic influence coefficients that describes the pressure change of the 2252 aerodynamic degrees-of-freedom. The mass, stiffness and aerodynamics coefficients matrices are combined using the P-K method which interconnects the structural and aerodynamic grids by splining interpolation and finds generalized aerodynamic matrix using the structural modal matrix [18]. The unsteady aerodynamics is approximated with a rational function to create a continuous-time aeroservoelastic state-space model of the airframe with 148 states. The general state-space form of the model is given by

$$\begin{Bmatrix}\dot{x}_p\\ \dot{x}_q\\ \dot{x}_{\omega 1}\\ \dot{x}_{\omega 2}\end{Bmatrix} = \begin{bmatrix}0 & I & 0 & 0\\ A_{21} & A_{22} & A_{23} & A_{24}\\ 0 & I & \omega_1 I & 0\\ 0 & I & 0 & \omega_2 I\end{bmatrix}\begin{Bmatrix}x_p\\ x_q\\ x_{\omega 1}\\ x_{\omega 2}\end{Bmatrix} + \begin{bmatrix}B_1\\ B_2\\ 0\\ 0\end{bmatrix}u \tag{1}$$

Eq. (1) represents a typical second order equation of motion with augmented state vector due to the rational functions approximation. The state vector consist of modal displacements, $x_{p,}$ modal velocities, $x_{q,}$ and two lags states, $x_{\omega 1}$, $x_{\omega 2,}$ for the unsteady aerodynamic rational function approximation. Moreover, each of the set of states is related with 5 rigid body modes (lateral, plunge, roll, pitch and yaw), 8 flexible modes (symmetric – anti-symmetric bending and torsion) and 24 secondary discrete degrees-of-freedom.

Finally, a set of state space matrices was generated in 2 knot increments from 40 to 100 KEAS (knots equivalent airspeed) with variable Mach at constant altitude of 3000 ft. [5]. The frequency and damping of the critical modes for the BFF vehicle are plotted at Fig. 2 as a function of airspeed.

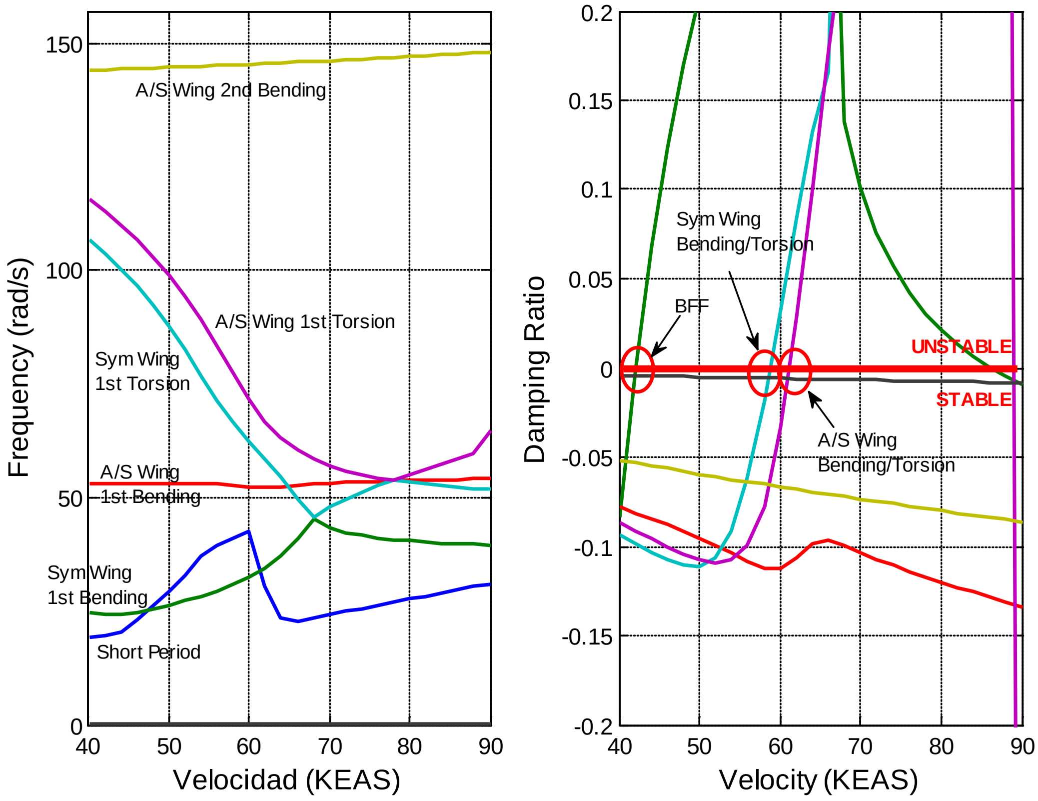

Fig. 2. Velocity/frequency/damping plot for BFF vehicle

The coupling of the short period with the symmetric wing bending produces BFF at 43 KEAS with a frequency of 24.3 rad/s. Flutter is also presented when the symmetric wing bending and torsion modes are coupled at an airspeed of 58 KEAS with frequency of 65 rad/s and when the anti-symmetric wing bending and torsion modes comes close in proximity at 61 KEAS and frequency of 69 rad/s. Clearly, the flight envelope of the open-loop vehicle is limited until 42 KEAS before it becomes unstable. Hence, an active control using control surfaces and structural feedback to stabilize and meet the desired handling qualities of the BFF vehicle is proposed.

MODEL REDUCTION

The linear, coupled state-space models of the vehicle generated across the flight envelope are function of the dynamic pressure and Mach. These state space models can be written as

$$\begin{bmatrix} \dot{x}(t) \\ y(t) \end{bmatrix} = \begin{bmatrix} A(\rho) & B(\rho) \\ C(\rho) & D(\rho) \end{bmatrix} \begin{bmatrix} x(t) \\ u(t) \end{bmatrix} \tag{2}$$

Where $A(\rho)$ is the state matrix, $B(\rho)$ is the input matrix, $C(\rho)$ is the output state matrix, $D(\rho)$ is the input feedthrought matrix and ρ is a vector that is function of time and corresponds to dynamic pressure and Mach in this example. The model is a *linear parameter-varying* (LPV) system. The current values of ρ are measurable by sensors in real-time but the future values are unknown [1].

Typically, LPV ASE models result in high-order state-space models. High-order controllers can be obtained using complex models but, the implementation of these high-order state controllers is usually avoided because numerical errors may generate undesirable behavior in the system. Hence, reduced-order LPV models are required for controller synthesis. Literature reports [7][10][19][24] that traditional reduction techniques for linear time invariant systems have been extended to LPV systems. However, the state transformation used depends on ρ, and it would in general be time-varying. This transformation $T(\rho)$ would introduce additional terms, that depend on the derivative of $T(\rho)$ with respect to time, in the state space model making the reduction problem harder. Hence, a model reduction technique that preserves the same state meaning across the flight is presented below. This technique will be useful for physical intuition and will ensure that control design methods can be easily scheduled.

Truncation

The focus of the control design is to actively control flutter and vehicle/wing vibration. Fig. 2 shows that flutter phenomena occur in a frequency bandwidth between 10-120 rad/s hence since the controller is only focused on these modes, the extremely slow dynamics can be eliminated from the model. Partitioning the state vector x, into $[x_1, x_2]'$, where x_2 contains the states to remove, the state-space equations become:

$$\begin{aligned} \dot{x}_1 &= A_{11}x_1 + A_{12}x_2 + B_1 u \\ \dot{x}_2 &= A_{21}x_1 + A_{22}x_2 + B_2 u \\ y &= C_1 x_1 + C_2 x_2 + Du \end{aligned} \tag{3}$$

For the BFF model the plunge mode is the only state that presents a slow dynamics. Then, the plunge mode is truncated and the reduced models match closely to the full order models at infinite frequency, retained the system behavior above the truncated frequency. Denoting the new state vector as x_r, the reduced model with 147 states, is given by

$$\begin{aligned}\dot{x}_r &= A_{11}x_r + +B_1 u \\ y &= C_1 x_r + Du\end{aligned} \tag{4}$$

Residualization

Residualization takes into account the interaction between slow and fast dynamics without including the dynamic effects of the fast dynamics. This is accomplished by letting the degrees of freedom to be removed from the model reach their steady state values instantaneously by setting their derivatives zero. For a general system with the space state structure in Eq. (3), the solution for the reduce model is given by

$$\begin{aligned}\dot{x}_1 &= \left(A_{11} - A_{12}A_{22}^{-1}A_{21}\right)x_1 + \left(B_1 - A_{12}A_{22}^{-1}\right)u \\ y &= \left(C_1 - C_2A_{22}^{-1}A_{21}\right)x_1 + \left(D - C_2A_{22}^{-1}B_2\right)u\end{aligned} \tag{5}$$

The state vector x_r is partitioned into $[x_1, x_2]'$, where x_2 corresponds to the lateral and yaw rigid body modes, the symmetric wing fore and aft, anti-symmetric wing 2nd bending and rotation modes and 18 discrete degrees-of-freedom with their respective derivatives and aerodynamic lags. These states are then residualized from the truncated models given in Eq. (4). As result, a total of 92 states are removed. Reduced models with 55 states are obtained across the entire flight envelope.

Modal Residualization

Lastly, a modal residualization is performed in order to eliminate high frequencies outside the bandwidth of interest that were retained after the truncation and residualization. A single modal transformation is applied to all the reduced linear models to preserve the states across the flight envelope. The state coordinate transformation, T, is computed using the residualized model at 90 KEAS, such that:

$$\begin{aligned}x_c &= Tx \\ \dot{x}_c &= TAT^{-1}x_c + TBu \\ y &= CT^{-1}x_c + Du\end{aligned} \tag{6}$$

A total of 12 high frequency modes are identified and residualized as in Eq. (5). Reduced models with the same 43 states are obtained across the flight envelope. The state transformation at 90 KEAS obtained the best accurate reduced models in the bandwidth of interest.

Fig. 3, Fig. 4 and Fig. 5 show the frequency response of the BFF vehicle from the body and outboard flaps to the right wing and aft accelerometers at three different airspeeds. Blue lines refer to the full order model, red lines correspond to reduced models with common set of states, and additionally, green lines represent a frequency weighted balanced truncation with a time-varying state transformation. It is observed that the frequency weighted balanced truncation can achieve models with less order than the proposed technique. However, the meaning of the states is lost at each flight condition.

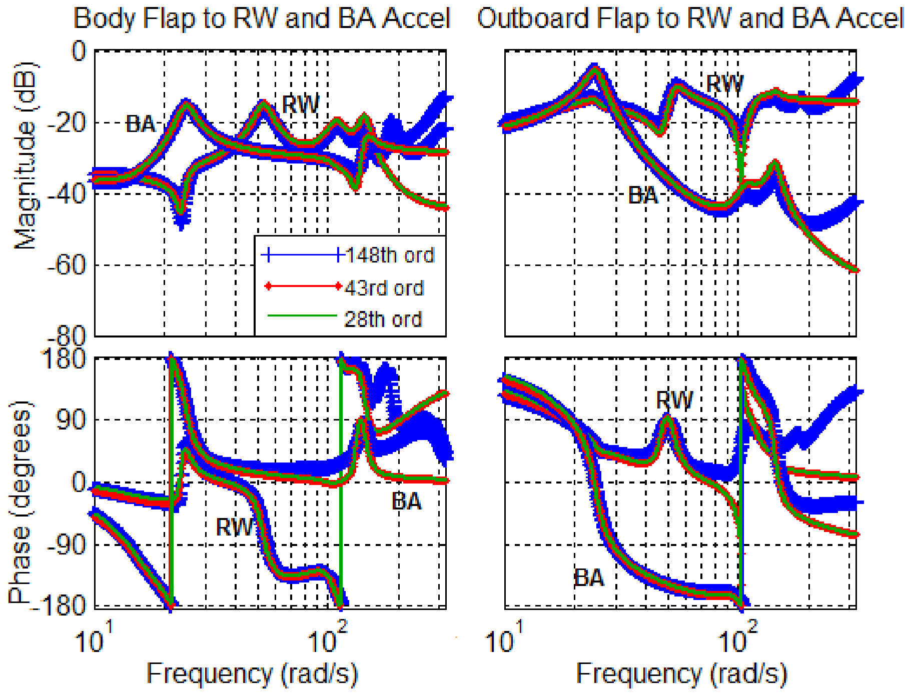

Fig. 3. Full and reduced order models from body and outboard flap to right wing (RW) and body aft (BA) accelerometers at 40 KEAS

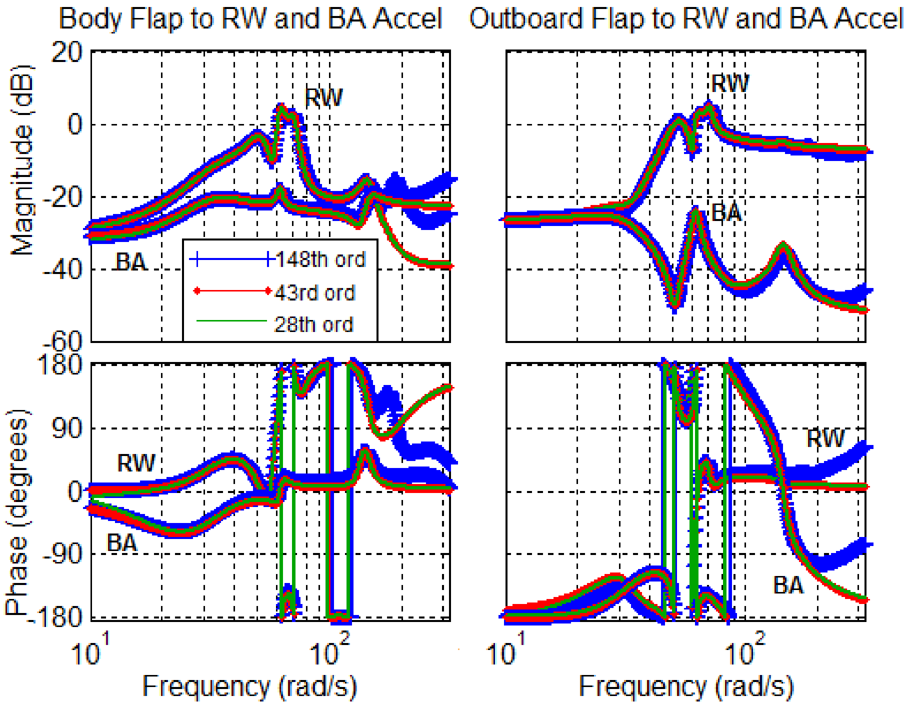

Fig. 4. Full and reduced order models from body and outboard flap to right wing (RW) and body aft (BA) accelerometers at 60 KEAS

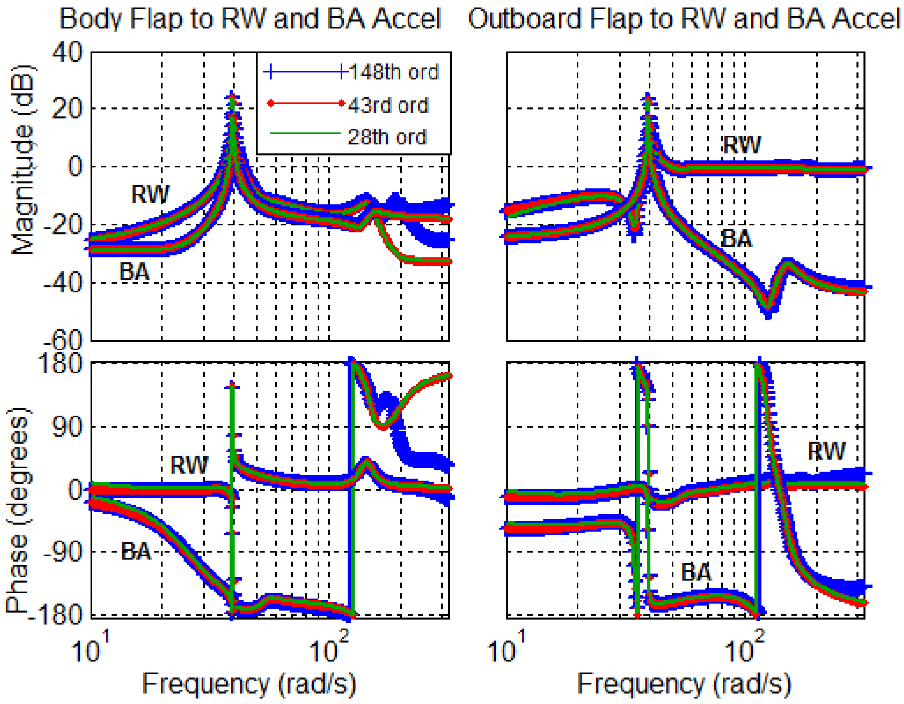

Fig. 5. Full and reduced order models from body and outboard flap to right wing (RW) and body aft (BA) accelerometers at 88 KEAS

The pole locations of the reduced and full order models are plotted in Fig. 6. It is observed that all the poles, stable and unstable, are present in the bandwidth of interest of the reduced model. Additionally, Fig. 7 and Fig. 8 show the stable time response from the body and outboard flap to the right wing (RW) accelerometer for both, full and reduced state order model at 40 KEAS.

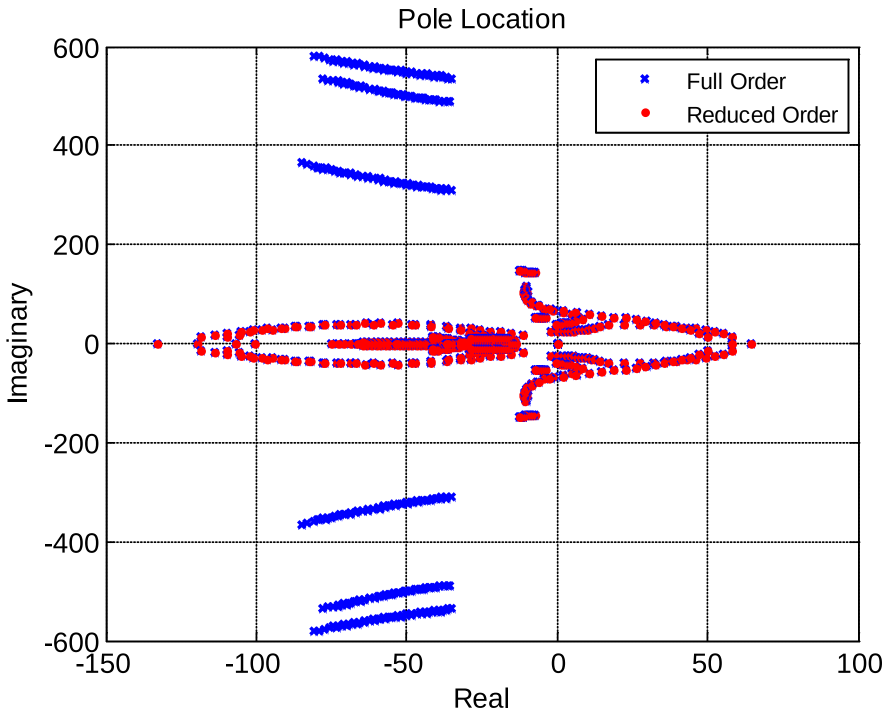

Fig. 6. Full and reduced order models pole location across the entire flight envelope

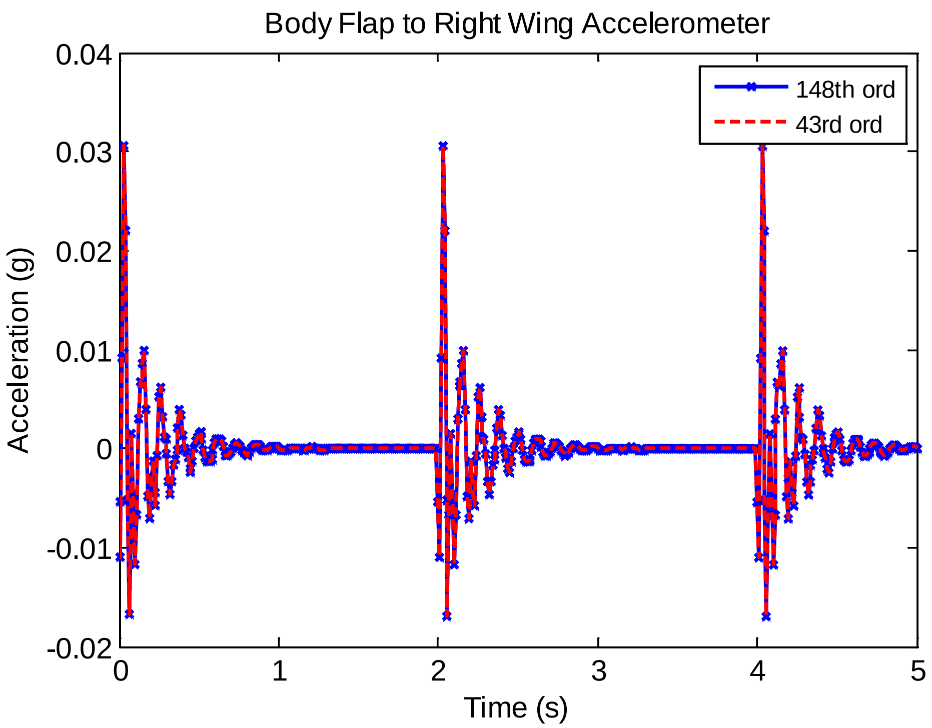

Fig. 7. Full and reduced order models from body and outboard flap to right wing (RW) and body aft (BA) accelerometers at 60 KEAS

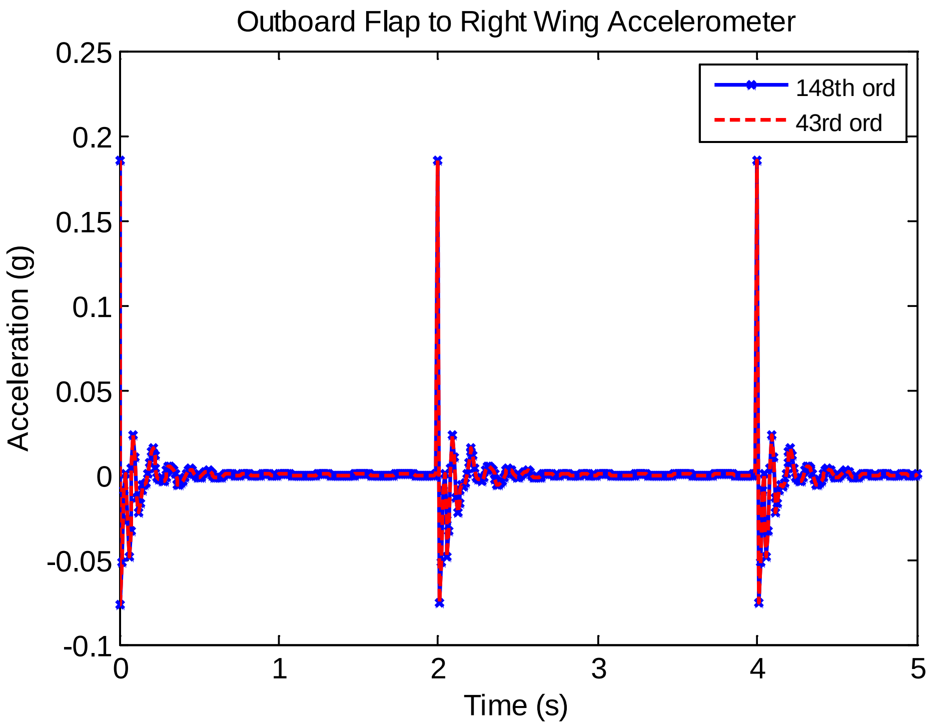

Fig. 8. Full and reduced order models from body and outboard flap to right wing (RW) and body aft (BA) accelerometers at 88 KEAS

CONCLUSIONS

A model reduction procedure for aeroservoelastic models based on modal truncation and residualization has been proposed. The proposed procedure is applied to a body freedom flutter (BFF) vehicle and consists of a truncation of uncoupled slow dynamics and a residualization of coupled modes outside the flutter frequencies of interest. Then, a modal transformation is applied to all the models in order to eliminate remaining high frequency modes. Finally, the full order models with 148 states were reduced to 43 states while retaining the main flutter dynamics and the same states characteristics across the flight envelope. Consequently, the proposed method can be used for design of linear parameter-varying controllers.

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