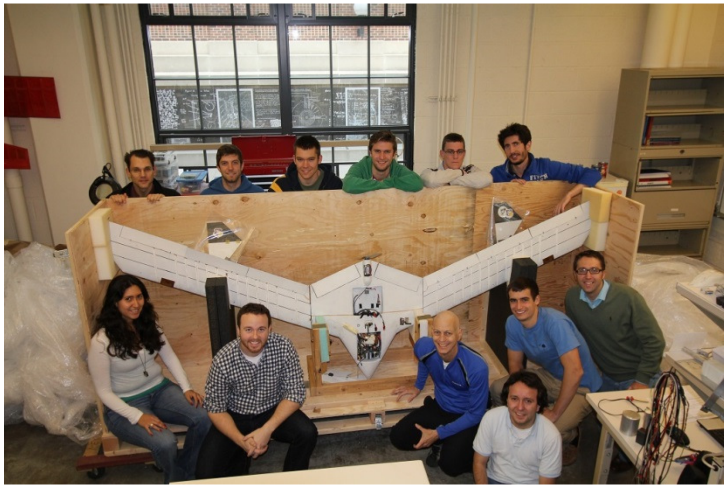
Design and Analysis of Safety Critical Systems

Peter Seiler and Bin Hu
Department of Aerospace Engineering & Mechanics
University of Minnesota

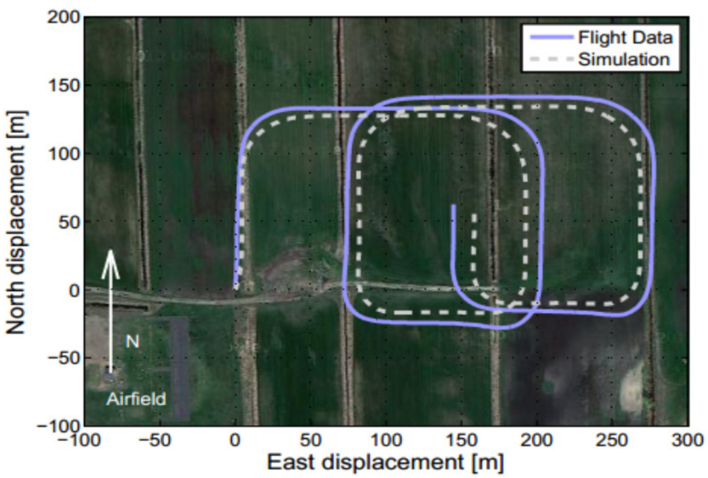
September 30, 2013

Uninhabited Aerial Systems (UAS)



Flight Research (UMN UAV Lab)
http://www.uav.aem.umn.edu/

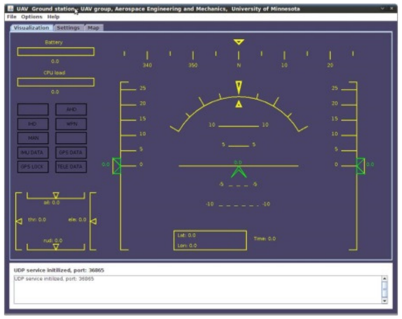
Public Safety (AeroVironment)



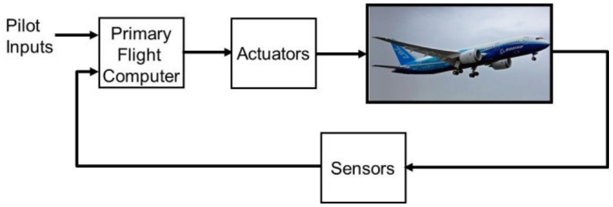
Agricultural Monitoring

Emergency Response (NASA/JPL)

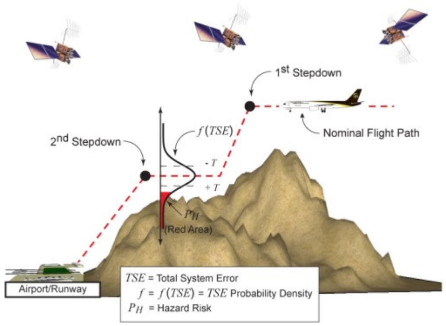
Design Challenges for Low-Cost UAS



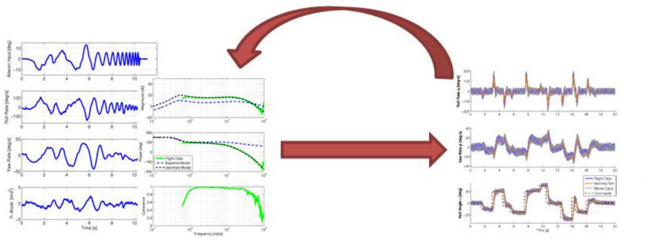
Human Factors



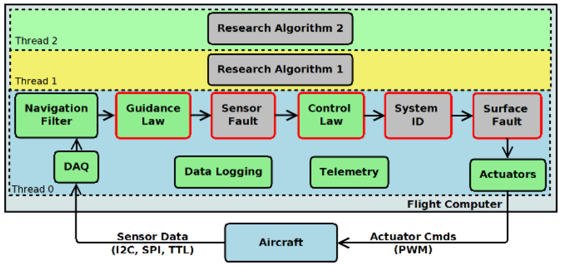
Guidance and Controls



Navigation

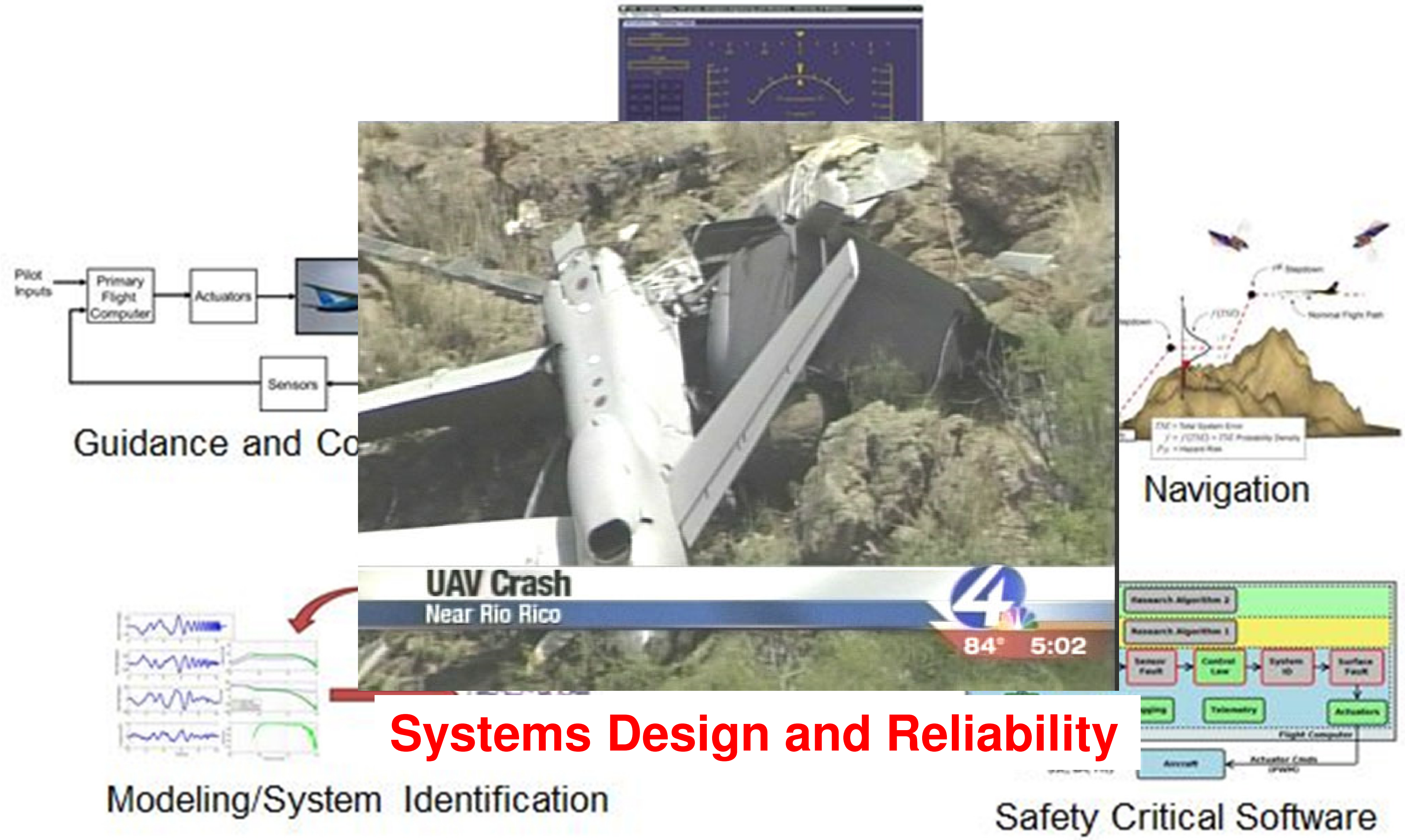


Modeling/System Identification



Safety Critical Software

Design Challenges for Low-Cost UAS



Recent Policy Changes

H. R. 658

One Hundred Twelfth Congress
of the
United States of America

AT THE SECOND SESSION

Begun and held at the City of Washington on Tuesday, the third day of January, two thousand and twelve

Increased reliability needed to integrate UAS into the national airspace

SEC. 332. INTEGRATION OF CIVIL UNMANNED AIRCRAFT SYSTEMS INTO NATIONAL AIRSPACE SYSTEM.

(a) REQUIRED PLANNING FOR INTEGRATION.—

(1) COMPREHENSIVE PLAN.—Not later than 270 days after the date of enactment of this Act, the Secretary of Transportation, in consultation with representatives of the aviation industry, Federal agencies that employ unmanned aircraft systems technology in the national airspace system, and the unmanned aircraft systems industry, shall develop a comprehensive plan to safely accelerate the integration of civil unmanned aircraft systems into the national airspace system.

Outline

- Existing design techniques in commercial aviation
  - Analytical redundancy is rarely used
  - Certification issues
- Tools for Systems Design and Certification
  - Motivation for model-based fault detection and isolation (FDI)
  - Extended fault trees
  - Stochastic false alarm and missed detection analysis
- Conclusions and future work

Outline

- **Existing design techniques in commercial aviation**
  - Analytical redundancy is rarely used
  - Certification issues
- Tools for Systems Design and Certification
  - Motivation for model-based fault detection and isolation (FDI)
  - Extended fault trees
  - Stochastic false alarm and missed detection analysis
- Conclusions and future work

Commercial Fly-by-Wire

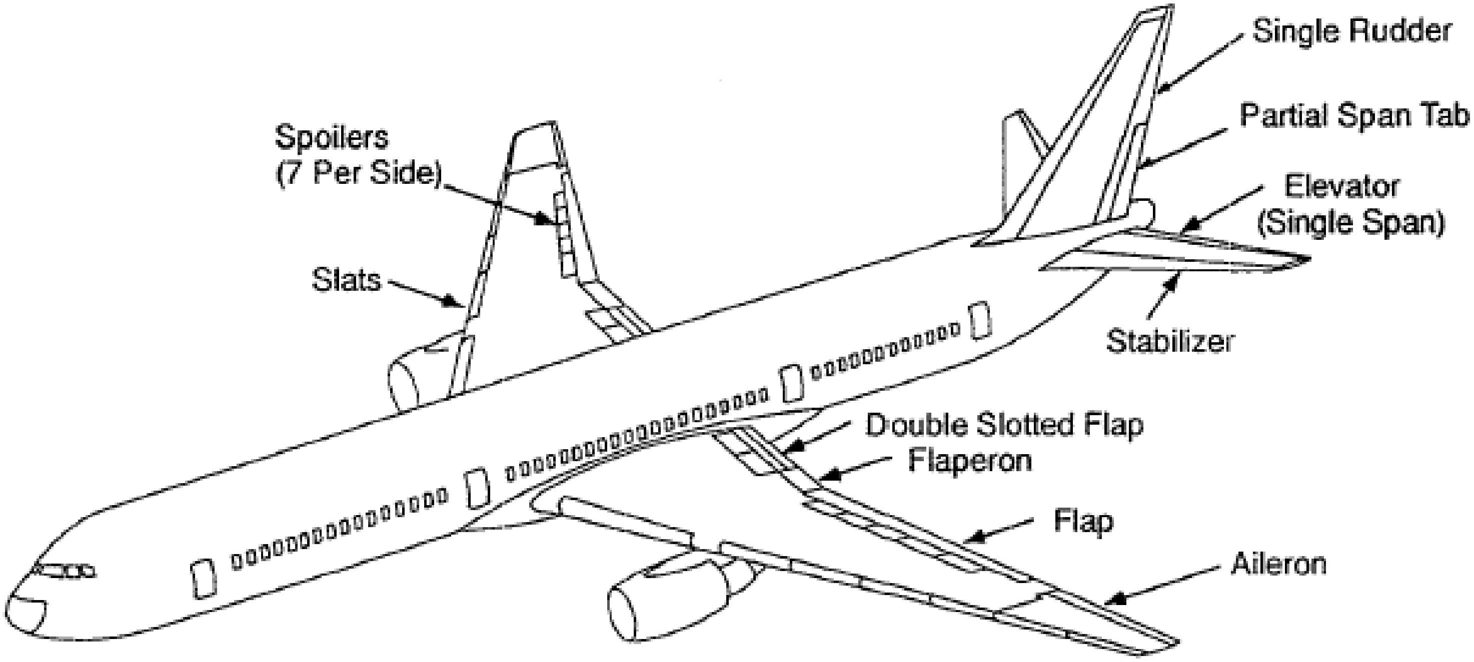
Boeing 787-8 Dreamliner

- 210-250 seats
- Length=56.7m, Wingspan=60.0m
- Range < 15200km, Speed< M0.89
- First Composite Airliner
- Honeywell Flight Control Electronics

Boeing 777-200

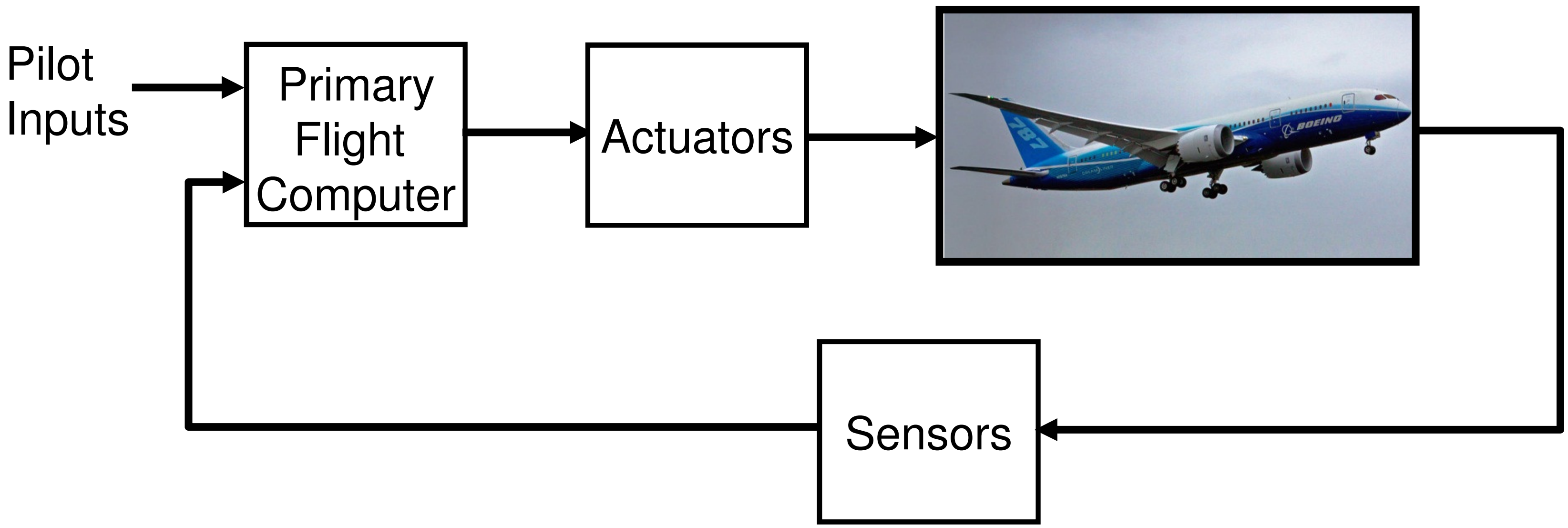
- 301-440 seats
- Length=63.7m, Wingspan=60.9m
- Range < 17370km, Speed< M0.89
- Boeing's 1st Fly-by-Wire Aircraft
- Ref: Y.C. Yeh, "Triple-triple redundant 777 primary flight computer," 1996.

777 Primary Flight Control Surfaces [Yeh, 96]



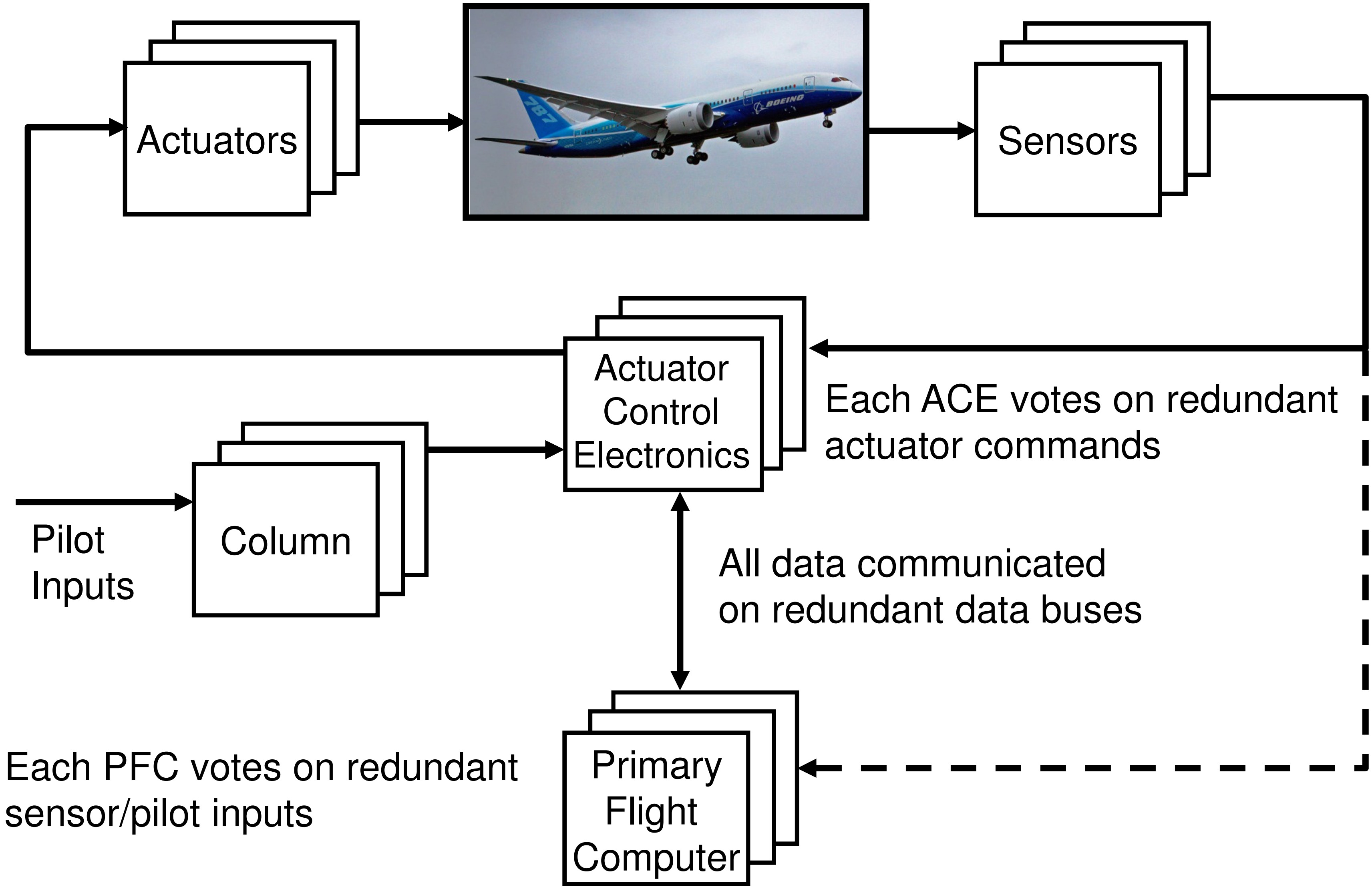
- Advantages of fly-by-wire:
  - Increased performance (e.g. reduced drag with smaller rudder), increased functionality (e.g. “soft” envelope protection), reduced weight, lower recurring costs, and possibility of sidesticks.
- Issues: Strict reliability requirements
  - $<10^{-9}$ catastrophic failures/hr
  - No single point of failure

Classical Feedback Diagram

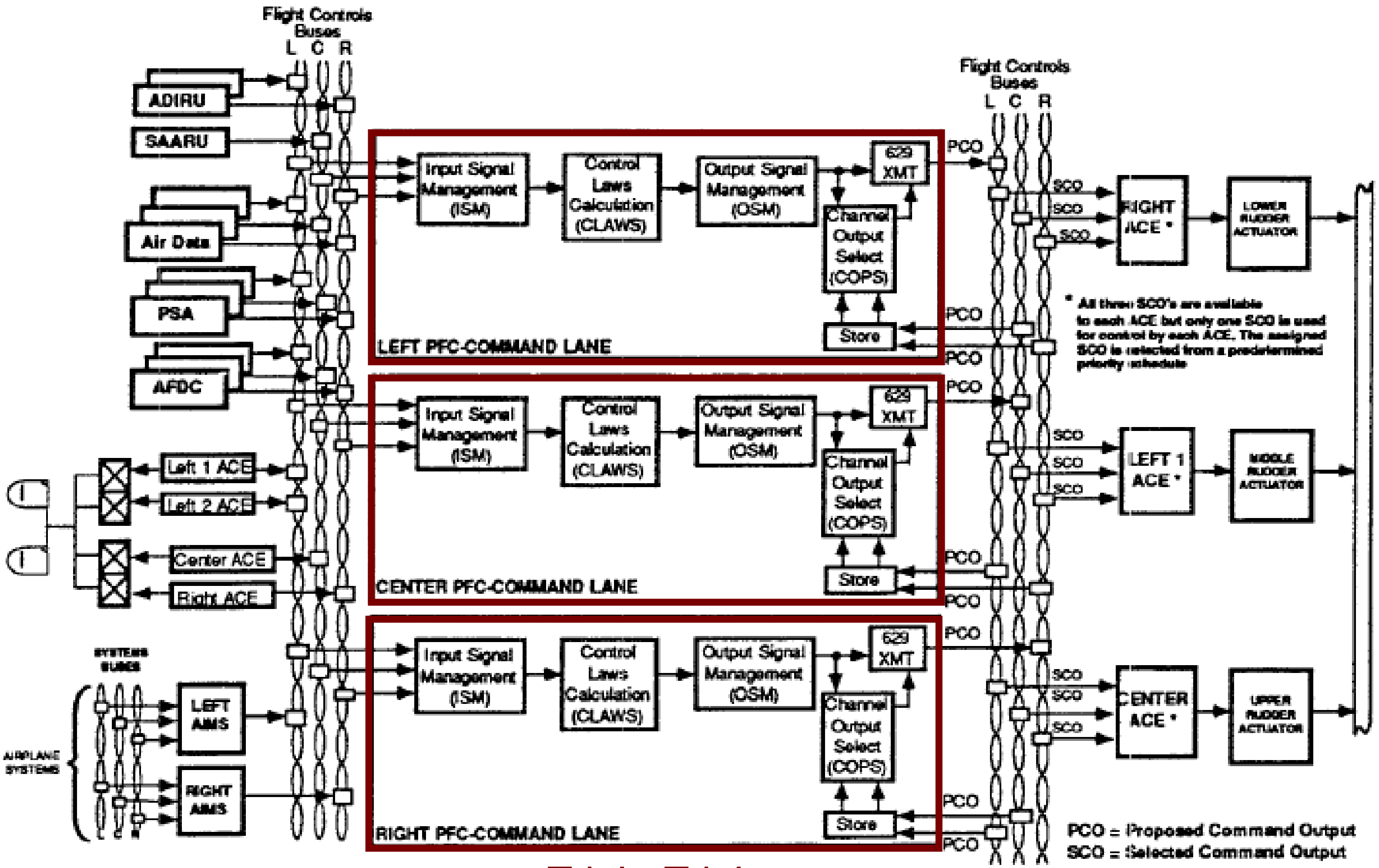


Reliable implementation of this classical feedback loop adds many layers of complexity.

Triplex Control System Architecture



777 Triple-Triple Architecture [Yeh, 96]



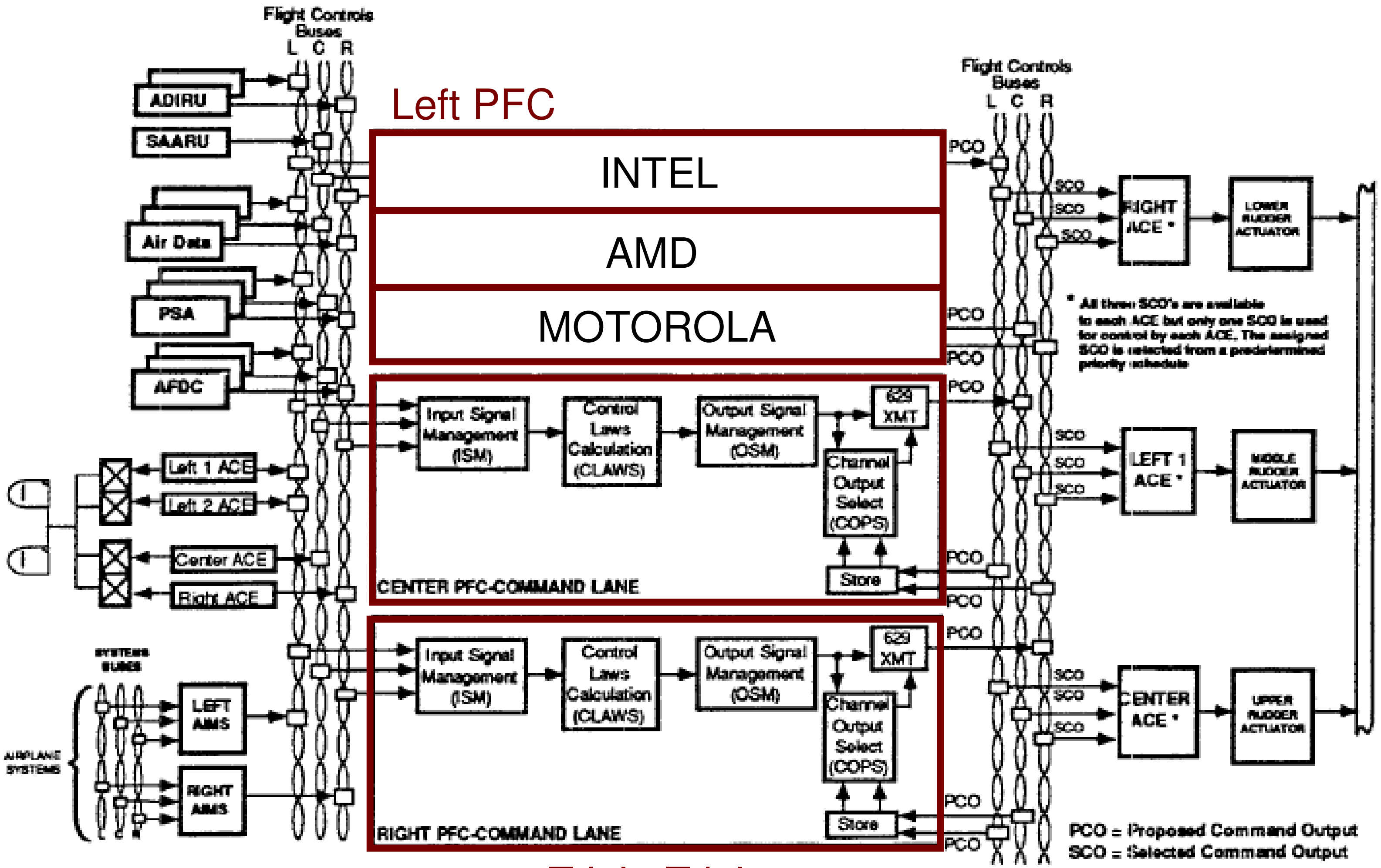
Sensors x3

Databus x3

Triple-Triple Primary Flight Computers

Actuator Electronics x4

777 Triple-Triple Architecture [Yeh, 96]



Sensors x3

Databus x3

Triple-Triple Primary Flight Computers

Actuator Electronics x4

Redundancy Management

- Main Design Requirements:
  - $< 10^{-9}$ catastrophic failures per hour
  - No single point of failure
  - Must protect against random and common-mode failures
- Basic Design Techniques
  - Hardware redundancy to protect against random failures
  - Dissimilar hardware / software to protect against common-mode failures
  - Voting: To choose between redundant sensor/actuator signals
  - Encryption: To prevent data corruption by failed components
  - Monitoring: Software/Hardware monitoring testing to detect latent faults
  - Operating Modes: Degraded modes to deal with failures
  - Equalization to handle unstable / marginally unstable control laws
  - Model-based design and implementation for software

Redundancy Management

- Main Design Requirements:
  - $< 10^{-9}$ catastrophic failures per hour
  - No single point of failure
  - Must protect against random and common-mode failures

- Basic Design Techniques
  - Hardware redundancy to protect against random failures
  - Dissimilar hardware / software to protect against common-mode failures
  - Voting: To choose between redundant sensor/actuator signals
  - Encryption: To prevent data corruption by failed components
  - Monitoring: Software/Hardware monitoring testing to detect latent faults
  - Operating Modes: Degraded modes to deal with failures
  - Equalization to handle unstable / marginally unstable control laws
  - Model-based design and implementation for software

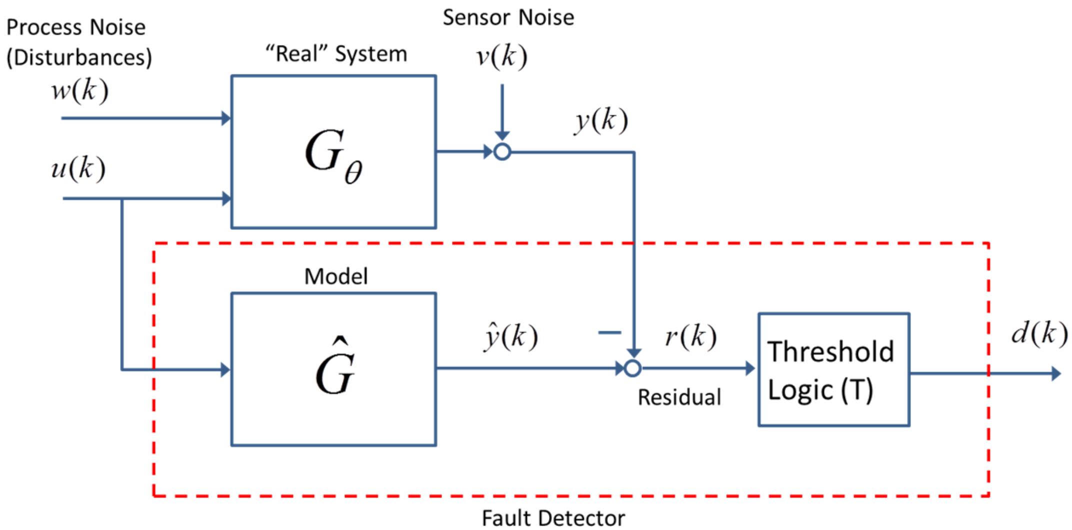
Outline

- Existing design techniques in commercial aviation
  - Analytical redundancy is rarely used
  - Certification issues
- **Tools for Systems Design and Certification**
  - Motivation for model-based fault detection and isolation (FDI)
  - Extended fault trees
  - Stochastic false alarm and missed detection analysis
- Conclusions and future work

Analytical Redundancy

Small UASs cannot support the weight associated with physical redundancy.

Approach: Use model-based or data-driven techniques to detect faults.



Parity-equation architecture (Wilsky)

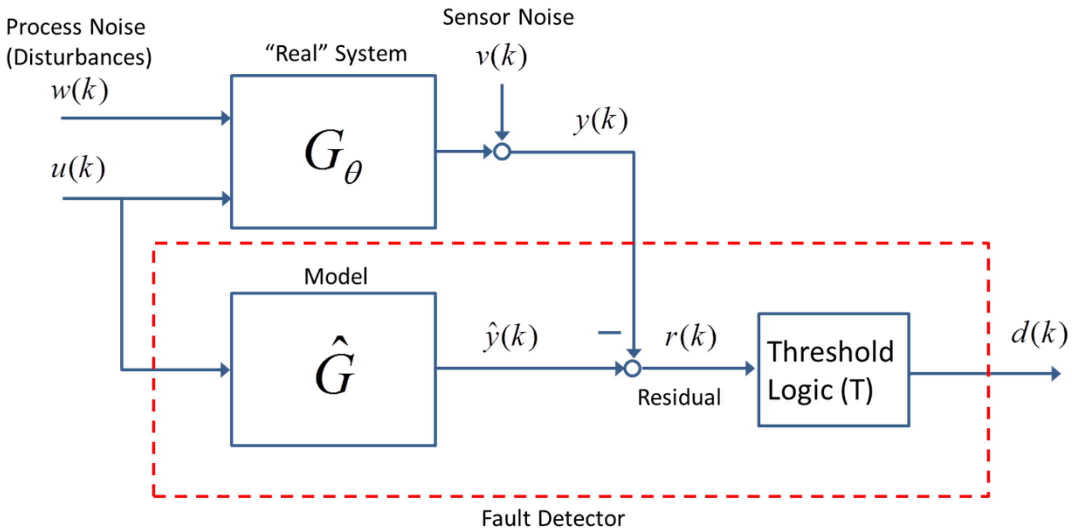
Analytical Redundancy

Small UASs cannot support the weight associated with physical redundancy.

Approach: Use model-based or data-driven techniques to detect faults.

Research Objectives:

- Hardware, models, data (Freeman, Balas)
- Advanced filter design
- Tools for systems design, analysis and certification



Parity-equation architecture (Wilsky)

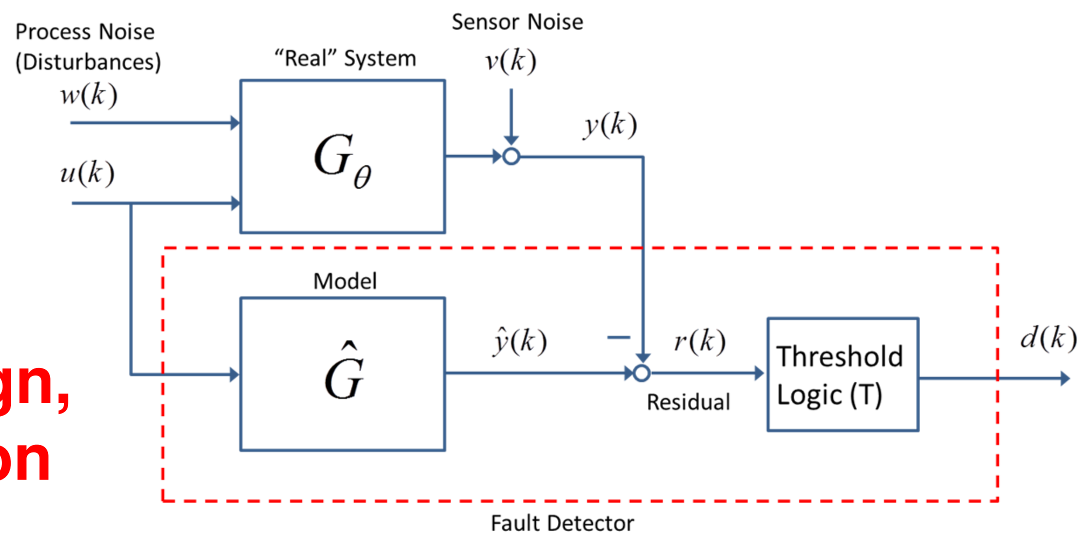
Analytical Redundancy

Small UASs cannot support the weight associated with physical redundancy.

Approach: Use model-based or data-driven techniques to detect faults.

Research Objectives:

- Hardware, models, data (Freeman, Balas)
- Advanced filter design
- **Tools for systems design, analysis and certification**



Parity-equation architecture (Wilsky)

Tools for Systems Design and Certification

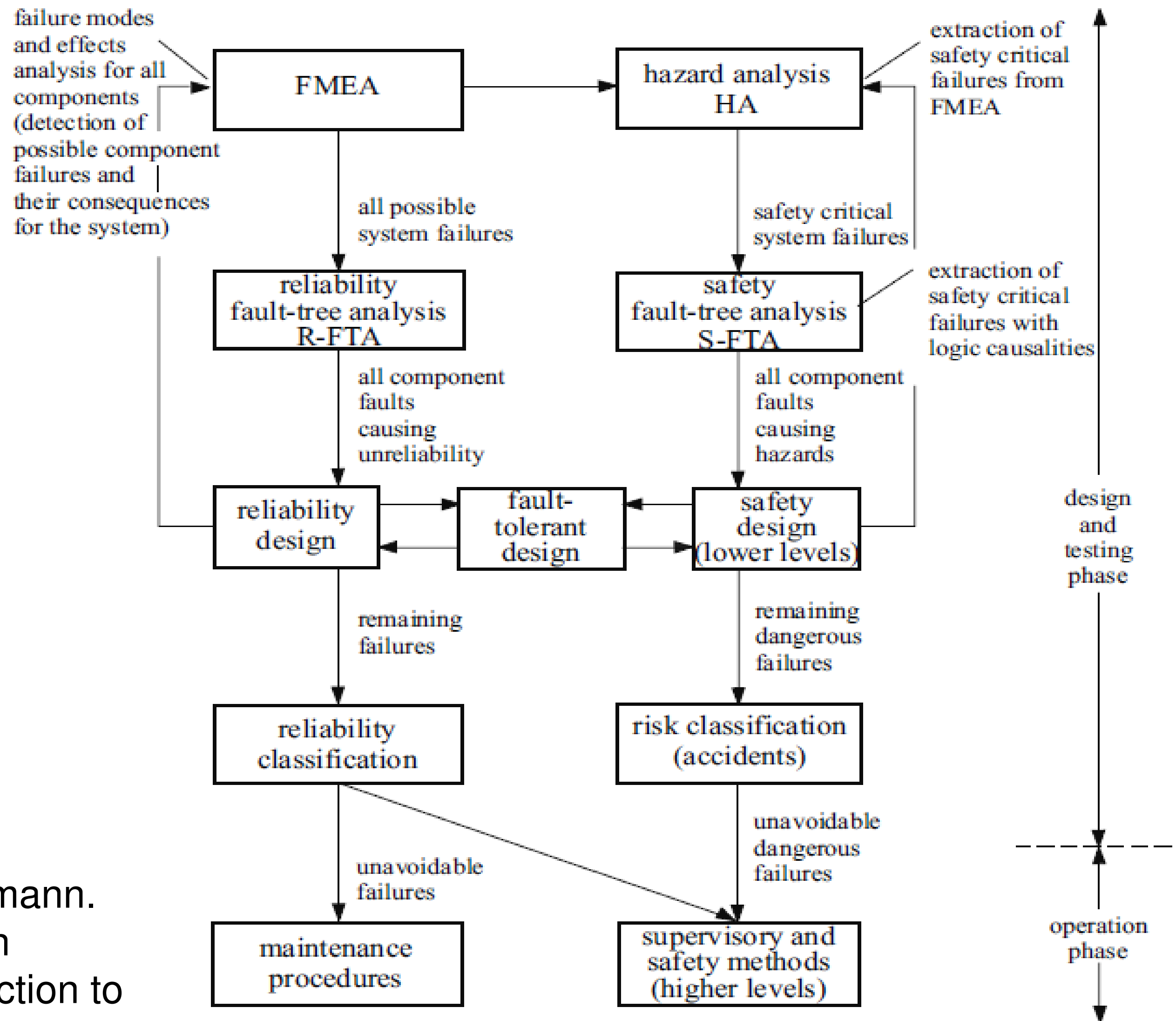
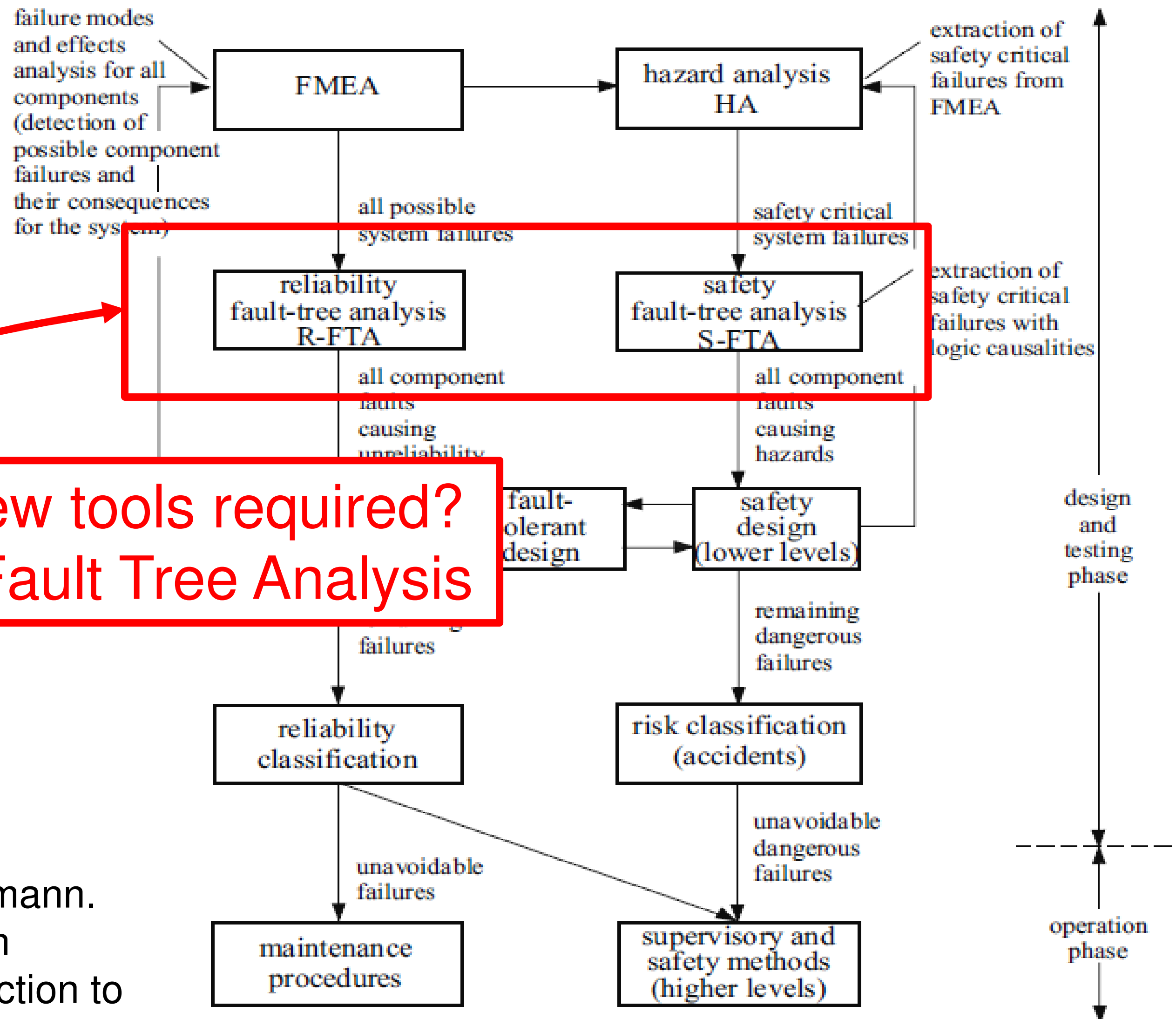


Diagram Reference: R. Isermann. Fault-Diagnosis Systems: An Introduction from Fault Detection to Fault Tolerance. Springer-Verlag, 2006.

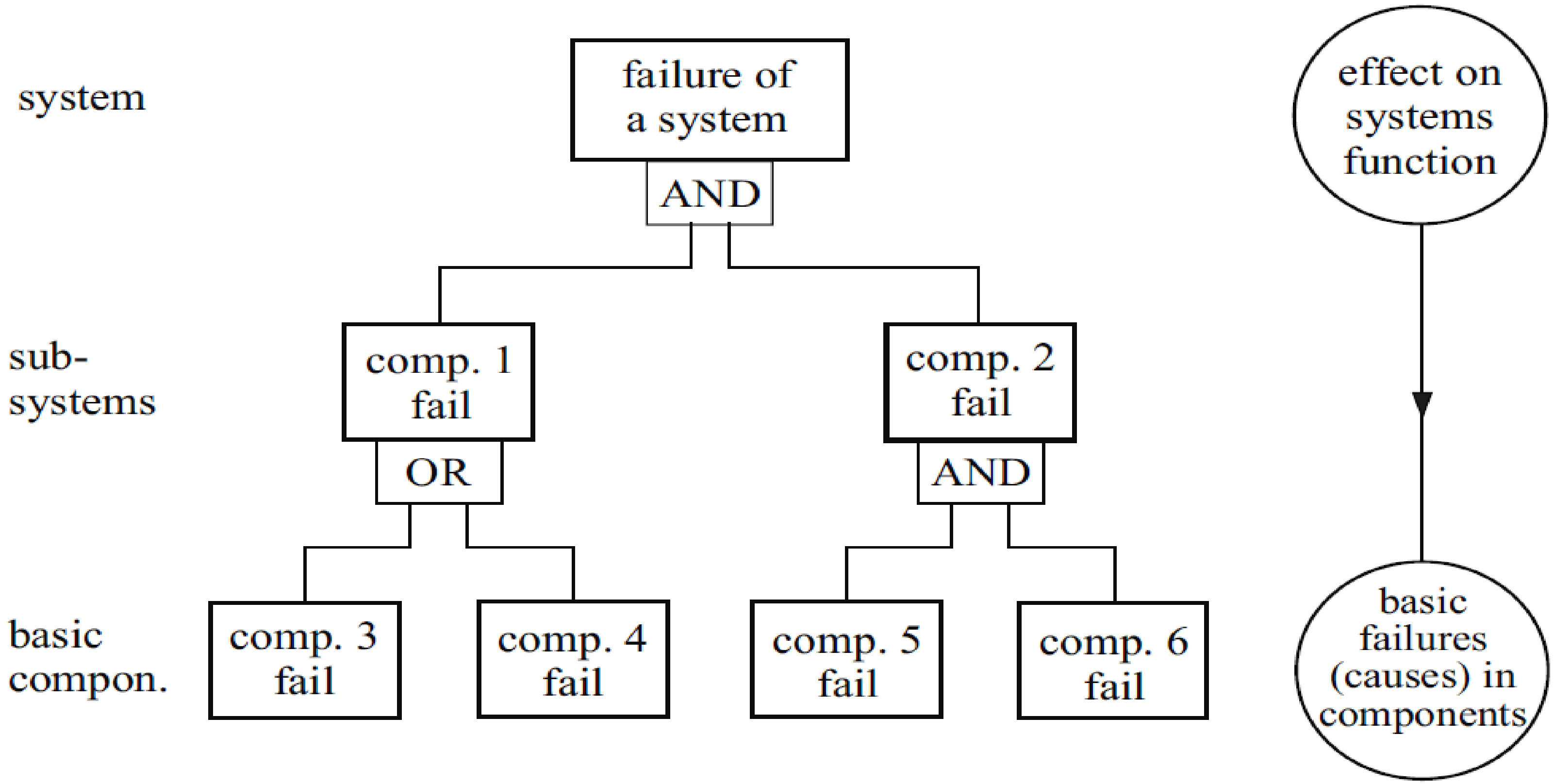
Tools for Systems Design and Certification



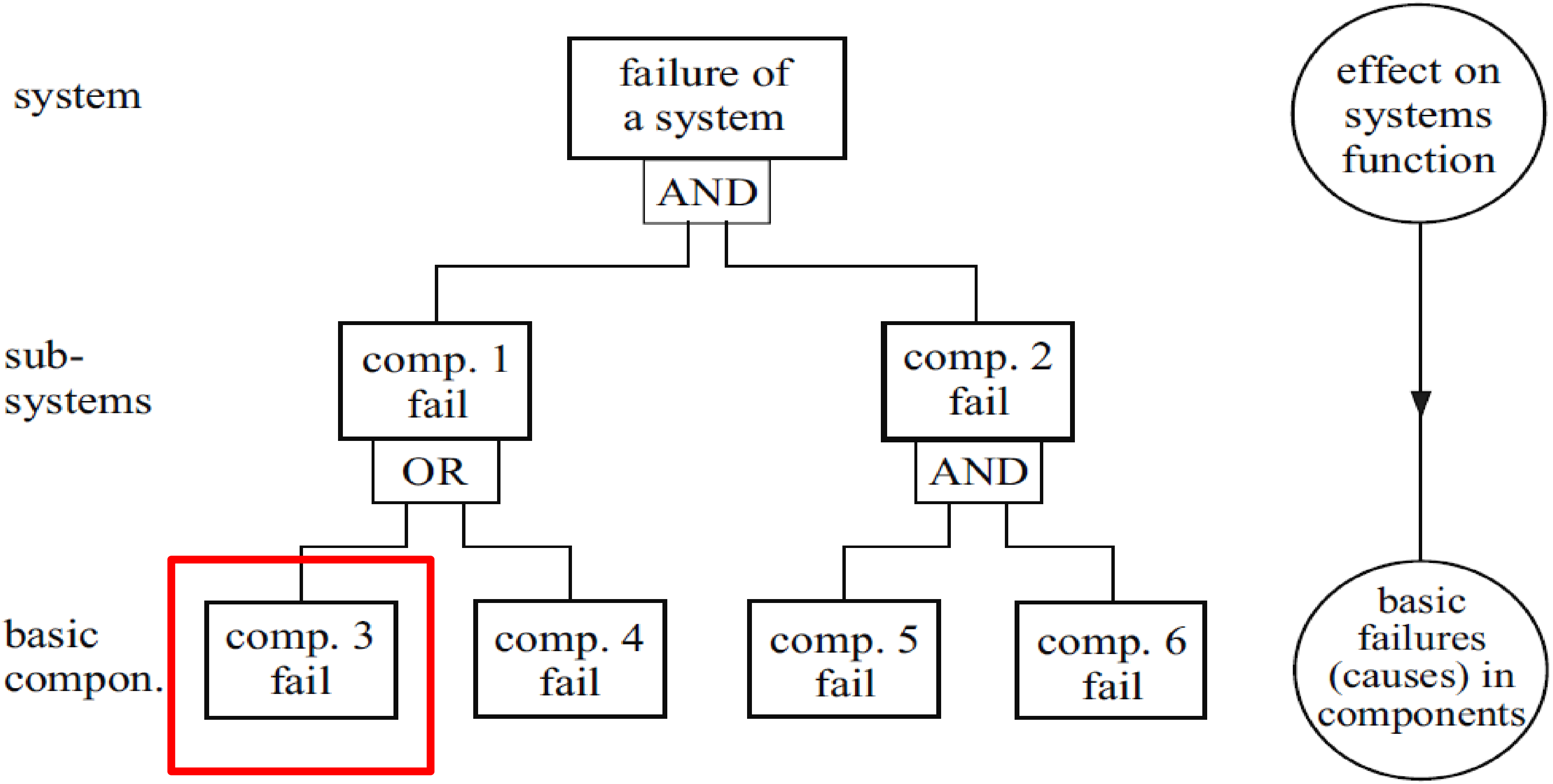
Why are new tools required?
Example: Fault Tree Analysis

Diagram Reference: R. Isermann. Fault-Diagnosis Systems: An Introduction from Fault Detection to Fault Tolerance. Springer-Verlag, 2006.

Fault Tree Analysis

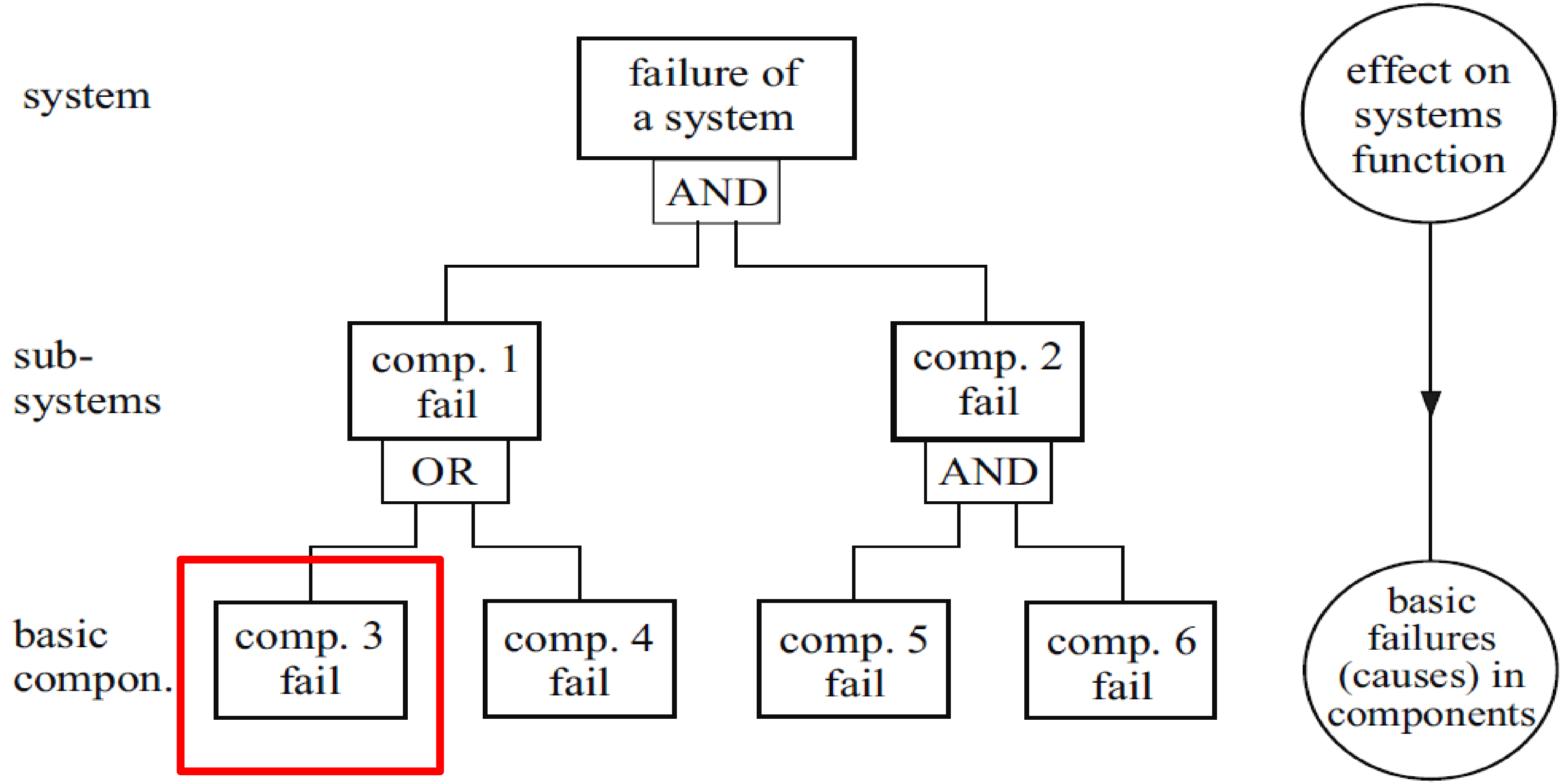


Fault Tree Analysis



Probability of hardware component failure can be estimated from field data.

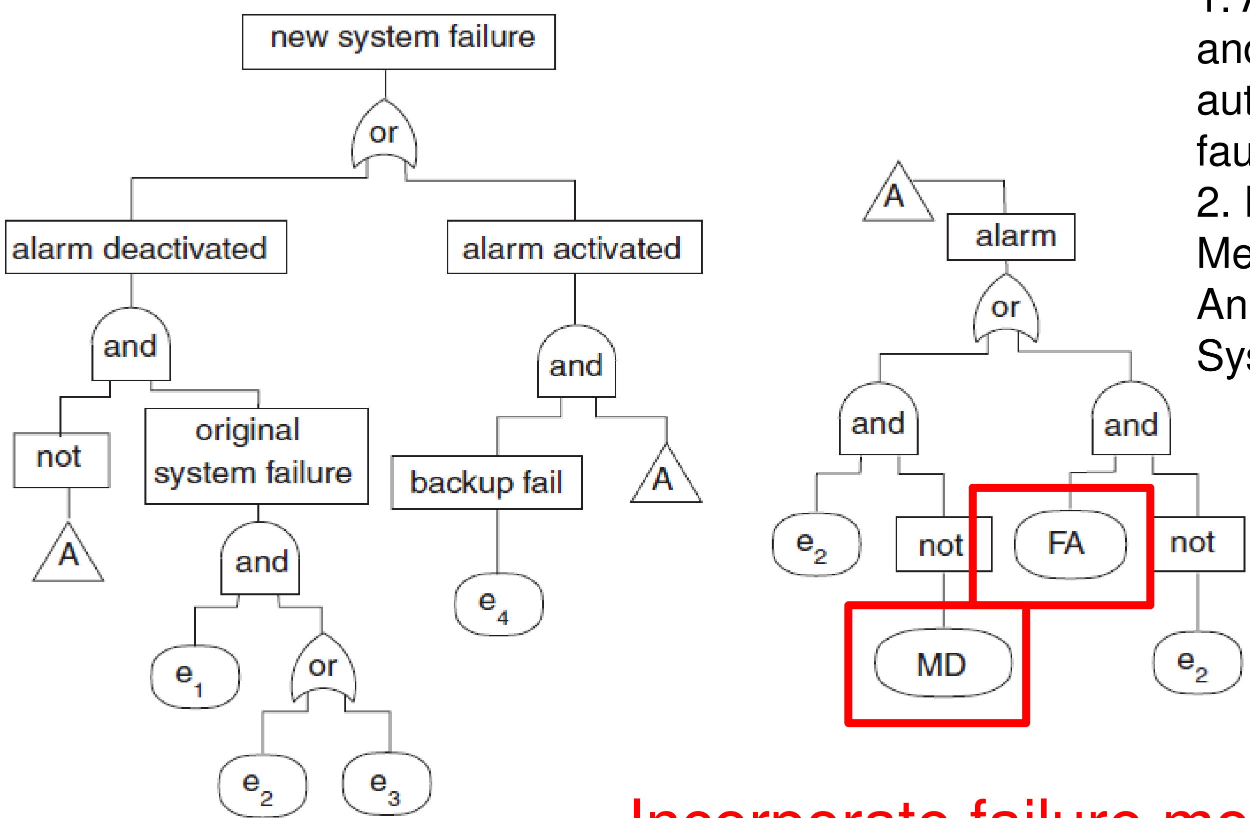
Fault Tree Analysis



Probability of hardware component failure can be estimated from field data.

Model-based fault detection introduces new failure models (false alarms, missed detections, etc.)

Extended Fault Tree Analysis

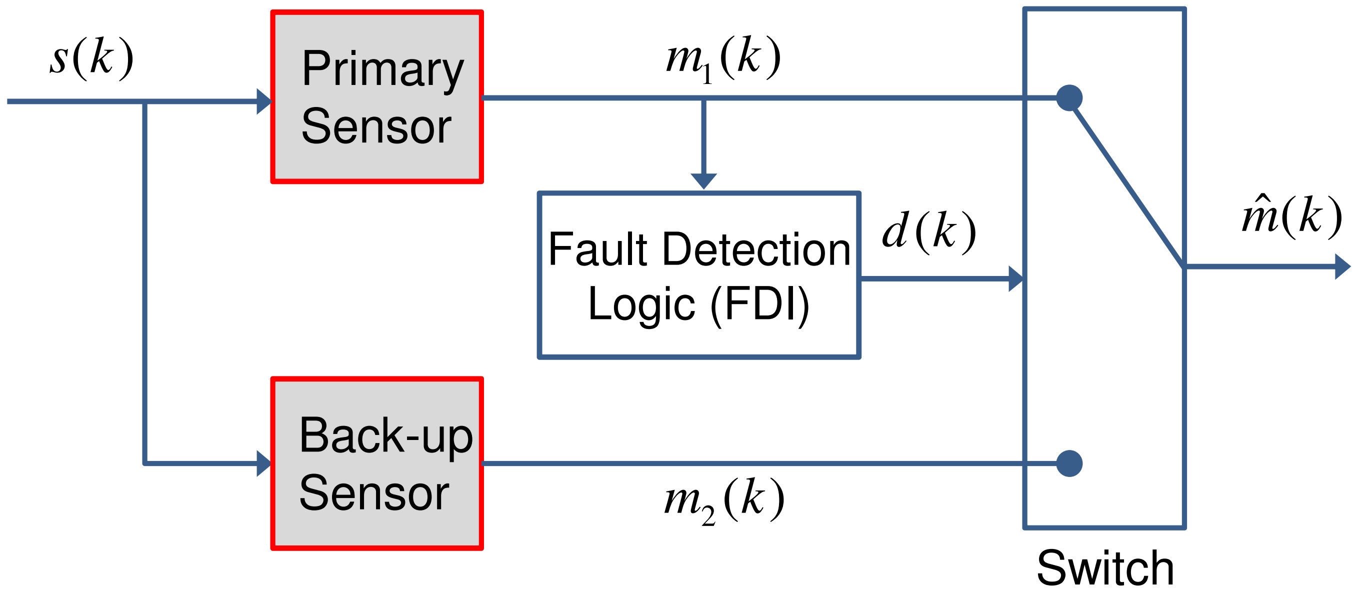


References
1. Aslund, Biteus, Frisk, Krysander, and Nielsen. Safety analysis of autonomous systems by extended fault tree analysis. IJACSP, 2007.
2. Hu and Seiler, A Probabilistic Method for Certification of Analytically Redundant Systems, SysTol Conference, 2013.

Incorporate failure modes due to false alarms and missed detections (per hour)

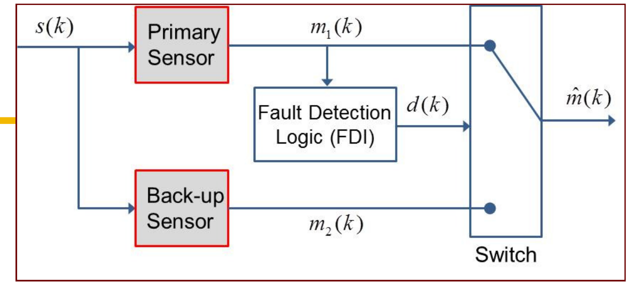
(Enumerate time-correlated failures and apply total law of probability)

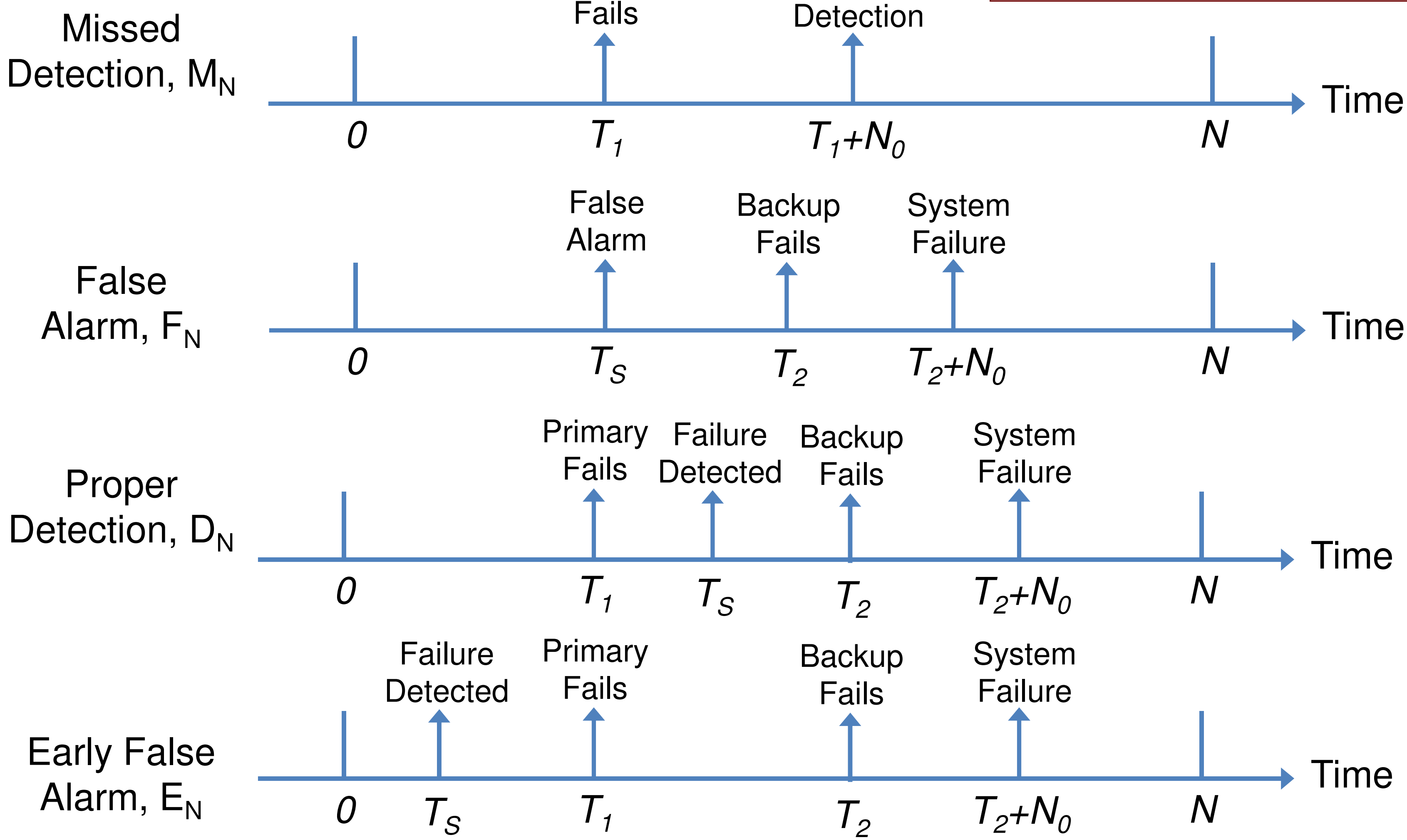
Example: Dual-Redundant Architecture



Objective: Compute reliability of system assuming sensors have a mean-time between failure of 1000Hrs.

Failure Modes





System Failure Rate

- Notation:

$\hat{q}$	Sensor failure per hour
$\hat{P}_F$	False alarm per hour
$\hat{P}_D$	Detection per failure

- Approximate system failure probability:

$$P_{S,N} \approx \hat{q}(1-\hat{P}_D) + \hat{P}_D\hat{q}^2 + \hat{P}_F\hat{q}(1-\hat{q})$$

System Failure Rate

- Notation:

$\hat{q}$ Sensor failure per hour

$\hat{P}_F$ False alarm per hour

$\hat{P}_D$ Detection per failure

- Approximate system failure probability:

$$P_{S,N} \approx \boxed{\hat{q}(1-\hat{P}_D)} + \boxed{\hat{P}_D\hat{q}^2} + \boxed{\hat{P}_F\hat{q}(1-\hat{q})}$$

Primary sensor fails + missed detection

Failure detected + Backup sensor fails

False alarm + Backup sensor fails

System Failure Rate

- Notation: $\hat{q}$ Sensor failure per hour

  $\hat{P}_F$ False alarm per hour

  $\hat{P}_D$ Detection per failure

Question: How can we compute these probabilities?

- Approximate system failure probability:

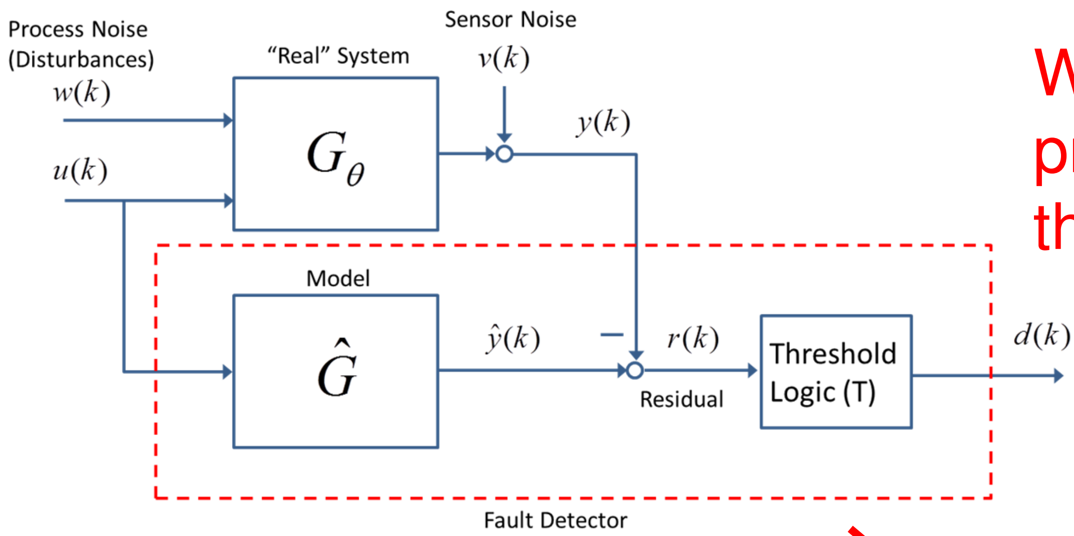
$$P_{S,N} \approx \boxed{\hat{q}(1-\hat{P}_D)} + \boxed{\hat{P}_D \hat{q}^2} + \boxed{\hat{P}_F \hat{q}(1-\hat{q})}$$

Primary sensor fails + missed detection

Failure detected + Backup sensor fails

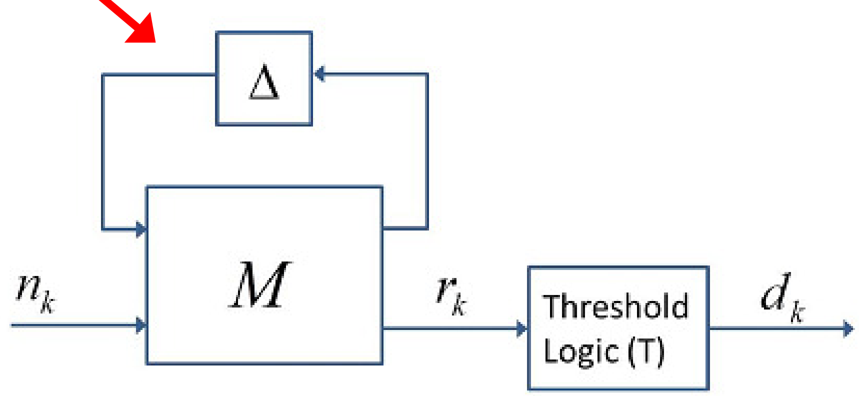
False alarm + Backup sensor fails

False Alarm Analysis



What is the conditional probability of an alarm given that no fault has occurred?

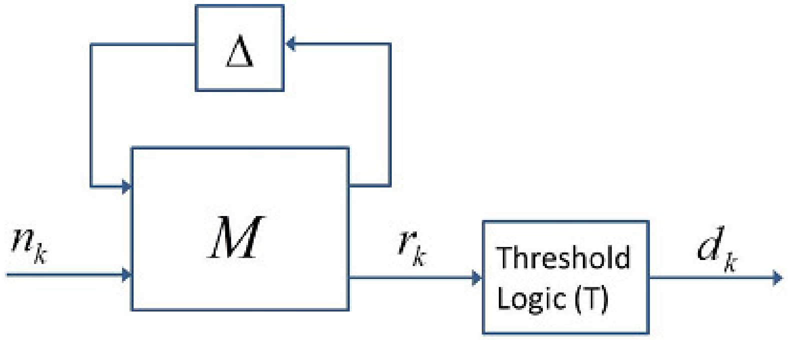
Abstraction: Discrete-time uncertain linear system driven by noise.



Problem Formulation

(Healthy) Dynamics for residual

$$x_{k+1} = Ax_k + Bn_k$$
$$r_k = Cx_k + Dn_k$$



Simple Thresholding

$$d_k := \begin{cases} 0 & \text{if } |r_k| \leq T \\ 1 & \text{else} \end{cases}$$

Objective:

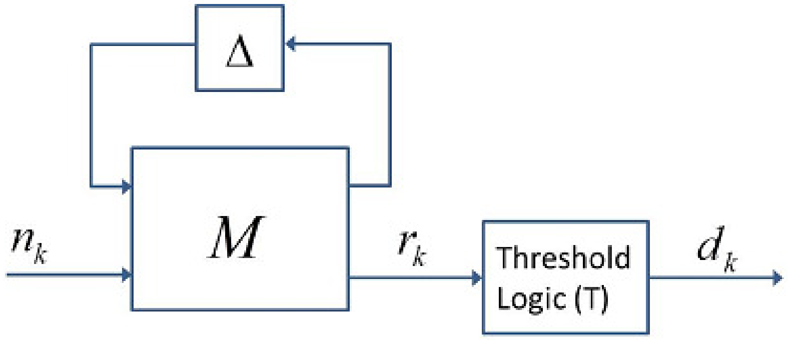
Assume n_k is a stationary Gaussian process and assume known dynamic model for residuals.

Compute the probability P_N that $|r_k| > T$ for some k in $\{1,\ldots,N\}$.

Problem Formulation

(Healthy) Dynamics for residual

$$x_{k+1} = Ax_k + Bn_k$$
$$r_k = Cx_k + Dn_k$$



Simple Thresholding

$$d_k := \begin{cases} 0 & \text{if } |r_k| \leq T \\ 1 & \text{else} \end{cases}$$

References

1. Glaz and Johnson. Probability inequalities for multivariate distributions with dependence structures. JASA, 1984
2. Hu and Seiler, Probability Bounds for False Alarm Analysis of Fault Detection Systems, Allerton, 2013.

Theorem:

There exist bounds γ_k ($k=1,\dots,N$) such that

1. $\gamma_k \geq P_N$
2. γ_k are monotonically non-increasing in k
3. γ_k requires evaluation of k-dim. Gaussian integrals

Results: Effects of Correlation

False Alarm Probabilities and Bounds for N=360,000

Neglecting correlations is accurate for small *a*

…but not for *a* near 1.

a	T	P_N	$1 - L_N^{(2)}$	$1 - L_N^{(1)}$
0	6.807	3.600×10^{-6}	3.600×10^{-6}	3.600×10^{-6}
0.7	9.531	3.587×10^{-6}	3.587×10^{-6}	3.598×10^{-6}
0.8	11.34	3.524×10^{-6}	3.524×10^{-6}	3.526×10^{-6}
0.9	15.62	3.167×10^{-6}	3.173×10^{-6}	3.200×10^{-6}
0.99	48.25	9.641×10^{-7}	1.177×10^{-6}	1.360×10^{-6}
0.999	152.2	1.395×10^{-7}	3.401×10^{-7}	4.446×10^{-7}

For each (a,T), $P_1 = 10^{-11}$ which gives $NP_1 = 3.6 \times 10^{-6}$

Residual Generation

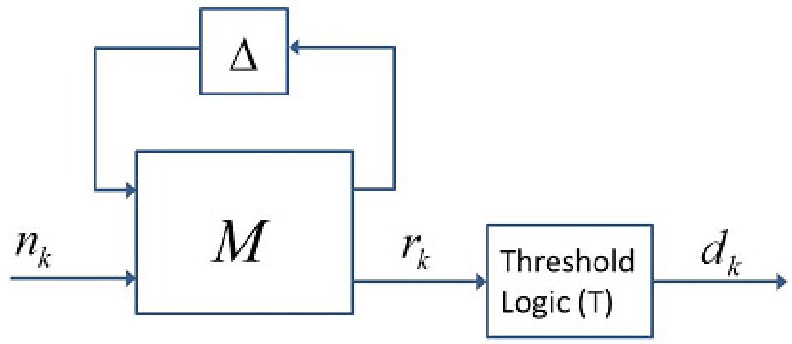
$$r_{k+1} = a r_k + n_k + f_k$$

Decision Logic

$$d_k = \begin{cases} 0 & \text{if } |r_k| \le T \\ 1 & \text{else} \end{cases}$$

Worst-case False Alarm Probability

Reference
Hu and Seiler, Worst-Case False Alarm Analysis of Aerospace Fault Detection Systems, Submitted to ACC, 2014.



Issue:
Model depends on unknown (uncertain) parameters, $\Delta \in \mathbf{\Delta}$.

Objective:
Compute the worst-case false alarm probability

$$P_N^* := \max_{\Delta \in \mathbf{\Delta}} P_N(\Delta)$$

Main Result:
Robust H_2 analysis results can be used to compute worst-case residual variance. This yields bounds on P_N*.

Conclusions

- Commercial aircraft achieve high levels of reliability.
  - Analytical redundancy is rarely used (Certification Issues)
  - Model-based fault detection methods are an alternative that enables size, weight, power, and cost to be reduced.
- Tools for Systems Design and Certification
  - Extended fault trees
  - Stochastic false alarm and missed detection analysis
  - Methods to validate analysis using flight test data (Hu and Seiler, 2014 AIAA)

Acknowledgments

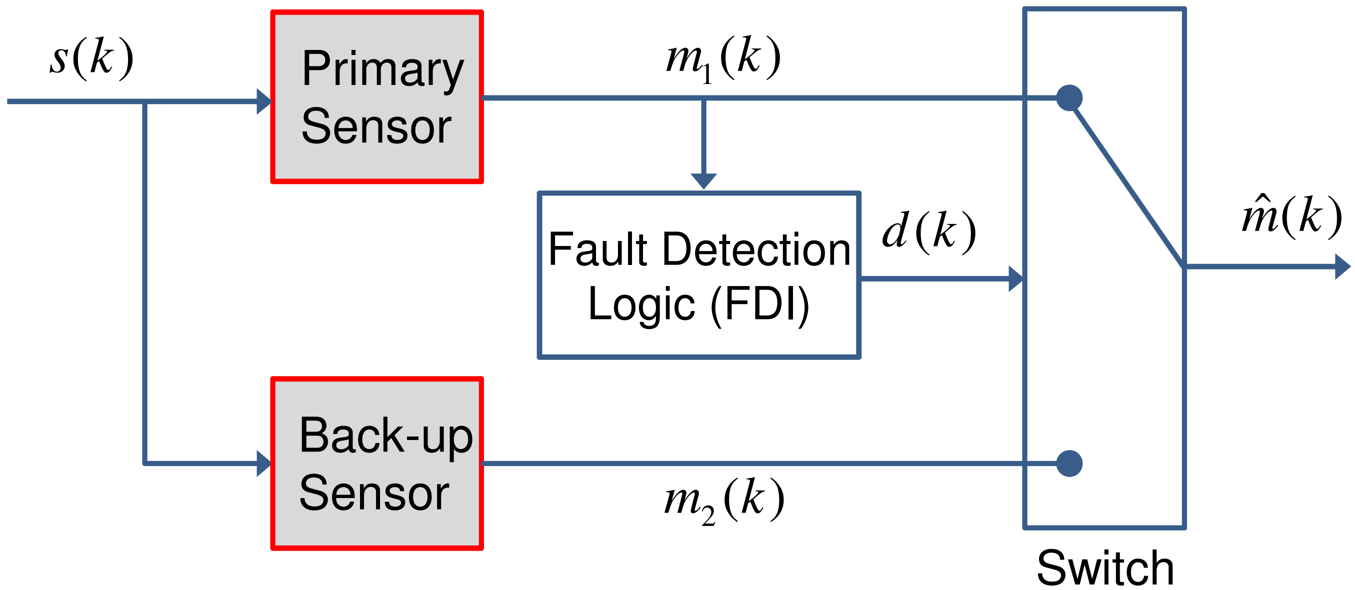
- NASA Langley NRA NNX12AM55A: “Analytical Validation Tools for Safety Critical Systems Under Loss-of-Control Conditions,” Technical Monitor: Dr. Christine Belcastro

- Air Force Office of Scientific Research: Grant No. FA9550-12-0339, "A Merged IQC/SOS Theory for Analysis of Nonlinear Control Systems,” Technical Monitor: Dr. Fariba Fahroo.

- NSF Cyber-Physical Systems: Grant No. 0931931, “Embedded Fault Detection for Low-Cost, Safety-Critical Systems,” Program Manager: Theodore Baker.

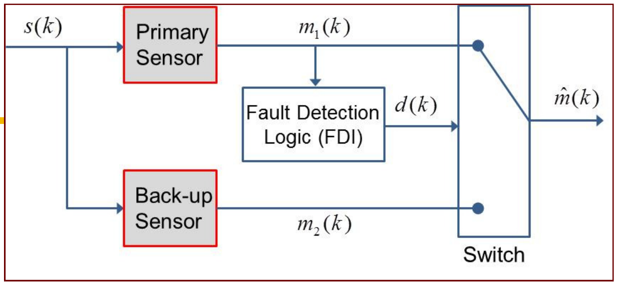
Backup Slides

Dual-Redundant Architecture



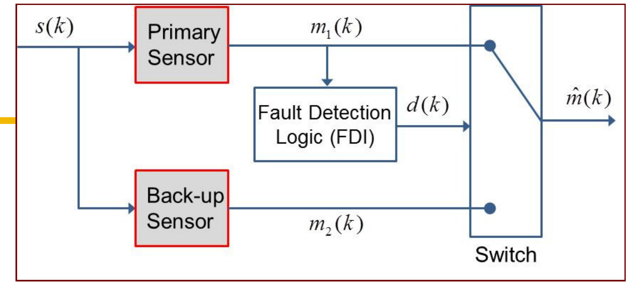
Objective: Efficiently compute the probability $P_{S,N}$ that the system generates "bad" data for N_0 consecutive steps in an N-step window.

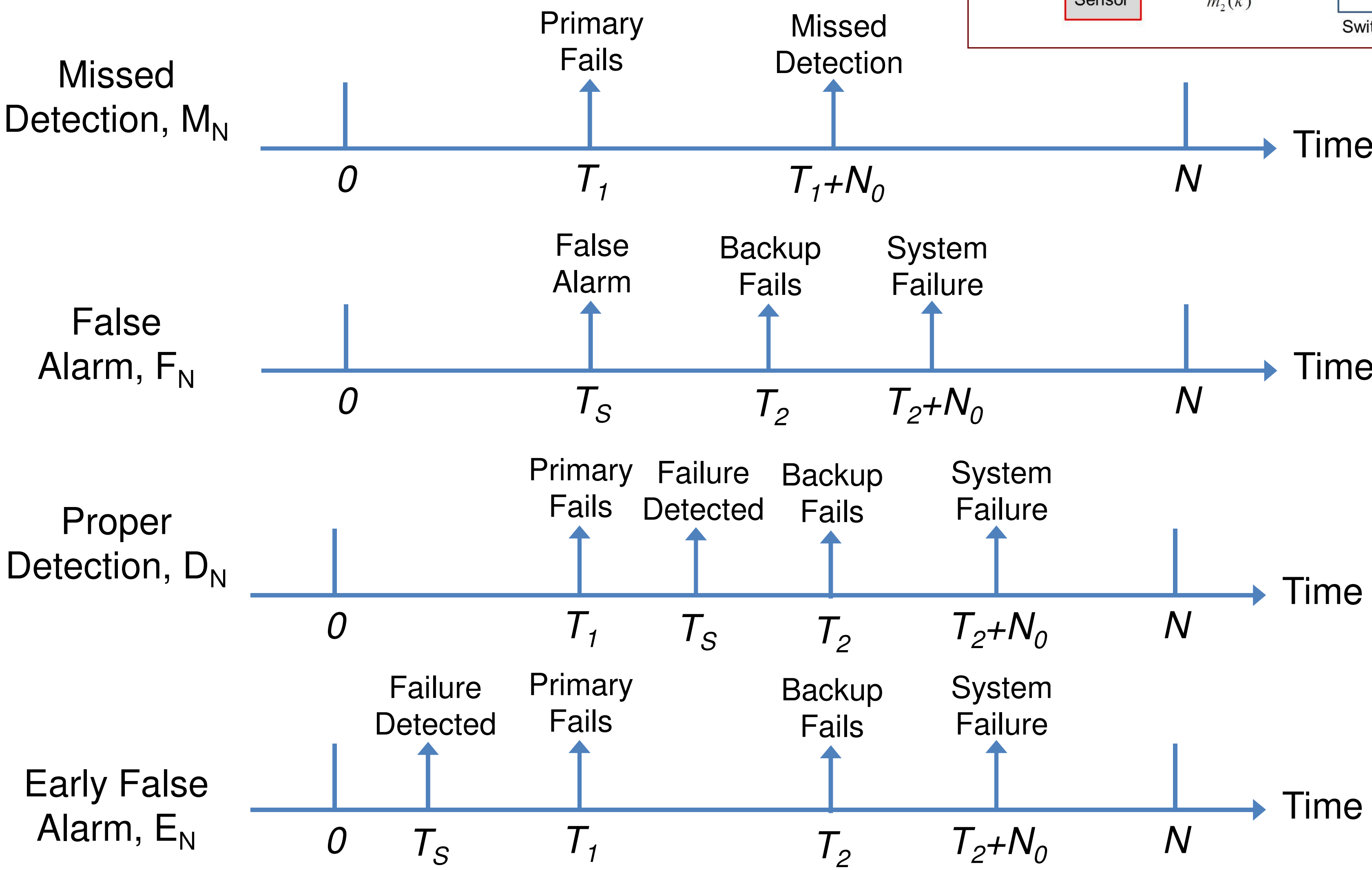
Assumptions



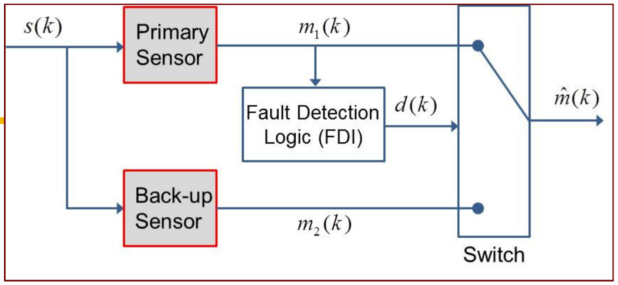
1. Knowledge of probabilistic performance
   a. Sensor failures: $P[\ T_i{=}k\ ]$ where T_i := failure time of sensor i
   b. FDI False Alarm: $P[\ T_S \leq N \mid T_1{=}N{+}1\ ]$
   c. FDI Missed Detection: $P[\ T_S \geq k{+}N_0 \mid T_1{=}k\ ]$
2. Neglect intermittent failures
3. Neglect intermittent switching logic
4. Sensor failures and FDI logic decision are independent
   - Sensors have no common failure modes.

Failure Modes





System Failure Probability



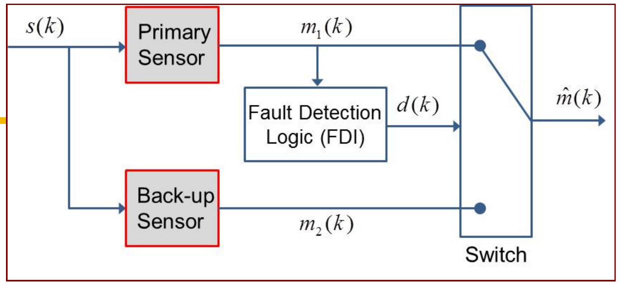
- Apply basic probability theory:

$$P_{S,N} = \Sigma_{k=1}^{N} Pr[T_S \geq k + N_0 \mid T_1 = k] Pr[T_1 = k]$$

$$+ Pr[T_S \leq N \mid T_1 = N+1] Pr[T_1 = N+1] Pr[T_2 \leq N]$$

$$+ \Sigma_{k=1}^{N} Pr[T_S < k + N_0 \mid T_1 = k] Pr[T_1 = k] Pr[T_2 \leq N]$$

System Failure Probability



- Apply basic probability theory:

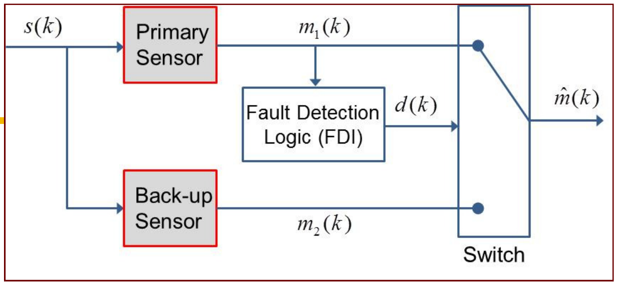
$$P_{S,N} = \Sigma_{k=1}^{N} Pr[T_S \geq k + N_0 \mid T_1 = k] \boxed{Pr[T_1 = k]}$$

$$+ Pr[T_S \leq N \mid T_1 = N+1] \boxed{Pr[T_1 = N+1] Pr[T_2 \leq N]}$$

$$+ \Sigma_{k=1}^{N} Pr[T_S < k + N_0 \mid T_1 = k] \boxed{Pr[T_1 = k] Pr[T_2 \leq N]}$$

- Knowledge of probabilistic performance
  a. Sensor failures: *$P[\, T_i = k \,]$* where T_i := failure time of sensor *i*

System Failure Probability

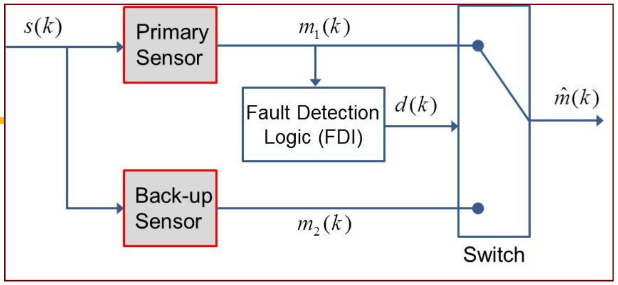


- Apply basic probability theory:

$$P_{S,N} = \Sigma_{k=1}^{N} Pr[T_S \geq k + N_0 \mid T_1 = k] \boxed{Pr[T_1 = k]}$$
$$+ \boxed{Pr[T_S \leq N \mid T_1 = N+1]} \boxed{Pr[T_1 = N+1] Pr[T_2 \leq N]}$$
$$+ \Sigma_{k=1}^{N} Pr[T_S < k + N_0 \mid T_1 = k] \boxed{Pr[T_1 = k] Pr[T_2 \leq N]}$$

- Knowledge of probabilistic performance
  a. Sensor failures: *$P[\ T_i=k\ ]$* where *T_i* := failure time of sensor *i*
  b. FDI False Alarm: *$P[\ T_S \leq N \mid T_1=N+1\ ]$*

System Failure Probability



- Apply basic probability theory:

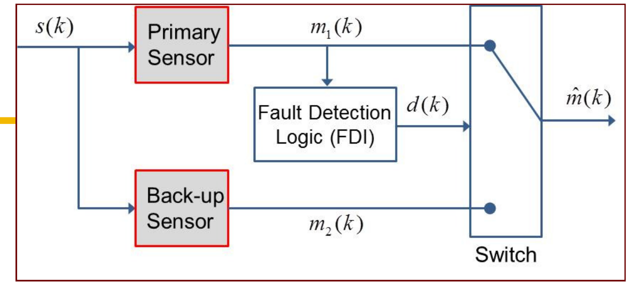
$$P_{S,N} = \Sigma_{k=1}^{N} \boxed{Pr[T_S \geq k + N_0 \mid T_1 = k]}\,\boxed{Pr[T_1 = k]}$$

$$+ \boxed{Pr[T_S \leq N \mid T_1 = N+1]}\,\boxed{Pr[T_1 = N+1]Pr[T_2 \leq N]}$$

$$+ \Sigma_{k=1}^{N} \boxed{Pr[T_S < k + N_0 \mid T_1 = k]}\,\boxed{Pr[T_1 = k]Pr[T_2 \leq N]}$$

- Knowledge of probabilistic performance
  a. Sensor failures: $P[\, T_i = k \,]$ where T_i := failure time of sensor i
  b. FDI False Alarm: $P[\, T_S \leq N \mid T_1 = N+1 \,]$
  c. FDI Missed Detection: $P[\, T_S \geq k + N_0 \mid T_1 = k \,]$

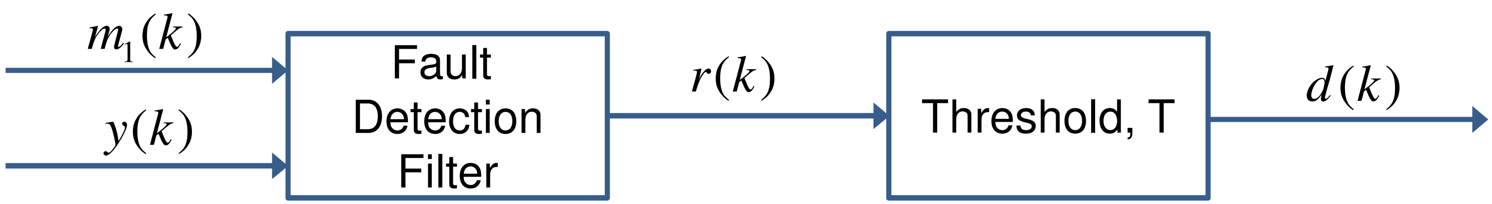
Example



- Sensor Failures: Geometric distribution with parameter *q*

$$q = 1 - e^{\frac{\Delta t}{MTBF}}$$

- Residual-based threshold logic



<u>Residual</u>

$$r(k+1) = n(k) + f(k)$$

f is an additive fault

n is IID Gaussian noise, variance=σ

<u>Decision Logic</u>

$$d(k) = \begin{cases} 0 & \text{if } |r(k)| \le T \\ 1 & \text{else} \end{cases}$$

Example

- Per-frame false alarm probability can be easily computed

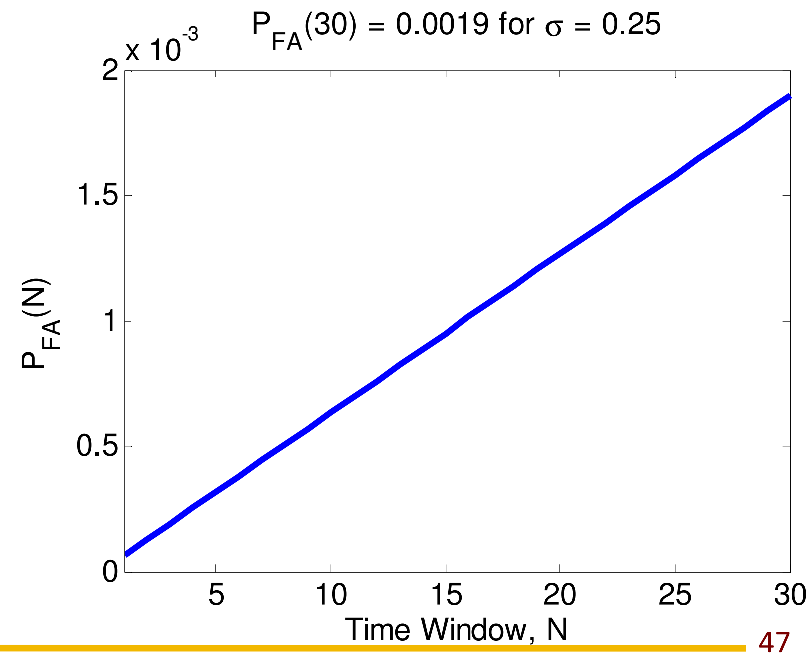
For each k, $r(k)$ is N(0,σ^2) : $P_F = \Pr\left[\, d(\mathrm{k}) = 1 \,\middle|\, \text{No Fault}\right] = 1 - \int_{-T}^{T} p(r)dr$

$$\Rightarrow \boxed{P_F = 1 - erf\left(T/\sqrt{2\sigma^2}\right)}$$

- Approximate per-hour false alarm probability

$$\boxed{P[T_S \le N \mid T_1 = N+1] = 1-(1-P_F)^N \approx NP_F}$$

Per-frame detection probability P_D can be similarly computed.



System Failure Rate

- Notation: $\hat{q} := Nq$ Sensor failure per hour

$$\hat{P}_F := NP_F$$ False alarm per hour

$$\hat{P}_D := 1 - (1 - P_D)^{N_0}$$ Detection per failure

- Approximate system failure probability:

$$P_{S,N} \approx \hat{q}(1 - \hat{P}_D) + \hat{P}_D \hat{q}^2 + \hat{P}_F \hat{q}(1 - \hat{q})$$

System Failure Rate

- Notation:

$$\hat{q} := Nq$$ Sensor failure per hour

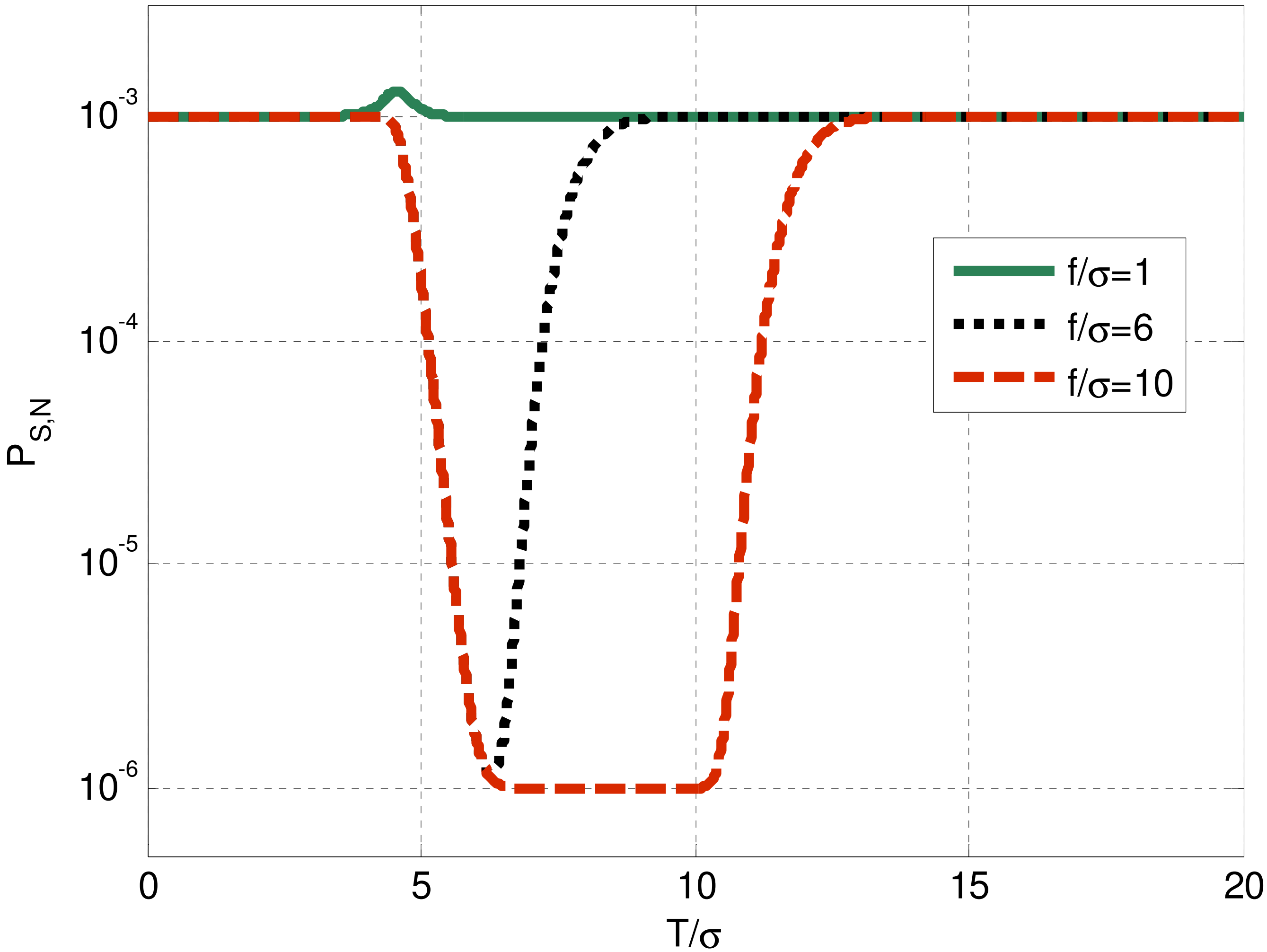
$$\hat{P}_F := NP_F$$ False alarm per hour

$$\hat{P}_D := 1 - (1 - P_D)^{N_0}$$ Detection per failure

- Approximate system failure probability:

$$P_{S,N} \approx \underbrace{\boxed{\hat{q}(1-\hat{P}_D)}}_{\text{Primary sensor fails + missed detection}} + \underbrace{\boxed{\hat{P}_D\hat{q}^2}}_{\text{Failure detected + Backup sensor fails}} + \underbrace{\boxed{\hat{P}_F\hat{q}(1-\hat{q})}}_{\text{False alarm + Backup sensor fails}}$$

System Failure Rate



Sensor mean time between failure = 1000hr
and N=360000 (= 1 hour at 100Hz rate)

Correlated Residuals

- Example analysis assumed IID fault detection logic.
- Many fault-detection algorithms use dynamical models and filters that introduce correlations in the residuals.

- **Question:** How can we compute the FDI performance metrics when the residuals are correlated in time?
  - FDI False Alarm: $P[\ T_S \leq N \mid T_1 = N+1\ ]$
  - FDI Missed Detection: $P[\ T_S \geq k+N_0 \mid T_1 = k\ ]$

False Alarm Analysis with Correlated Residuals

- Problem: Analyze the per-hour false alarm probability for a simple first-order fault detection system:

Residual Generation (*0<a<1*)

$$r_{k+1} = ar_k + n_k + f_k$$

f is an additive fault

n is IID Gaussian noise, variance=1

Decision Logic

$$d_k = \begin{cases} 0 & \text{if } |r_k| \le T \\ 1 & \text{else} \end{cases}$$

Residuals are correlated in time due to filtering

- The *N-step false alarm probability* P_N is the conditional probability that d_k=1 for some 1≤k≤N given the absence of a fault.

$$P_N = 1 - \int_{-T}^{T} \cdots \int_{-T}^{T} p_R(r_1, \ldots, r_N)\, dr_1 \cdots dr_N$$

There are N=360000 samples per hour for a 100Hz system

False Alarm Analysis

- Residuals satisfy the Markov property:

$$r_{k+1} = ar_k + n_k + f_k \Rightarrow p\left(r_{k+1} \middle| r_1, \cdots, r_k\right) = p\left(r_{k+1} \middle| r_k\right)$$

$$\Rightarrow p_R\left(r_1, \cdots, r_k\right) = p\left(r_k \middle| r_{k-1}\right) \cdots p\left(r_2 \middle| r_1\right) \cdot p_1\left(r_1\right)$$

- P_N can be expressed as an N-step iteration of 1-dimensional integrals:

$$P_N = 1 - \int_{-T}^{T} \cdots \int_{-T}^{T} p_R(r_1, \ldots, r_N) dr_1 \cdots dr_N \Rightarrow$$

$$\boxed{\begin{aligned} & f_N(r_N) = 1 \\ & f_{N-1}(r_{N-1}) = \int_{-T}^{T} f_N(r_N) p(r_N | r_{N-1}) dr_N \\ & \vdots \\ & f_1(r_1) = \int_{-T}^{T} f_2(r_2) p(r_2 | r_1) dr_2 \\ & P_N = 1 - \int_{-T}^{T} f_1(r_1) p_1(r_1) dr_1 \end{aligned}}$$

This has the appearance of a power iteration $A^N x$

False Alarm Probability

- **Theorem:** Let λ_1 be the maximum eigenvalue and ψ_1 the corresponding eigenfunction of

$$\lambda_1 \psi_1(x) = \int_{-T}^{T} \psi_1(y) p(y \mid x) dy$$

Then $P_N \approx c\lambda_1^{N-1}$ where $c = \langle 1, \psi_1 \rangle$

- **Proof**
  - This is a generalization of the matrix power iteration
  - The convergence proof relies on the Krein-Rutman theorem which is a generalization of the Perron-Frobenius theorem.
  - For a=0.999 and N=360000, the approximation error is 10^{-156}

Ref: B. Hu and P. Seiler. False Alarm Analysis of Fault Detection Systems with Correlated Residuals, Submitted to IEEE TAC, 2012.