

Validating Uncertain Aircraft Simulation Models Using Flight Test Data

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Reliability validation of safety-critical flight control systems requires accurate simulation models of aircraft. This paper proposes a simple approach to validate such models efficiently using flight data. A statistical model validation analysis is shown to be equivalent to a robust control analysis for simple linear systems. The analysis is extended to nonlinear systems in conjunction with Monte Carlo simulations to validate an uncertain aircraft simulation model. This approach is demonstrated using the University of Minnesota Unmanned Aerial Vehicle flight test platform.

I. Introduction

Ensuring the reliability of safety-critical flight control systems is a major challenge for the aerospace industry. Typical safety requirements mandate failure rates on the order of 10^{-9} failures per flight hour.^{1,2} The difficulty associated with meeting these requirements has increased due to a growing reliance on advanced technologies. Developing accurate models of aircraft is key to validating control systems more efficiently. Such models embody the vehicle dynamics during all stages of flight, including take-off, cruise, and landing. Many components of the models are also characterized by uncertainty. The end result is a complex, uncertain, nonlinear aircraft simulation model. Its accuracy must be validated based on ground, bench, and flight test data in order to use the model, in turn, to validate safety-critical flight control systems.

Model validation using experimental data has been widely considered across many industries. A variety of approaches have been proposed for a diverse set of applications, each with its particular advantages and restrictions. Optimization-based methods that link model validation to robust control are analytically rigorous, but applicable only to simple models and subject to computational limits.³⁻⁶ Statistical methods are broadly applicable to many problems, but generally suffer from a lack of providing definite guarantees.⁷⁻¹¹ Empirical methods developed purely based on engineering experience are useful, but not ideally suited to analyze complex and integrated systems.¹² A simple approach to model validation with rigorous analytic ties and broad applications would be useful to the aerospace community. This paper proposes such an approach by linking a statistical analysis to standard robust control metrics.

The proposed approach determines whether a time history of flight data can be represented by an uncertain aircraft simulation model. Theil's Inequality Coefficient (TIC), originally developed for economic forecasting,⁷ has been used to evaluate similarity between flight and simulation time histories.^{9,10} This paper proves that the TIC is related to a robust control metric for simple linear systems. In the linear case, evaluating the TIC is equivalent to analyzing model uncertainty using robust control tools. Unlike most robust control tools, however, the TIC is applicable to nonlinear systems. The direct ties between the TIC and model uncertainty cannot be preserved in the nonlinear case, and Monte Carlo simulations are required as a substitute. The task of model validation amounts to comparing predicted TIC statistics from the Monte Carlo simulations versus the TIC relating the flight data to the nominal simulation. If a confidence bound on the Monte Carlo TIC exceeds the flight data TIC, then the flight data is represented by the model. This analysis is demonstrated using the University of Minnesota Unmanned Aerial Vehicle (UAV) platform.¹³

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II. Model Validation Analysis

A simple and broadly applicable metric is desired to validate the accuracy of aircraft simulation models. The aerospace community, however, has not yet agreed upon such a metric. Individual groups have developed their own, tailored metrics that meet the needs of their particular applications. The scattered focus within the community, and consequently the lack of a standardized metric, is one of the main roadblocks to forming a unified approach to model validation. The TIC is a candidate metric with the potential to unify model validation techniques. A theorem is given herein that relates this metric to standard robust control metrics for simple linear systems. The analysis is extended to nonlinear systems, culminating in the validation of an aircraft simulation model using flight test data.

A. Proposed Metric

Theil's Inequality Coefficient was originally developed in the 1970s as tool for economic forecasting.⁷ In general, it measures the similarity between two time histories of data on a normalized scale from 0 to 1. Values of the TIC near 0 indicate strong similarity between the two time histories. Values near 1 represent the worst-case deviation, where one of the time histories is constant and zero. For applications considered in this paper, the TIC is defined by the following relationship:

$$\text{TIC} = \frac{\sqrt{\frac{1}{n} \sum_{i=1}^n (x_i - \tilde{x}_i)^2}}{\sqrt{\frac{1}{n} \sum_{i=1}^n x_i^2} + \sqrt{\frac{1}{n} \sum_{i=1}^n \tilde{x}_i^2}}, \quad (1)$$

where x is a simulation time history, \tilde{x} is a time history obtained in flight, and n is the number of data samples. Values of $\text{TIC} < 0.25$ have been shown to correspond to accurate models of aircraft when comparing simulation to flight data.^{9,10} The normalized scale associated with the TIC is a significant advantage over standard error metrics, such as the direct \mathcal{L}_2 norm of the error $e = x - \tilde{x}$. By scaling the TIC based on the \mathcal{L}_2 norms of the two time histories being compared, the analysis is decoupled from signal units and amplitudes.

In simple cases, the TIC is related to uncertainty metrics commonly used in robust control. Consider a single-input, single-output (SISO) aircraft model $P(j\omega)$ governed by linear time-invariant (LTI) dynamics. This nominal model is perturbed with multiplicative input uncertainty:

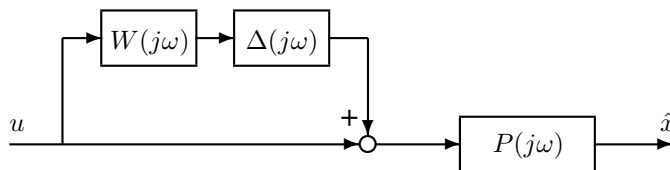


Figure 1. SISO LTI system with multiplicative input uncertainty.

Here, $\Delta(j\omega)$ is an unknown complex number, norm bounded by 1 for all frequencies. The weight $W(j\omega)$ is a stable transfer function that measures the amount of uncertainty entering the system. Time history \hat{x} is one of the possible perturbed outputs consistent with the uncertainty description. Although not shown in Figure 1, time history x is the nominal output, characterized by $\Delta(j\omega) = 0$. Assume that the true system dynamics $\tilde{P}(j\omega)$ lie within a multiplicative uncertainty set centered at $P(j\omega)$. This set is defined as:

$$\mathcal{M} := \{ P(j\omega)[1 + \Delta(j\omega)W(j\omega)] : \|\Delta(j\omega)\|_\infty \leq 1 \}. \quad (2)$$

The true aircraft response \tilde{x} is therefore a subset of the set of all possible outputs \hat{x} , as defined by the uncertainty description in the model.

The objective is to interpret the TIC using standard robust control metrics. Evaluating Equation 1 using the perturbed simulation response \hat{x} (instead of flight response \tilde{x}) yields the TIC predicted by the model. When $\Delta(j\omega) = 0$, which is the nominal case, $\hat{x} = x$ and the predicted TIC is 0. The worst-case deviation corresponds to 100% uncertainty, and the predicted TIC is 1. An upper-bound on the predicted TIC depends on the amount of uncertainty in the model, as described by the weight $W(j\omega)$. Since $\tilde{P}(j\omega) \in \mathcal{M}$, this is also an upper bound on the TIC relating the nominal simulation to the true aircraft response.

Theorem 1 Suppose $P(j\omega)$ is a SISO LTI system, $\tilde{P}(j\omega) \in \mathcal{M}$, and assume $\|W(j\omega)\|_\infty \leq 1$. Then

$$\text{TIC} \leq \frac{\|W(j\omega)\|_\infty}{2 - \|W(j\omega)\|_\infty}.$$

Moreover, there exists a model in \mathcal{M} that achieves the TIC upper-bound.

Proof: Consider a continuous time version of the TIC given in Equation 1, where $x(t)$ is the nominal simulation response and $\tilde{x}(t)$ is the true aircraft response:

$$\text{TIC} = \frac{\|x(t) - \tilde{x}(t)\|}{\|x(t)\| + \|\tilde{x}(t)\|}. \quad (3)$$

Using Parseval's theorem, Equation 3 is converted to the frequency domain. Since $\tilde{P}(j\omega) \in \mathcal{M}$ and therefore $\tilde{x} \in \hat{x}$, the TIC is bounded and re-written in terms of $u(j\omega)$, $P(j\omega)$, $\Delta(j\omega)$, and $W(j\omega)$. The result is shown by the following, where the frequency domain argument $j\omega$ is omitted for clarity:

$$\text{TIC} \leq \frac{\|Pu - P(1 + \Delta W)u\|}{\|Pu\| + \|P(1 + \Delta W)u\|}. \quad (4)$$

Because $P(j\omega)$ is SISO, the nominal aircraft dynamics are factored out eliminated from Equation 4. Similarly, the input u is eliminated. As a result, the TIC is independent of the nominal aircraft dynamics and the input, as shown by the following:

$$\text{TIC} \leq \frac{\|\Delta W\|}{1 + \|1 + \Delta W\|}. \quad (5)$$

The TIC is bounded once more by applying the triangle inequality to the denominator:

$$\text{TIC} \leq \frac{\|\Delta W\|}{\|2 + \Delta W\|}. \quad (6)$$

The maximum upper-bound is found by choosing $\Delta(j\omega)$ in order to maximize the numerator and minimize the denominator, yielding the result in Theorem 1.

It remains to be shown that the upper-bound is achievable. A pair of $\Delta(j\omega)$ and $W(j\omega)$ are selected to show that Equations 5 and 6 can be equivalent. Suppose that $W(j\omega)$ is a complex number $re^{j\phi}$, where $0 < r \leq 1$ and the phase ϕ is arbitrary. Then $\Delta(j\omega)$ is selected as $-e^{-j\phi}$, which yields $\Delta(j\omega)W(j\omega) = -r$. Applying this to Equation 5 results in the following:

$$\text{TIC} = \frac{\|-r\|}{1 + \|1 - r\|} = \frac{r}{2 - r}, \quad (7)$$

which is equivalent to the bound in Equation 6 and completes the proof. ■

The existence of a tight upper-bound on the TIC increases confidence in this metric as a useful tool for model validation. Connecting the upper-bound to a standard uncertainty description aligns the TIC with a worst-case gain analysis from robust control. At this time, the analytically rigorous ties between the TIC and robust control are only proven for SISO LTI systems. There is potential to derive a related proof for multiple-input, multiple-output (MIMO) LTI systems. Most importantly, however, the existence of rigorous ties between the TIC and robust control increase confidence that TIC analysis will provide meaningful insight when applied to uncertain nonlinear systems.

B. Extension to Nonlinear Systems

The direct connection between the TIC and the uncertainty description is lost when considering nonlinear systems. An upper-bound for the TIC is not readily available in this case. However, the intuition upon which the analysis is founded remains. Monte Carlo simulations are required to identify an upper confidence bound on the TIC. Here, the uncertainty is randomly sampled and simulations are performed using the input from the flight data. The TIC is evaluated for each Monte Carlo run against the nominal simulation to obtain a distribution of TIC values predicted by the model. If the TIC relating the flight data to the nominal

simulation is bounded by the predicted TIC distribution from the Monte Carlo analysis, it is concluded that the model is valid.

As a very simple example to demonstrate the Monte Carlo analysis, let $P(s)$ be the nominal system described by the following second-order transfer function:

$$P(s) = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}, \quad (8)$$

where the natural frequency of this system is $\omega_n = 10$ rad/s, and the damping ratio is $\xi = 0.3$. To represent 25% input gain uncertainty, weighting function $W(s) = 0.25$ is used. The uncertain system is randomly sampled and simulated 1000 times with a unit step input. The Monte Carlo simulation results are shown on the left in Figure 2. The TIC is computed for each simulation run against the nominal simulation, resulting in a distribution of TIC values predicted by the model. This distribution is collected into a histogram, shown on the right in Figure 2.

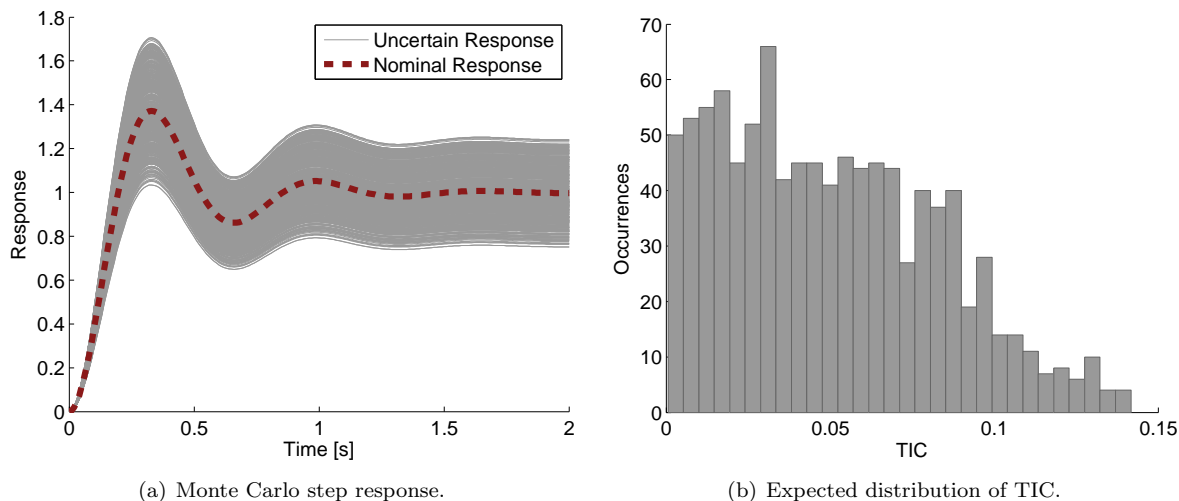


Figure 2. Example of Monte Carlo TIC analysis.

Since the nominal system used in the Monte Carlo analysis is linear, it is possible to apply Theorem 1 to obtain a theoretical upper-bound on the TIC. Given $W(s) = 0.25$, the maximum TIC is 0.14. This matches closely with the maximum TIC found by the Monte Carlo simulations.

III. Validating an Aircraft Model

The proposed model validation approach is demonstrated using the University of Minnesota Unmanned Aerial Vehicle (UAV) flight test platform. This research platform is readily available for guidance, navigation, and flight control experiments. A nonlinear simulation model for the test vehicle is also available.

A. Flight Test Platform

The University of Minnesota UAV Research Group has developed a low-cost experimental test platform with advanced research capabilities in areas such as guidance, navigation, and flight control. At the center of the platform is a flight test vehicle along with a simulation environment. The flight test vehicle operates on a custom real-time software suite. A baseline flight control system, including algorithms for attitude and navigation state estimation, has been developed to enable autonomous flight.¹⁴ The simulation environment includes a model of the nonlinear aircraft dynamics, a software-in-the-loop (SIL) simulation, and a hardware-in-the-loop (HIL) simulation. The real-time software suite, simulation environment, and baseline flight control system are version controlled, documented, and available in the open source.¹³

The flight test vehicle is based on a modified version of the Ultra Stick 25e hobby airframe. It is powered by an electric motor and has a 1.5 kg mass, a 1.27 m wingspan, and a 17 m/s cruise speed. The vehicle has conventional fixed-wing aircraft geometry with elevator, aileron, and rudder control surfaces. Figure 3 shows

the vehicle being prepared for a flight test. The vehicle is equipped with a flight computer and an array of sensors, including an inertial measurement unit (IMU) and a GPS sensor. The IMU provides measurements of the angular rates and translational accelerations. An extended Kalman filter is implemented to estimate the attitude and position of the aircraft by fusing the IMU and GPS sensor measurements. The full system architecture is summarized by the diagram on the right in Figure 3.

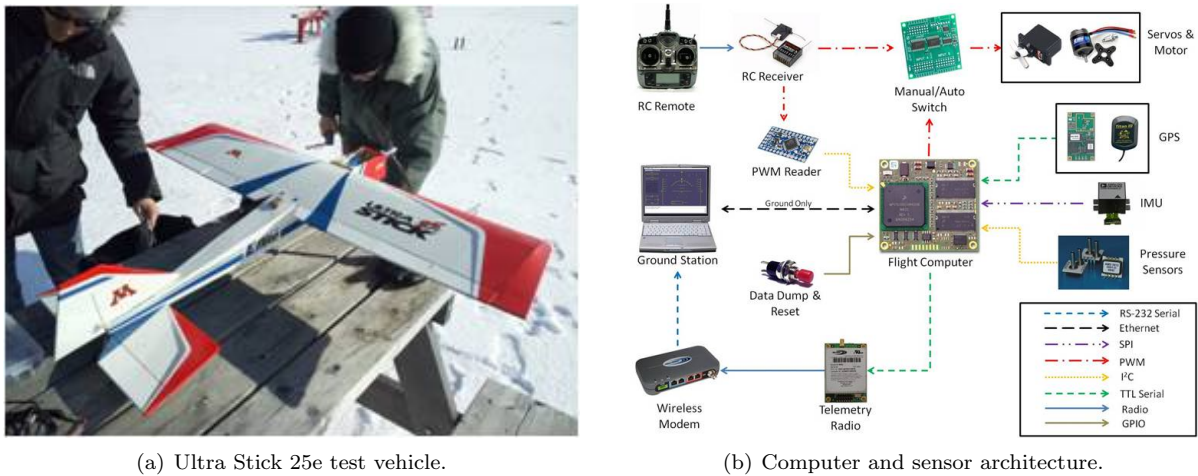


Figure 3. University of Minnesota UAV flight test platform.

A simulation environment has been developed to facilitate efficient testing and validation of control algorithms in preparation for flight experiments. The nonlinear model of the aircraft dynamics was determined using system identification.¹⁵ The SIL simulation takes advantage of this model and allows for verification of the control software prior to flight testing. Similarly, the HIL simulation is used to check the integrity of the compiled software on the flight computer. The simulation environment is built for a generic fixed-wing aircraft, where configuration files are used to define aircraft-specific parameters. Development of the environment is ongoing, and the most recent versions are available online in a subversion repository.¹³

A simple and reliable baseline control algorithm has been developed as a benchmark for the UAV platform. For more details on this design, please see Reference 14. The controller was validated through simulation and flight testing. It currently serves as the standard controller used for any research experiment that requires closed-loop control. The design has a two-tiered structure: an inner-loop attitude controller and an outer-loop flight management system. For the model validation analysis in this paper, only the inner-loop controller is considered. The objective of the inner-loop system is to track desired attitude pitch and roll angles of the aircraft, while damping out oscillations present in the open-loop dynamics. The flight test results used to verify controller performance are shown in Figure 4. The results are compared to the nominal simulation.

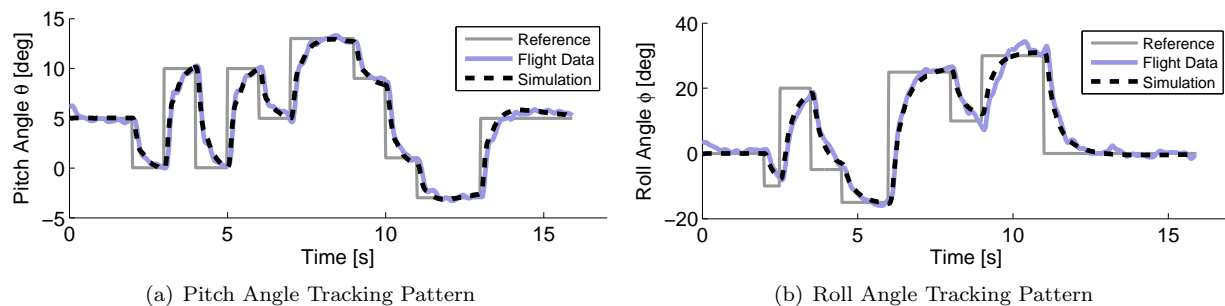


Figure 4. Inner-loop control performance flight test vs. simulation.

Qualitatively, the results in Figure 4 show that the simulation model is able to accurately predict aircraft behavior. However, the model must be validated more rigorously using the proposed TIC analysis. The application example in this paper focuses on the lateral/directional. To account for possible cross-coupling

effects, a 2-by-2 input uncertainty matrix is used for the aileron and rudder channels. 25% multiplicative input uncertainty is considered in the direct channels, and 10% in the cross-coupling channels. Additionally, input disturbances are modeled by random noise signals with a 5 degree amplitude, roughly capturing the effects of turbulence.

B. Validation Analysis

A set of repeated flight experiments, similar to those presented in Figure 4, were performed to obtain data for model validation. A step-type pattern was commanded as the roll angle reference. The collected flight test results are superposed and shown together in Figure 5. Randomly sampling the uncertainty, 1000 Monte Carlo simulations were performed. The Monte Carlo simulation results are shown in the background, while the nominal simulation result is shown on top.

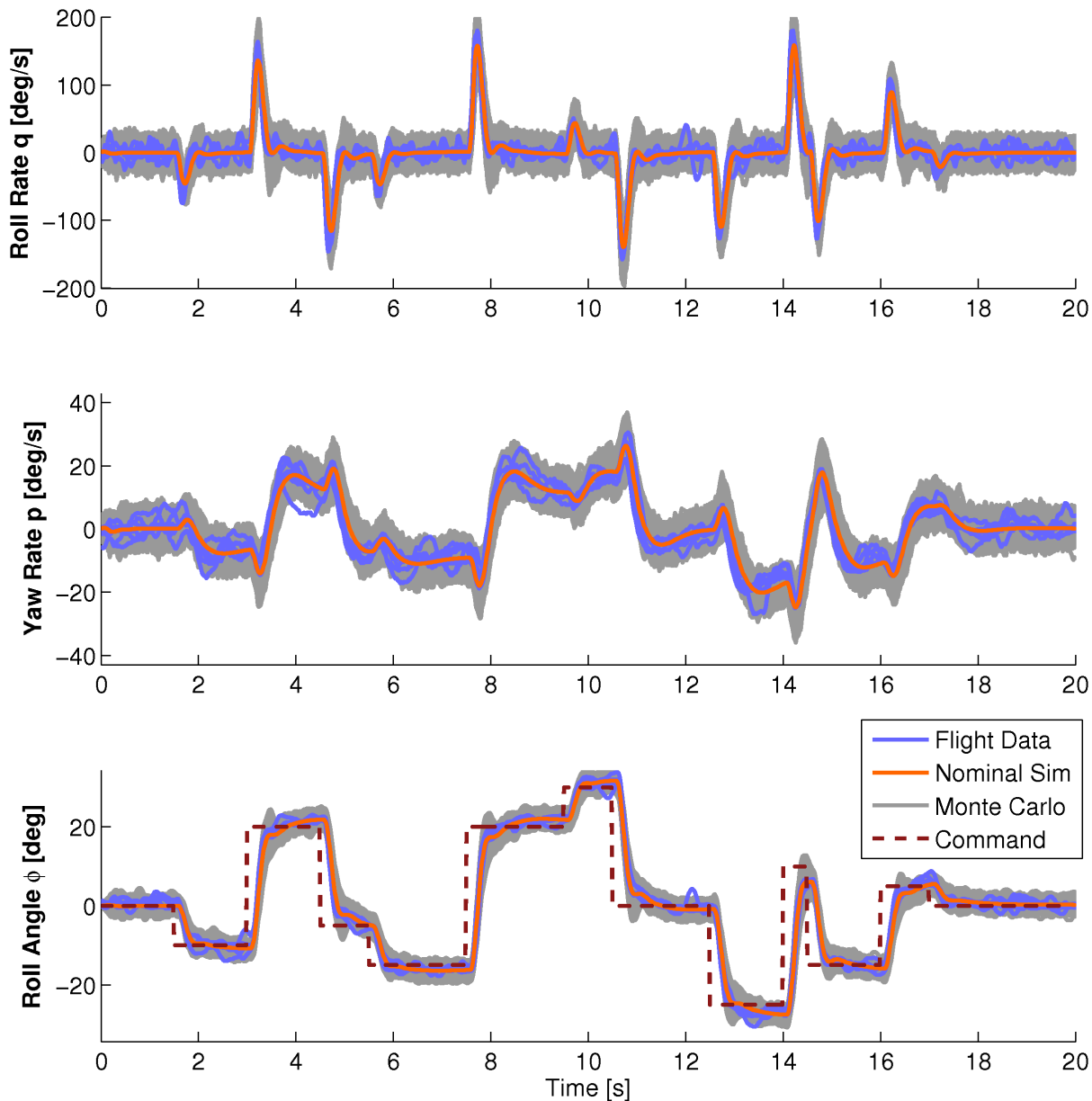


Figure 5. Monte Carlo simulation results versus flight data.

The TIC is computed between each Monte Carlo run and the nominal simulation result. The collected

distributions of TIC values, as predicted by the uncertain model, are shown as histograms in Figure 6. The roll rate, yaw rate, and roll angle distributions are shown separately. TIC values for each flight test result versus the nominal simulation are highlighted by the markers.

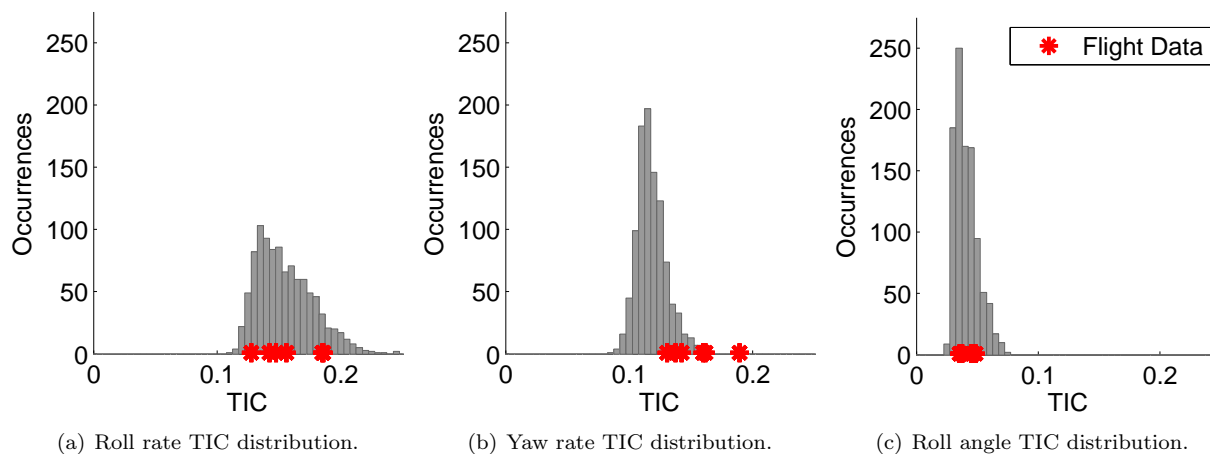


Figure 6. Monte Carlo TIC distribution results versus flight data TIC.

The TIC distribution results provide definitive insights into the accuracy of the uncertain simulation model. In the roll axis, the model is very accurate. The accuracy is qualitatively noted by the close match of the time histories shown in Figure 5. The TIC results in Figures 6(a) and 6(c), however, provide rigorous quantitative evidence of this match. It is clear that the Monte Carlo simulation TIC values bound the TIC values computed from the flight data. On the contrary, the opposite is true for the yaw axis. The time history results in Figure 5 qualitatively suggest that the simulation is accurate in this axis. However, the TIC results in Figure 6(b) clearly indicate that the flight data TIC exceeds that of the Monte Carlo simulations. In this case, the yaw rate model cannot be validated. It appears that the turbulence in the flight data exceeds the amount of random variation predicted by the simulation. One solution to validate the yaw model is to increase the turbulence entering this axis.

IV. Conclusion

A model validation approach with direct applications to aircraft simulation models was proposed and demonstrated using flight test data. Furthermore, the approach was shown to have rigorous ties to standard robust control metrics, thus increasing confidence in the validity of the technique. Continuing work is required to extend the ties between the Theil Inequality Coefficient and robust control analysis to multiple-input, multiple-output linear systems. Even in its infancy, however, the proposed approach shows great potential in simplifying and unifying model validation within the aerospace community.

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