Fundamental Limitations of Preview for Wind Turbine Control

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Wind turbine preview control has recently attracted attention in the literature. This is due to significant load attenuation and power optimization benefits that can be obtained by incorporating wind preview measurements into the turbine control laws. However, the trade-offs between performance, preview time, and actuation rate limits are not well understood. This paper investigates the fundamental limits of performance for the Region 3 rotor speed regulation problem. An optimal control problem is formulated in discrete-time using linearized turbine models subject to pitch actuator rate constraints. Linear programming is used to numerically compute the optimal control input for a given wind profile. The solution of the linear program provides a bound on the performance achievable by any controller. These results are validated on H_∞ controllers designed to incorporate various amounts of preview wind information. H_∞ controllers are simulated on a higher-fidelity, nonlinear turbine model. The linear programming analysis accurately predicts the performance versus preview time characteristics of these H_∞ controllers.

I. Introduction

The power produced by a single wind turbine is proportional to the square of the blade length. This has driven the wind power industry to turbines of enormous size. Unfortunately the tower and blade flexibility becomes significant at these larger dimensions resulting in higher structural loads. At high wind speeds, the structural loads are controlled by appropriately pitching the blades in response to wind gusts. The blade pitch actuators have restrictive rate limits due to the large blade inertia. As a result, it is not possible to respond to fast changing wind gusts. Another issue is accurate measurements of the wind speed at the turbine location are typically not available. Anemometers are located on the back of each turbine nacelle but the rotating blades corrupt these wind speed measurements. However, advanced sensors, e.g. LIDAR,[1] are able to measure the incoming wind field. These measurements can be used to provide preview information to the closed-loop controller.

The initial work by Laks, et al.[3] demonstrated that significant improvements in load attenuation and speed regulation can be achieved by utilizing preview wind measurements. Various methodologies have been studied for wind preview control including disturbance accommodating control,[2] H_∞ control,[2,3] and model predictive control (MPC).[4,5] Numerical studies are typically used to investigate the impact of different sensor models and additional preview time. For example, Laks et al.[3] presented worst case gains of full-information H_∞ controllers versus preview time and blade pitch rate limits. In addition, Monte Carlo simulations were performed on high fidelity, nonlinear turbine models to understand the impact of preview time. These results provide useful insight about the effects of additional preview information. However, the results depend on specific design choices, e.g. the control methodology and design weights. Additional insight can be gained by studying the optimal blade pitch response for specific wind disturbance rejection problems.

The objective of this paper is to understand the fundamental trade-offs between performance, wind preview time, and blade pitch rate limits. Region 3 rotor speed regulation is formulated as a discrete-time optimal control problem. The objective is to use preview information to reject a step wind gust subject to

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blade pitch rate constraints. Linear programming (LP) is used to numerically compute the optimal collective blade pitch response which minimizes the infinity-norm of the rotor speed error. The solution of the linear program provides a bound on the performance achievable by any controller.

The approach described in this paper has close ties to the work by Boyd and Barratt[6] and Dahleh and Diaz-Bobillo.[7] Boyd and Barratt use the Youla parametrization to formulate wide classes of linear control design problems with time and frequency domain constraints as convex optimizations. Solution of the convex optimizations can be used to determine if there exists a controller that satisfies the given design constraints. The work contained in this paper is influenced by these studies of fundamental performance limits. The work by Dahleh and Diaz-Bobillo considers the L_1 optimal control problem: Find a control K that minimizes the closed loop induced L_∞ (peak) norm from disturbances to errors. Linear programming techniques are used to solve for the optimal controller. This paper also uses on L_∞ (peak) signal norms and LP techniques. The L_∞ norm of the error signal is minimized for a given disturbance signal rather than for a class of L_∞ norm bounded disturbance signals. This enables analysis of the optimal control action based on a particular wind trajectory. In addition, the focus of this paper is on understanding the limits of performance and no effort has been made to synthesize feedback controllers. Implementation of the proposed control actions synthesized by LP techniques can be realized in MPC style but this is not investigated in this paper.

We envision that the proposed approach can be employed in various ways. First, given a wind profile it is possible to analyze the optimality of various control designs. Second, the impact of the preview wind information versus controller performance can be studied in time-domain. Lastly, linear programming results can be used to develop insight about controller behavior.

The remainder of the paper is structured as follows: Section II formulates an optimal rotor speed tracking problem with wind preview information as an LP. Section III applies this framework to understand the fundamental trade-offs of wind preview control on a low-order, rotor inertia model of National Renewable Energy Laboratory's (NREL) CART3 turbine.[8] Section IV uses the proposed framework to understand the performance trade-offs for H_∞ controllers designed on a more realistic, higher-order model of the CART3. The performance of the H_∞ controllers versus preview time is evaluated using NREL's nonlinear FAST turbine simulation package. These results are compared with the performance trade-offs computed using the LP framework. Conclusions are presented in Section V.

II. Problem Formulation

An optimal rotor speed tracking problem with preview wind measurements is formulated and solved as a linear program in Section A. Generalizations of this formulation are discussed in Section B.

A. Rotor Speed Tracking

This section considers a rotor speed tracking problem in above-rated wind conditions (Region 3). The optimal control formulation aims to minimize the peak variations in rotor speed. Rotor speed tracking and its peak error is important for three reasons. First, it ensures that the generated power in Region 3 is maintained at its rated value. Second, minimizing the peak rotor speed error is crucial for avoiding generator over-speed problem. Third, variations in rotor speed are correlated to the various structural loads on the turbine. Reducing variations in rotor speed typically leads to reduced blade, tower and gearbox loads. Eliminating sharp peaks in the structural loads reduce possibility of premature fatigue due to high stress cycles. This is an important design factor for large, commercial wind turbines that contain highly flexible structures.

It is assumed the turbine dynamics can be linearized around a constant wind speed to obtain a discrete-time linear time-invariant system of the form:

$$\begin{aligned} x[k+1] &= Ax[k] + Bu[k] + B_d u_d[k] \\ e[k] &= Cx[k] \end{aligned} \tag{1}$$

where $x[k+1] \in \mathbb{R}^n$ is the turbine state, $u[k] \in \mathbb{R}$ is the collective blade pitch, $u_d[k] \in \mathbb{R}$ is the uniform hub-height horizontal wind speed. Each of these quantities is measured relative to their trim value, e.g. $u[k]$ is the difference between the collective blade pitch and its trim value. $e[k] \in \mathbb{R}$ is the rotor speed tracking error, i.e. the difference between the current rotor speed and the rated rotor speed. These linear models are typically obtained from higher fidelity, nonlinear turbine models that involve tower and blade structural modes, models of aeroelastic behavior and turbulent wind profiles. Linearization is used to obtain lower fidelity, control-oriented design models. In general, the turbine dynamics depend on rotor position and have a non-steady trim trajectory even under steady wind conditions. These trim trajectories are periodic with period equal to one rotor period. A linear time-varying (LTV) model is obtained by linearizing the high-fidelity nonlinear model on a grid of rotor positions. Two common approaches exist in literature for converting LTV models to LTI models. These methods are simple averaging of the LTV matrices over one rotor period and the use of multi-blade coordinate transformation[9,10] (MBC) followed by averaging of the resulting matrices. Benefits and drawbacks of these methods are discussed in various resources in the literature.[10–12] It is assumed that one these methods has been used to obtain the discrete-time LTI turbine model in Equation (1).

If the wind disturbances and collective blade pitch remain at their trim values ($u[k] = 0$ and $u_d[k] = 0$), the turbine will reach the rated rotor speed in steady state ($e[k] \to 0$ as $t \to \infty$) [a]. Wind gusts ($u_d[k] \neq 0$) will impact the turbine and perturb the rotor speed from its rated value. Actuator rate limits will prevent the blades from instantaneously moving to reject this disturbance. Intuitively preview information of the wind disturbance can be used to (partially) overcome the actuator rate limitations and reduce the effect of the wind on the rotor speed error. The objective of this section is to formulate a simple optimal control problem that provides insight into the benefits and limitations of wind preview information.

Let $N > 0$ denote the number of steps of available wind preview information. The essence of the performance vs. preview trade-off is captured by the following control problem:

$$
\begin{aligned}
p^* := \min_{u[1],\ldots,u[T-1]} \quad & ||e||_\infty \\
\text{subject to:} \quad & \text{Equation (1) with } x[0] = 0, u[0] = 0 \\
& u_d[k] = \begin{cases} 0 & \text{if } k < N \\ v & \text{if } k \geq N \end{cases} \\
& |u[k] - u[k-1]| \leq r \text{ for } k = 1, \ldots, T-1
\end{aligned} \tag{2}
$$

where $||e||_\infty := \max_{0 \leq k \leq T} e[k]$ is the peak rotor speed error over the window $0 \leq k \leq T$ where $T > N$. This optimal control problem assumes that the turbine state, collective blade pitch and wind are all initially at their trim values. A step wind gust of magnitude v occurs at time $k = N$. The actuator rate constraints are modeled by a bound of r (degs) on the change in the collective blade pitch between discrete sample times. The objective is to find the optimal collective blade pitch $u[1], \ldots, u[T-1]$ that minimizes the peak rotor speed error. N denotes the preview time in the sense that the collective blade pitch can begin responding at time $k = 1$ to a gust at time N. This problem formulation solves for the optimal control input over the entire horizon $1 \leq k \leq T-1$, i.e. it assumes knowledge of the entire wind profile. Technically a controller with N steps of preview would only have access to $u_d[j]$ for $1 \leq j \leq N$ at time $k = 1$. A more precise formulation would require the controller to model the wind profile beyond the N-step prediction window. For step wind disturbances, this more precise formulation is equivalent to Equation (2) when the controller uses a persistence model for the wind profile beyond the prediction window.

The error signal defined in Equation (1) with initial condition $x[0] = 0$ has a closed form solution of:

$$
e[k] = C \sum_{n=0}^{k-1} A^{k-n-1} B u[n] + C \sum_{n=0}^{k-1} A^{k-n-1} B_d u_d[n] \quad \text{for } k \geq 1 \tag{3}
$$

[a] The turbine dynamics are assumed to be stable, i.e. all eigenvalues of the state matrix A have magnitude < 1.

This response can be written in matrix form as:

$$\begin{bmatrix} e[2] \\ e[3] \\ e[4] \\ \vdots \\ e[T] \end{bmatrix} = \begin{bmatrix} CB & 0 & 0 & \dots & 0 \\ CAB & CB & 0 & \dots & 0 \\ CA^2B & CAB & CB & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & 0 \\ CA^{k-1}B & CA^{k-2}B & CA^{k-3}B & \dots & CB \end{bmatrix} \begin{bmatrix} u[1] \\ u[2] \\ u[3] \\ \vdots \\ u[T-1] \end{bmatrix} + \begin{bmatrix} CB_d & 0 & 0 & \dots & 0 \\ CAB_d & CB_d & 0 & \dots & 0 \\ CA^2B_d & CAB_d & CB_d & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & 0 \\ CA^{k-1}B_d & CA^{k-2}B_d & CA^{k-3}B_d & \dots & CB_d \end{bmatrix} \begin{bmatrix} u_d[1] \\ u_d[2] \\ u_d[3] \\ \vdots \\ u_d[T-1] \end{bmatrix} \tag{4}$$

Let $\bar{e}$ denote the stacked vector of $e[2], ..., e[T]$ that appears in Equation (4). Similarly, let $\bar{u}$ and $\bar{u}_d$ denote the stacked vectors of $u[k]$ and $u_d[k]$ for $1 \leq k \leq T-1$. Note that the assumptions of $x[0] = u[0] = u_d[0] = 0$ imply $e[1] = 0$. Define M_1 and M_2 as the Toeplitz matrices that multiply $\bar{u}$ and $\bar{u}_d$ in Equation (4). Now the optimization described in Equation (2) can be formulated as:

$$\begin{aligned} \min_{\bar{u},\gamma} \quad & \gamma \\ \text{subject to:} \quad & -\gamma \leq M_1\bar{u} + M_2\bar{u}_d \leq \gamma \\ & -r\mathbf{1} \leq M_3\bar{u} \leq r\mathbf{1} \\ & u_d[k] = \begin{cases} 0 & \text{if } k < N \\ v & \text{if } k \geq N \end{cases} \end{aligned} \tag{5}$$

where γ is a slack variable, $\mathbf{1} \in \mathbb{R}^{T-1}$ is a vector of ones, and M_3 is the difference operator given by the matrix in Equation (6).

$$M_3 = \begin{bmatrix} 1 & 0 & 0 & \dots & 0 \\ -1 & 1 & 0 & \dots & 0 \\ 0 & -1 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & -1 & 1 \end{bmatrix} \tag{6}$$

This problem has a linear cost subject to linear equality constraints. This is a linear programming problem and more details can be found in the textbook by Vandenberghe and Boyd.[13] Linear programs (LPs) are convex problems and efficient software exists to solve problems with thousands of constraints and decision variables.

B. Generalizations

The optimization problem presented in Equation (2) can be generalized in various ways to represent other turbine control problems of interest. First, arbitrary wind disturbance profiles can be modeled within the LP framework. This would be useful to investigate the performance for more realistic, turbulent wind profiles. In this case, the LP would solve for the optimal control input with respect to the specified, fixed wind profile. Second, blade, tower and gearbox bending loads can be incorporated into the constraints and/or cost function. This generalization would be useful for modeling an individual pitch control problem in Region 3 where the objective is to minimize the bending loads. The peak norm, i.e. $||y||_\infty$ norm, could be used to measure the peak bending load. However, some initial investigations have indicated that the total variation as $TV = \sum_{n=0}^{k-1} |y[n+1] - y[n]|$ is better suited for measuring fatigue. The total variation is a measure of the variation of a signal from one time step to the next one. Minimization of this quantity results in smoother bending loads and reduced variations. In the ideal case, zero total variation corresponds to a constant load and hence zero fatigue. Hence there is a good correlation between minimization of the total variation and the damage equivalent loads that are commonly used for evaluation of the performances of controllers for load attenuation. Optimizations involving TV in the constraints and/or objective function can be formulated as LPs. To summarize, the LP framework can be used to investigate the effect of preview time on more complicated wind turbine control problems involving constraints and objective functions that depend on rotor speed tracking, bending loads, and/or blade pitch deflection and rate limits.

III. Limits of Performance of One-State Turbine Model

This section analyzes a rotor speed tracking problem using the linear programming framework. This analysis is based on a one-state inertia model that captures the dynamics of the turbine rotor. The objective is to use wind gust preview information for Region 3 (above rated) wind conditions subject to blade pitch rate constraints. The wind turbine considered for this analysis is the three-bladed Controls Advanced Research Turbine (CART3)[8] located at the National Wind Technology Center (NWTC). CART3 is a 40 m diameter turbine with 600 kW rated power. A one-state LTV model of the turbine is obtained from a higher complexity nonlinear model in NREL's Fatigue, Aerodynamics, Structures and Turbulence modeling (FAST)[14] code. The linearization is obtained at uniform hub-height wind speed of 18 m/s. The trim rotor speed is 3.881 rad/s and the trim blade pitch angle is 16.5 deg. Since the rotor dynamics do not have a strong dependence on rotor position, the LTV system matrices are averaged over one rotor period to obtain an LTI model. The resulting one-state model is:

$$\begin{aligned} \Delta\dot{\omega}_r(t) &= a\Delta\omega_r(t) + b\Delta\beta(t) + c\Delta v(t) \\ e(t) &= \Delta\omega_r(t) \end{aligned} \tag{7}$$

where $\Delta\omega_r(t)$ is the rotor speed error in rad/s, $\Delta\beta(t)$ is the difference between collective pitch angle and trim pitch in deg, and $\Delta v(t)$ is the uniform hub-height wind perturbation in m/s.

One-state rotor models typically involve nonlinear power or torque coefficient curves with respect to rotor speed, blade pitch and wind inputs. The parameters a, b and c represent the linearizations of these curves with respect to rotor speed, blade pitch and wind inputs respectively. CART3 is a pitch-to-feather turbine and it sheds the excess power in wind by increasing blade pitch angles. This implies $b < 0$ at the trim pitch angle. The power curve of CART3 has a negative slope with respect to tip-speed ratio λ at this point. Tip-speed ratio is defined as $\lambda = \frac{\omega_r R}{v}$ where R is the rotor radius. Hence the extracted power decreases with increasing rotor speed or decreasing wind speed. This implies $a < 0$ and $c > 0$. The numerical values of these parameters are $a = -0.2771$, $b = -0.0527$ and $c = 0.0731$.

The system model given in Equation (7) is discretized using a bilinear (Tustin) transformation at a sampling time of $T_s = 0.01s$. The resulting discrete-time turbine model is:

$$\begin{aligned} \Delta\omega_r[k+1] &= 0.997\Delta\omega_r[k] - 5.26 \times 10^{-4}\Delta\beta[k] + 7.30 \times 10^{-4}\Delta v[k] \\ e[k] &= \Delta\omega_r[k] \end{aligned} \tag{8}$$

It is assumed that T_{prev} seconds or $N := \frac{T_{prev}}{T_s}$ time-steps of preview information is available where N is assumed to be an integer. This problem is analyzed for a 2 m/s step wind gust for the sake of developing insight into optimal controller behavior. Although step wind disturbances cannot fully capture the effects of turbulent wind conditions, this wind profile can be used to obtain insights into the effects of preview time. The CART3 has a blade pitch rate limit of 18 deg/s but this analysis is conducted for a pitch rate limit of 6 deg/s. This is done for two reasons. First, a controller designed to yield 6 deg/s peak pitch rate on step wind gusts is expected to use higher pitch rates in turbulent wind conditions. Second, this analysis only considers collective blade pitch actuation. If individual blade pitch controllers are employed, some actuation capacity should be reserved for the individual pitch actions. At the sample time of $T_s = 0.01$ s, the 6 deg/s rate limit is equivalent to a discrete-time rate bound of $r = 0.06$ deg per sample time.

The linear program in Equation (5) is solved for the one-state turbine model using SeDuMi[15] in MATLAB. Solutions are obtained for preview times of $T_{prev} = 0, 0.05, 0.1, \ldots, 0.7$ seconds. It is assumed that the turbine is initially at trim, i.e. $\Delta\omega_r[0] = 0$, $\Delta\beta[0] = 0$ and $\Delta v[0] = 0$. The wind gust is assumed to occur at $t = 1$ s. Solutions for 0 $s, 0.15$ s and 0.45 s preview times are presented in Figure 1. Results with preview times larger than 0.45 s yield same results as the 0.45 s preview case. The top plot in Figure 1 shows the blade pitch angles about the trim pitch ($\Delta\beta$). The middle plot shows the blade pitch rates. The pitch rates are plotted with their equivalent continuous-time values using the expression $(\Delta\beta[k] - \Delta\beta[k-1])/T_s$.

There are four key points in this figure. First, a small amount of preview information can lead to significant reductions in peak rotor speed error. For small preview times, the optimal pitch action for

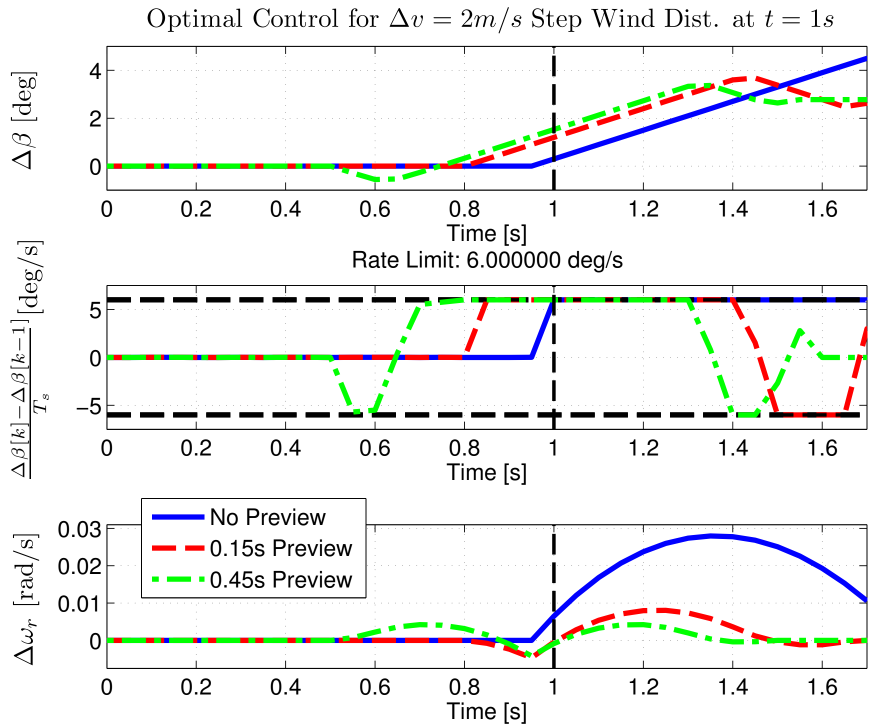

Figure 1. Optimal control action for $min||\Delta\omega_r||_\infty$ with varying preview wind amounts

this pitch-to-feather turbine is to increase the pitch angle with the maximum pitch rate as soon as the information about gust enters the system. The control action pitches the blades toward their steady state values and slows down the rotor in preparation for the wind gust. Second, there is a diminishing return from increasing preview information. Only a limited improvement in performance is observed with 0.45 s of preview compared to the results for 0.15 s of preview. Third, with 0.45 s of preview the turbine shows an interesting behavior of initially pitching in the negative direction. The blades are initially pitched away from their steady state values at the maximum rate. Then the blades are pitched in the other direction again at their maximum rate. This generates a second positive peak in the rotor speed error signal prior to the wind gust. The two peaks and one valley in the rotor speed signal are equal. With the initial speed-up of the rotor, a smaller negative peak in the rotor speed error is achieved at around $t = 1$ s with a larger pitch angle before the gust. This larger pitch angle in turn results in a smaller peak after the gust.

The last key point is that increasing the preview amount beyond 0.45 s yields no benefits. This is confirmed by the peak rotor speed error vs. preview time plot (blue curve) shown in Figure 2. H_∞ (red dash) curve will be described in the Section IV. The peak rotor speed error (blue curve) shows small improvements above 0.15 s of preview and no additional improvements above 0.45 s. This indicates a fundamental performance limit imposed by the actuator pitch rate constraint that cannot be overcome via preview information. Since all the peaks on the error signal were already equal in magnitude, a larger pitch angle before the gust would generate a larger negative peak. Therefore extra preview information yields no benefit.

IV. H_∞ Preview Control for CART3 Turbine

The previous section used the LP framework to analyze the performance limits of a simple, one-state rotor inertia model for a wind turbine. This section focuses on a more realistic design for a higher-order turbine model that includes structural modes. Specifically, a H_∞ preview controller is designed for the NREL's CART3 turbine. The architecture for the H_∞ preview controller is based primarily on the work by Laks, et al.[3] The performance vs. preview trade-offs are investigated with Monte Carlo simulations on a high fidelity, nonlinear simulation model using both step and turbulent wind profiles. The performance

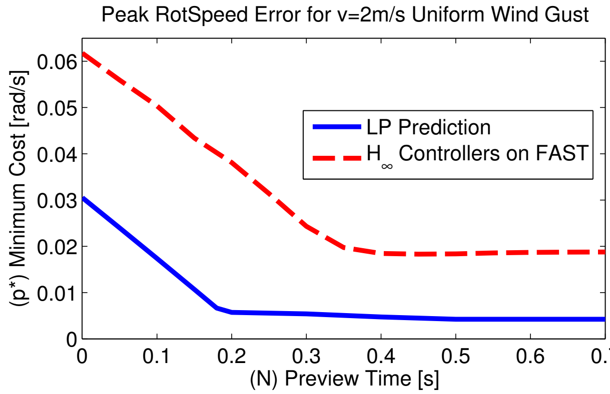

Figure 2. Peak rotor speed error versus preview times for a uniform step wind disturbance of $\Delta v = 2\ m/s$

trends for this more realistic example are compared with the trends predicted by the analysis on the simpler one-state model.

The CART3 turbine is modeled with the nonlinear Fatigue, Aerodynamics, Structures and Turbulence (FAST) simulation code[14] developed by the National Renewable Energy Lab (NREL). The model data for the CART3 turbine was obtained from A. Wright.[8] FAST can model on-shore wind turbines with a total of 22-24 degrees of freedom. This full order model includes first and second tower fore-aft and side-to-side bending modes, first and second flapwise bending modes of blades, first edgewise bending modes of blades, drivetrain torsion, generator position and nacelle yaw angle. The basic FAST code does not include models for pitch actuator dynamics and rate limits. Dynamics of the pitch actuators of the CART3 can be represented with first-order models with time constants of $\tau = 1/30s$.[3] These actuator models were added to FAST for nonlinear simulations. In addition, the CART3 pitch actuators have a rate limit of 18 $^o/sec$ and these rate limits were also modeled within FAST.

A low order model of the CART3 was used for the control design. FAST has the capability to freeze specified modes-of-freedom to obtain reduced-order models. The control design model contained modes for generator speed, tower first fore-aft and blade first flapwise bending modes. The first order actuator models were included but the rate limits were neglected for the design. An LTV model of this reduced-order model turbine was obtained from FAST by linearizing the nonlinear dynamics around a periodic trajectory. This LTV model was converted into an LTI model using the MBC transformation and averaging as described in the work by Bir.[10] Finally, the turbine model was discretized using a bilinear (Tustin) transformation with a sample time of $T_s = 0.025s$. This discretization step was needed for modeling of the wind preview information.

A H_∞ preview controller is designed for the reduced-order, discrete-time LTI model of the CART3. The control objective is rotor speed tracking in Region 3 (above rated wind speeds). The control input is the collective pitch angle of three blades. The measurements for control are the rotor speed, collective blade bending moment, and the tower fore-aft bending loads. Cyclic blade bending moments are not included in the collective pitch controller design since collective pitch control has limited authority on the cyclic loads. In addition, it is assumed that the controller has access to preview measurements of the average wind speed across the rotor disk. The preview measurements are modeled by augmenting the wind disturbance input of the design model with N delays. The controller has access to a measurement of the wind disturbance input to the chain of N delays. As a result the controller has a measurement of the wind disturbance with a preview of NT_s seconds prior to its impact on the turbine. The amount of preview available to the H_∞ controller is adjusted by changing the number of delays N.

Signal-weighted H_∞ control designs were performed for a variety of preview times N using the system interconnection shown in Figure 3. The block labeled "LTI Turbine Model" is the discrete-time LTI design model without the actuator dynamics and the N steps of delay on the wind input. The system interconnection contains weights for performance, input uncertainty, measurement noise, actuator usage, and wind disturbance. These weights are briefly described and, unless stated otherwise, controller designs for all values of preview time share the same weights. The weights were initially specified in continuous-time and then converted to discrete-time using a bilinear (Tustin) transformation and sample time of $T_s = 0.025s$. The continuous and discrete-time transfer functions for each weight are provided in Table 1. The performance weight is block diagonal $W_{perf} = diag(W_{BladeM}, W_{TowerM}, W_{\omega_r})$ with the individual blocks penalizing collective blade bending moment, tower bending moment, and rotor speed tracking, respectively. The performance penalty on rotor speed error signal W_{ω_r} is chosen to attenuate lower-frequency tracking errors. The performance penalty on collective blade bending moment W_{BladeM} emphasizes attenuation of the blade bending moment at middle to high frequencies. This choice is made because the DC and lower frequency components of the blade bending moments due to persistent wind disturbances cannot be attenuated. The penalty weight on the tower bending W_{TowerM} is chosen to add extra damping at the tower bending moment frequency. The input disturbance W_{in} models increasing dynamic uncertainty at high frequencies. The weight W_{nois} is a 2×2 diagonal weight that models noise on the rotor speed and wind speed. The measurement noise weight on rotor speed sensor is 0.4. This constant weight roughly models noise corresponding to 10% of its respective value under trim operating conditions. The noise weight on the wind speed measurements is 0.1. The weight on the wind disturbance W_{wind} was chosen based on the spectral content of rotor-averaged wind speeds of a 5% turbulent wind data. The actuator weight W_{act} is a high pass filter that penalizes high frequency control effort. This weight includes a gain K that is chosen as a function of the wind preview time N. The value of K is tuned through simulations to obtain a closed-loop peak pitch rate of 6 deg/s for a 2 m/s uniform wind gust input for all H_∞ controllers that incorporate different preview lengths. This ensures that controllers for all preview times have roughly the same actuator usage. Values of gain K versus preview time are provided in Table 2.

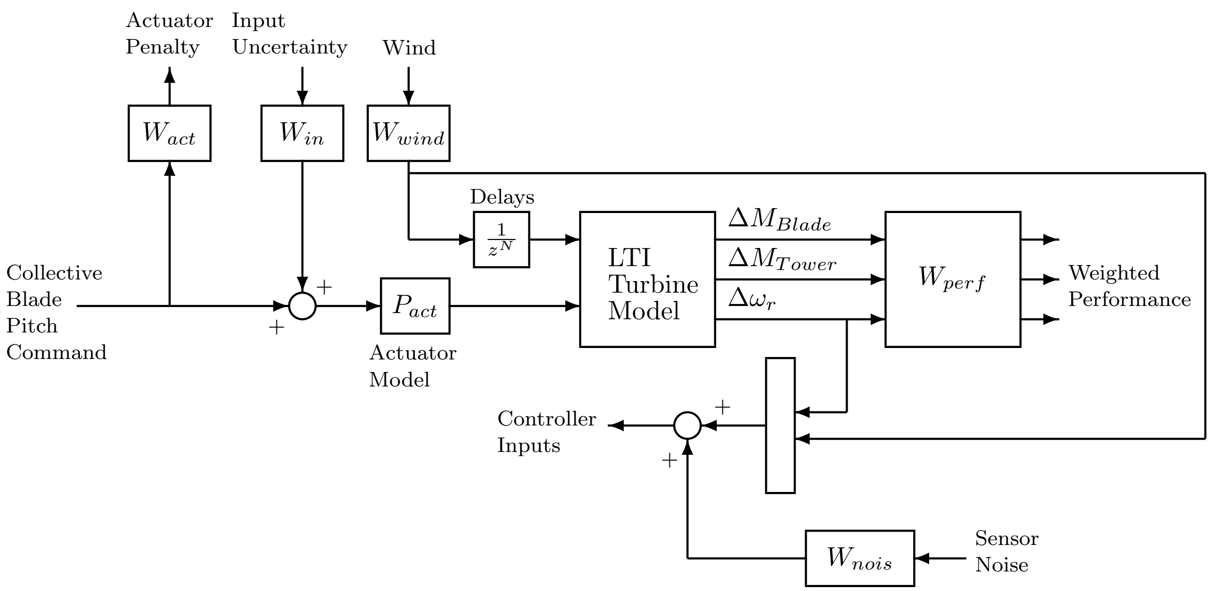

Figure 3. System Interconnection for H_∞ Collective Pitch Controller Design

The effect of preview time on the closed-loop performance for the H_∞ preview controllers was evaluated on the nonlinear FAST CART3 model. All degrees of freedom available in FAST for onshore turbines were used in the simulations except rotor-teeter and furling DOF which are not employed by CART3. The model included the first order actuator dynamics and rate limits. Simulations were conducted with step wind inputs as well as turbulent wind conditions. TurbSim,[17] developed at NWTC, was used to generate the turbulent wind data. The parameters for generating the turbulent wind data are taken from the work by Laks, et al.[4] and are listed in the Table 3. These wind conditions are considered to be realistic for the NWTC site where the CART3 is located.

The closed-loop response of the nonlinear CART3 system to a 2 m/s uniform wind disturbance is shown in Figure 4 for H_∞ controllers with different preview lengths. The step wind gust occurs at time $t = 50$ s. The oscillations observed in the pitch rate (bottom plot) is caused by the different sample times employed by the controller and the simulation model. The simulation sample time is 10 times faster of the controller sample time $T_s = 0.025$ s. The control input is held fixed between these 10 time steps. The pitch rate peaks occur at each control input update. As the first-order actuator model response approaches its steady state

Weight	Continuous Time	Discrete Time
W_{ω_r}	$\frac{0.02s+1}{5s+1}$	$\frac{0.006484z-0.001496}{z-0.995}$
W_{BladeM}	$1\times10^{-4}\frac{2s+0.01732}{s+1.732}$	$\frac{0.0001958z-0.0001957}{z-0.9576}$
W_{TowerM}	$2\times10^{-5}\frac{0.01333s+1}{0.2s+1}$	$\frac{2.43\times10^{6}z-7.84\times10^{8}}{z-0.882}$
W_{in}	$0.02\frac{s+1}{0.01333s+1}$	$\frac{0.784z-0.765}{z-0.0323}$
W_{n,ω_r}	0.4	0.4
$W_{n,wind}$	0.1	0.1
W_{wind}	$0.25\frac{0.01667s+1}{0.25s+1}$	$\frac{0.0278z-0.00397}{z-0.905}$
W_{act}	$K(N)\frac{0.1667s+1}{0.01667s+1}$	$K(N)\frac{6.14z-5.29}{z-0.143}$

Table 1. Performance Weights for H_∞ Preview Control Design

Preview Time (s)	0	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	≥ 0.50
Gain K	2.55	2.47	2.39	2.15	1.91	1.59	1.11	0.80	0.67	0.61	0.57

Table 2. Values of gain K used in actuator penalty weight W_{act}

value, the pitch rate approaches zero.

The results in Figure 4 show that significant reductions in peak rotor speed error are obtained by incorporating wind preview measurements. The H_∞ controller with 0.6 s preview yields very similar results to the controller with 0.45 s preview and is not plotted. Figure 2, discussed previously in Section III, plots the peak rotor speed errors (red dash curve) as a function of preview time for the FAST simulations with the H_∞ controllers. The results from nonlinear simulations show a similar trend to the linear programming solution. In particular, small amounts of preview yields large improvements in rotor speed tracking. The peak rotor speed deviation declines linearly with the preview time. However, there is a limit to the performance improvements that can be achieved with preview. The H_∞ controllers tuned to use the same peak pitch rate of 6 deg/s indicate that there is little to no improvement after the preview time of 0.45 s in the FAST simulations. This is in good agreement with the previous numerical results obtained on the simple, one-state rotor model.

Peak rotor speed errors, peak blade pitch rates and blade damage equivalent loads obtained with H_∞ controllers on nonlinear FAST simulations are shown in Figure 5. The results for step wind disturbances are presented on the left and the results with turbulent wind conditions are on the right. Turbulent wind conditions represent a more realistic operating condition for the turbine. The results in Figure 5 show good agreement in the performance vs. preview time trends for the step and turbulent wind conditions. Both

Parameter	Value
Mean Wind Speed	18 m/s
Vertical Wind Shear (α_0)	0.110
Vertical Stability (Ri_{TL})	-0.18
Mean Friction Velocity (U_{*D})	0.682 m/s

Table 3. Atmospheric parameters used in TurbSim for generating turbulent wind data

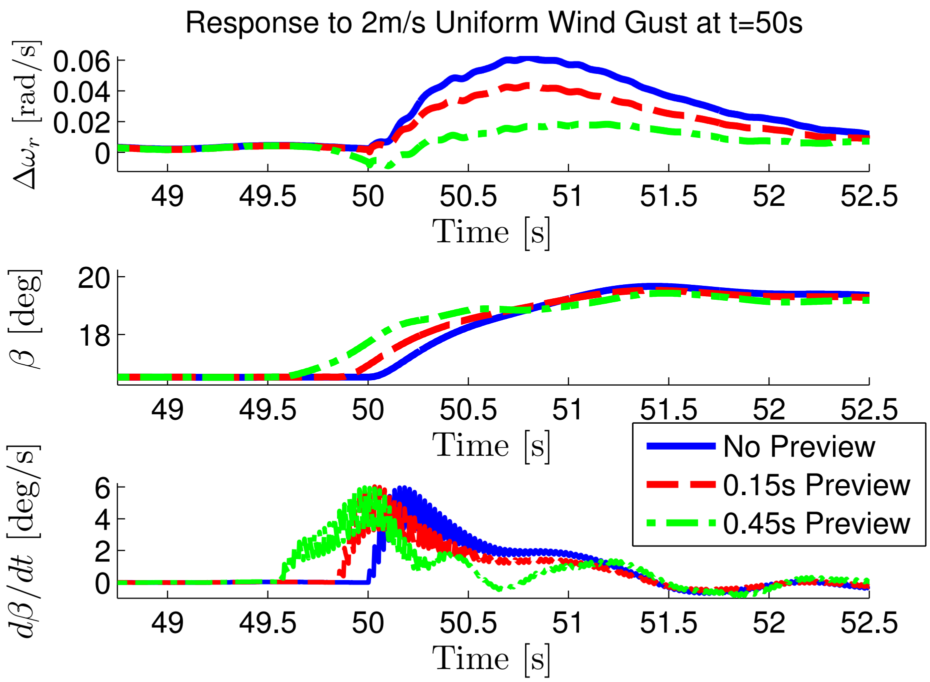

Figure 4. Closed-loop response to a $2\,m/s$ step uniform wind gust

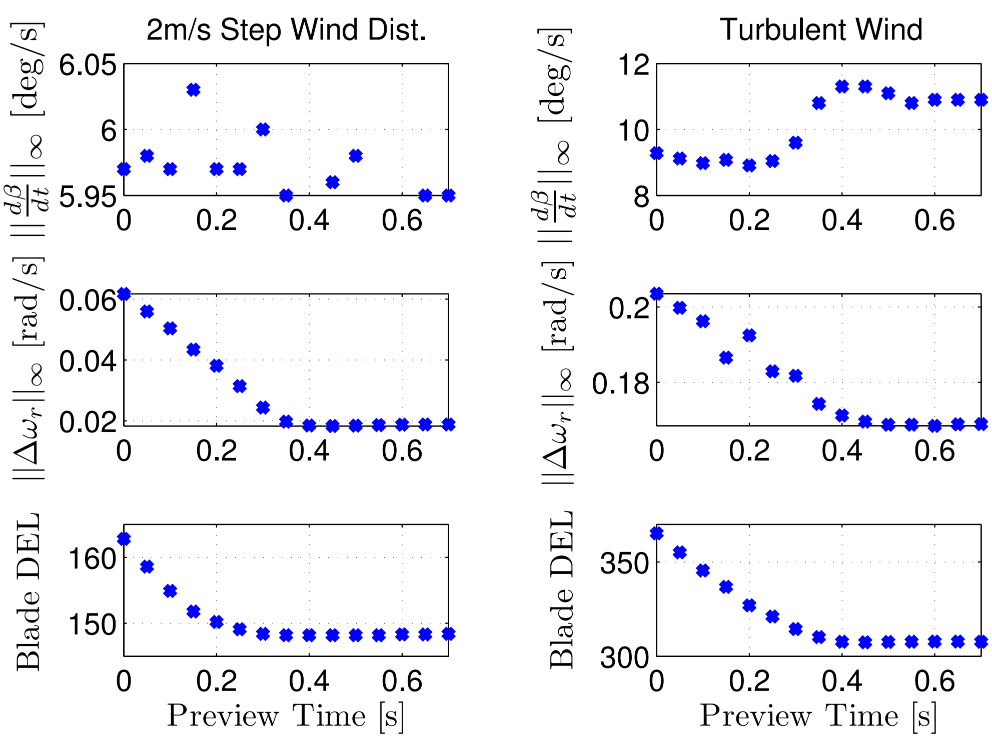

Figure 5. Performance metrics vs. preview time for step wind (left column) and turbulent wind (right column) conditions for H_∞ controllers on FAST. Blade damage equivalent loads shown are the averages of the three blades

rotor speed tracking and blade fatigue are reduced significantly by the use of preview wind measurements. The peak rotor speed errors and the blade damage equivalent loads decrease linearly for small preview times. This linear decrement flattens out roughly around the same preview amounts for both step wind disturbances and turbulent wind conditions. Once again the benefits of preview measurements are negligible for sufficiently large preview times of about 0.45 s. This agrees with the predictions obtained from linear programing solutions in the previous section.

V. Conclusions

This paper investigated the performance limitations of a Region 3 rotor speed tracking problem with preview wind measurements. An optimal control problem that minimizes peak rotor speed errors subject to pitch rate constraints was converted into a linear program. The solution of this linear program provides a bound on performance achievable by any controller. Solutions of linear programs provided good predictions for the performance vs. preview time trade-offs for an H_∞ controller simulated on a higher-order, nonlinear turbine model. Future work includes application of this framework to more general wind turbine control problems. In particular, the total variation of the collective blade moments appears to have good correlation with blade damage equivalent loads. Constraints on structural loads can be combined with rotor speed tracking objectives by formulating LPs with L_∞ (peak) and total variation norms. Solutions for such problems can provide insights into the use of preview for more general turbine control problems.

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