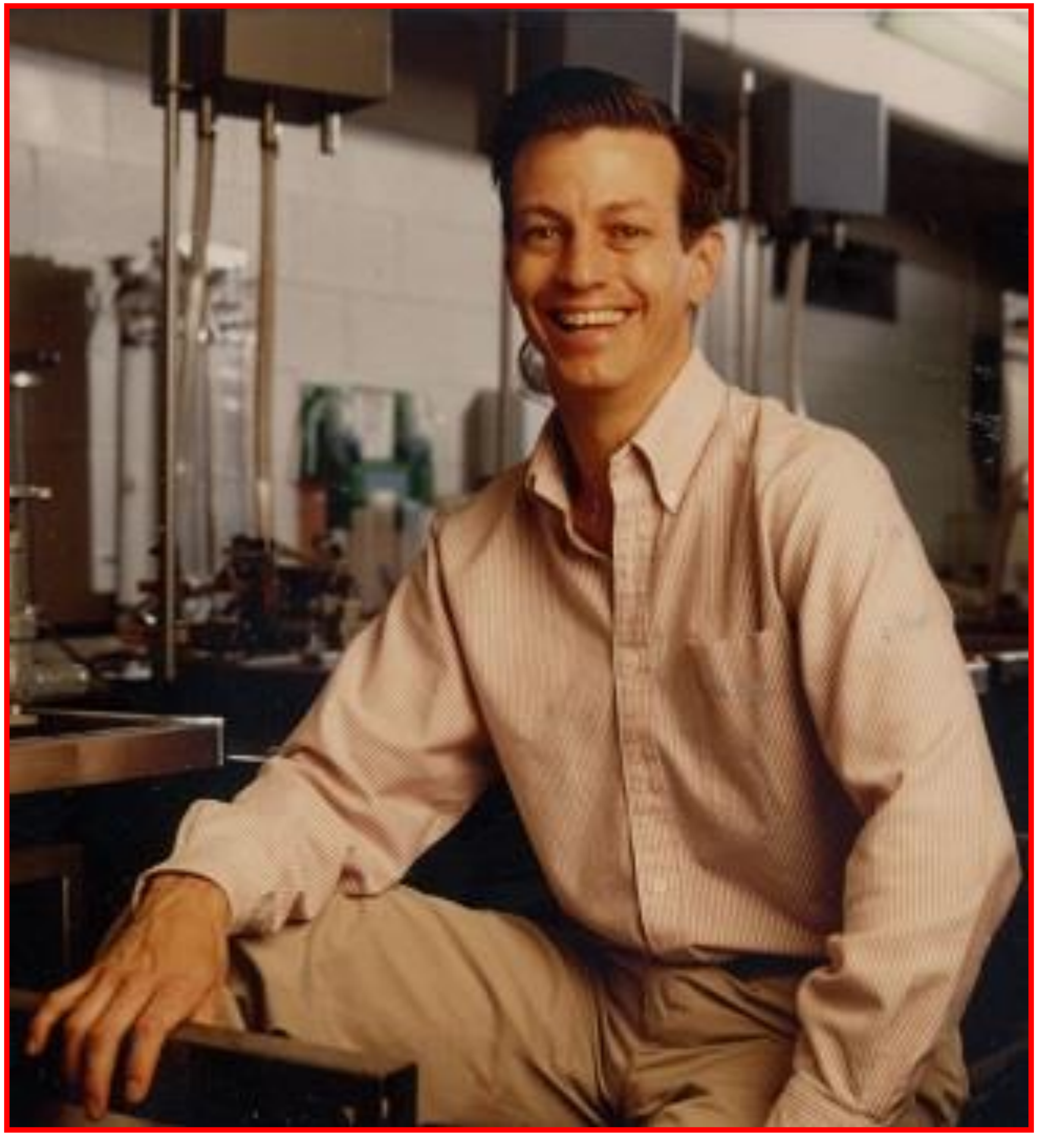
Robust Analysis and Synthesis for Linear Parameter Varying Systems

Peter Seiler
University of Minnesota

Gary J. Balas (Sept. 27. 1960 – Nov. 12, 2014)



Gary and Andy Packard

Spreading the Word

MUSYN Robust Control Theory
Short Course (Start: 1989)

MUSYN

ROBUST MULTIVARIABLE CONTROL: THEORY AND APPLICATION USING μ-TOOLS

AUGUST 4 - 7

MUSYN is pleased to announce the latest short course in robust multivariable control design. A detailed, four day instructional workshop will be taught August 4-7 by three researchers in the field: Prof. John C. Doyle, Prof. Andy Packard and Prof. Gary J. Balas. The short course provides the attendees with an introduction to robust multivariable control using H_∞ and μ analysis and design techniques.

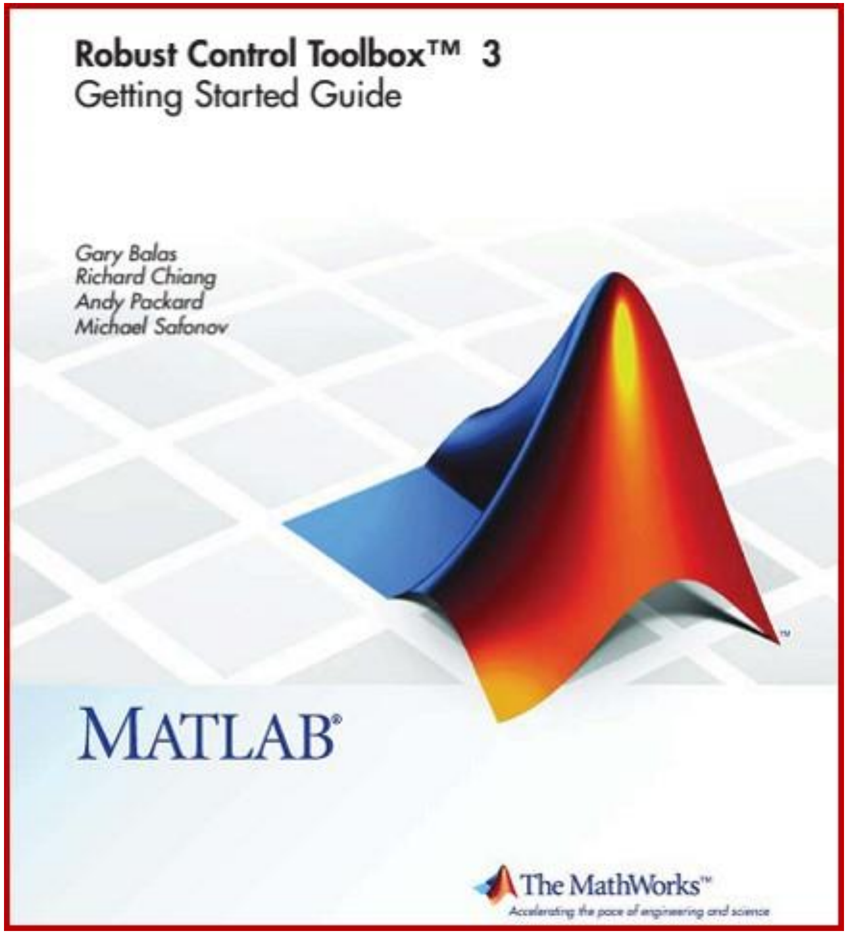
In the past three years over 200 people from industry, government laboratories and academia have attended this course. Locations have included: Los Angeles, Minneapolis, NASA Langley Research Center, Cambridge University, and Delft University, The Netherlands.

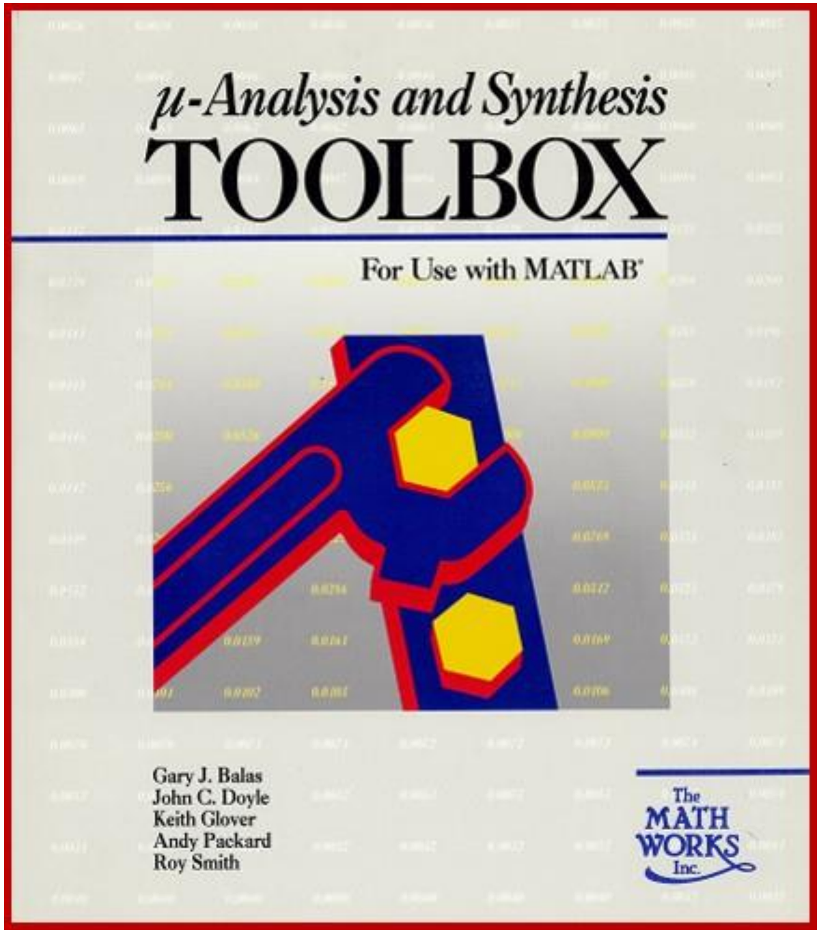
The course has been updated to reflect the latest advances in theory and software. The course covers: various models of uncertainty for components, motivation of "structured uncertainty models," analysis of effects of structured uncertainty using the structured singular value (μ), real/complex μ analysis, controller design using H_∞ and μ techniques, and example applications. Theoretical understanding of the subject material as well as its application to practical problems is emphasized.

Participants will learn and use the μ-Analysis and Synthesis Toolbox (μ-Tools) control design package in conjunction with MATLAB to apply the course material to application areas which include: flight control systems for advanced aircraft, space shuttle lateral axis control system, and vibration attenuation of flexible structures. Each application lecture will discuss modeling of the physical system, formulation of the control problem, application of μ and H_∞ techniques and corresponding results. The participants will have an opportunity to analyze and design control laws for each example with the μ-Tools software following the talks.

Software Development

μ-Analysis and Synthesis (*μ*-Tools) Matlab Toolbox (1990)





μ-Tools merged with the Matlab Robust Control Toolbox (2004)

LPVTools: Matlab Toolbox for LPV Systems

- Developed by MuSyn: Balas, Packard, Seiler, Hjartarson
  - Funded by NASA SBIR contract #NNX12CA14C
  - Contract Monitor: Dr. Martin J. Brenner, NASA Armstrong.
- Goal: Unified framework for grid/LFT based LPV
  - Modeling, Synthesis, Analysis, and Simulation
  - Compatible with Control Toolbox, Robust Control Toolbox, & Simulink using Matlab object-oriented programming.
  - Full documentation (manual, command line, Matlab “doc”)

- LPVTools is freely available under a GNU Affero GPL
  - Google Search: SeilerControl
  - www.aem.umn.edu/~SeilerControl/software.shtml

Aeroservoelasticity

Abhineet Gupta
Aditya Kotikalpudi

Sally Ann Keyes
Adrià Serra Moral

Gary Balas
(9/27/60 – 11/12/14)

Brian Taylor (UAV Lab Director)
Chris Regan

Harald Pfifer
Julian Theis

Performance Adaptive Aeroelastic Wing (PAAW)

- Goal: Suppress flutter, control wing shape and alter shape to optimize performance
  - Funding: NASA NRA NNX14AL36A
  - Technical Monitor: Dr. John Bosworth
  - Two years of testing at UMN followed by two years of testing on NASA's X-56 Aircraft

Aurora FLIGHT SCIENCES

CMSoft

Schmidt & Associates

VirginiaTech Invent the Future®

LM BFF

UMN Mini-Mutt

LM/NASA X-56

Outline

- Linear Parameter Varying (LPV) Systems
- Applications
  - Flexible Aircraft
  - Wind Farms
- Theory for LPV Systems
  - Robustness Analysis
  - Model Reduction

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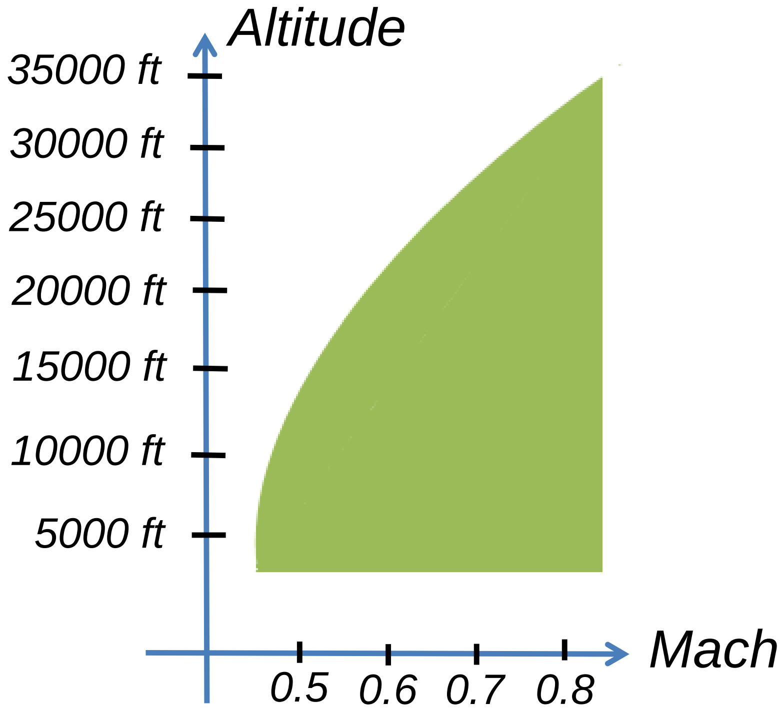
Modeling for Aircraft Control



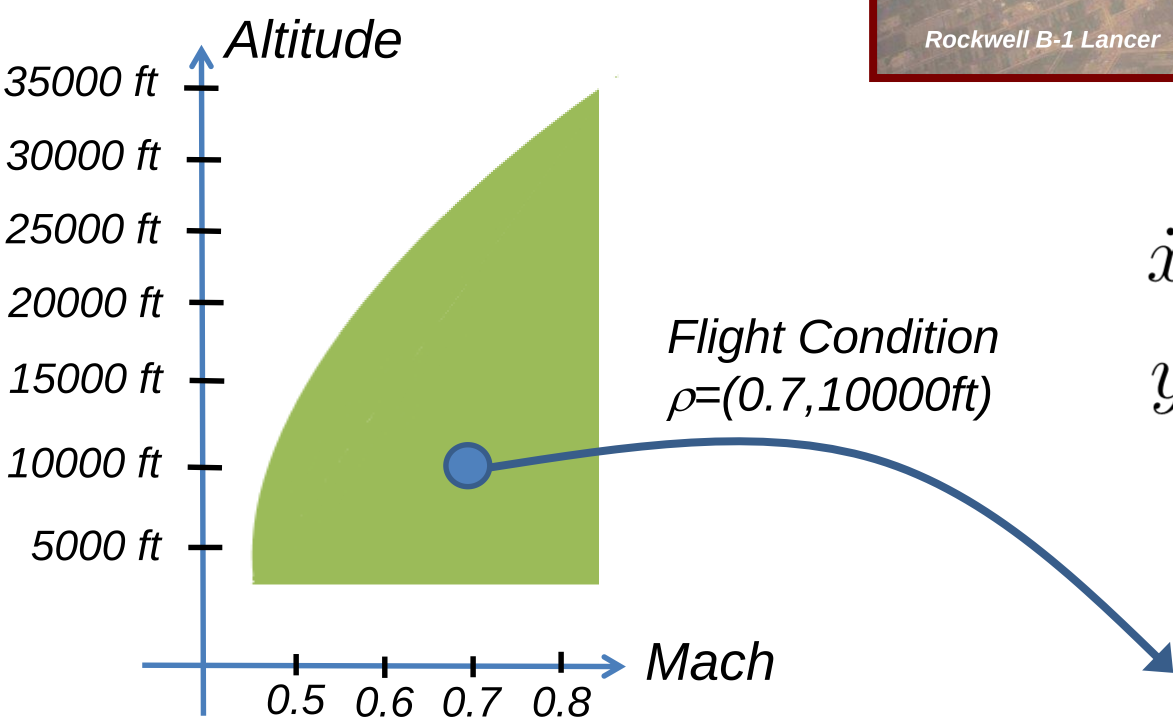
Flight Envelope

Nonlinear ODE

$$\dot{x}(t) = f(x(t), u(t))$$

$$y(t) = h(x(t), u(t))$$

Modeling for Aircraft Control



Flight Envelope

Nonlinear ODE

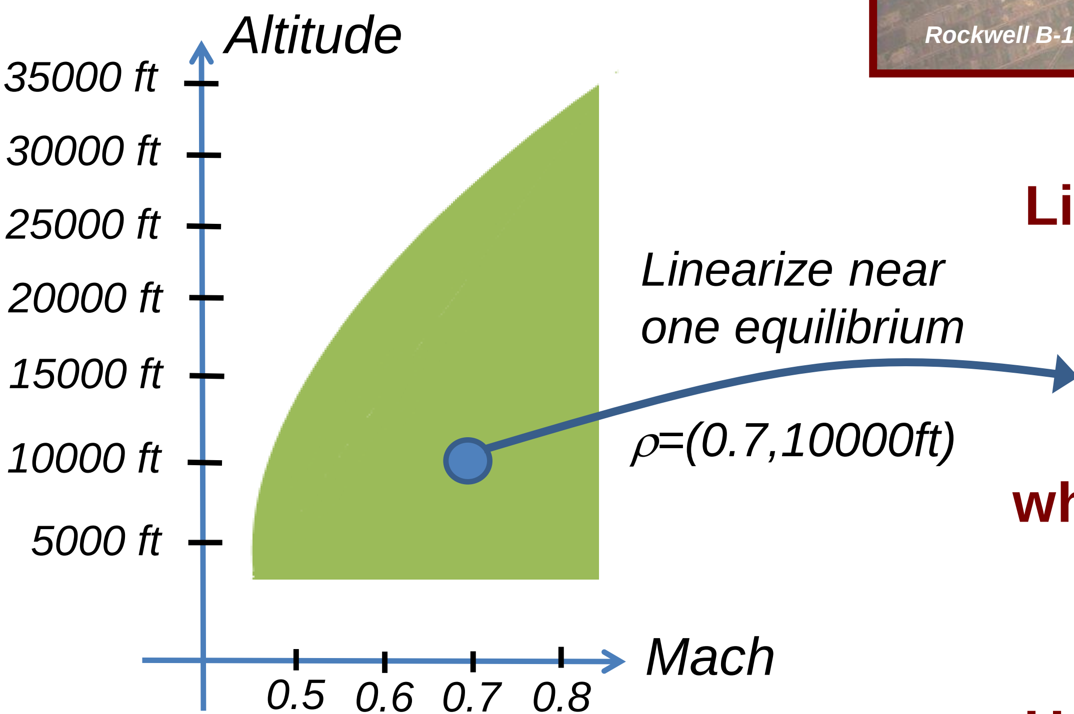
$$\dot{x}(t) = f(x(t), u(t))$$
$$y(t) = h(x(t), u(t))$$

Equilibrium Condition

$$0 = f(\bar{x}, \bar{u})$$
$$\bar{y} = h(\bar{x}, \bar{u})$$

Modeling for Aircraft Control

Rockwell B-1 Lancer (Photo: US Air Force)



Flight Envelope

Linear Time Invariant (LTI)

$$\dot{\delta}_x(t) = A\ \delta_x(t) + B\ \delta_u(t)$$
$$\delta_y(t) = C\ \delta_x(t) + D\ \delta_u(t)$$

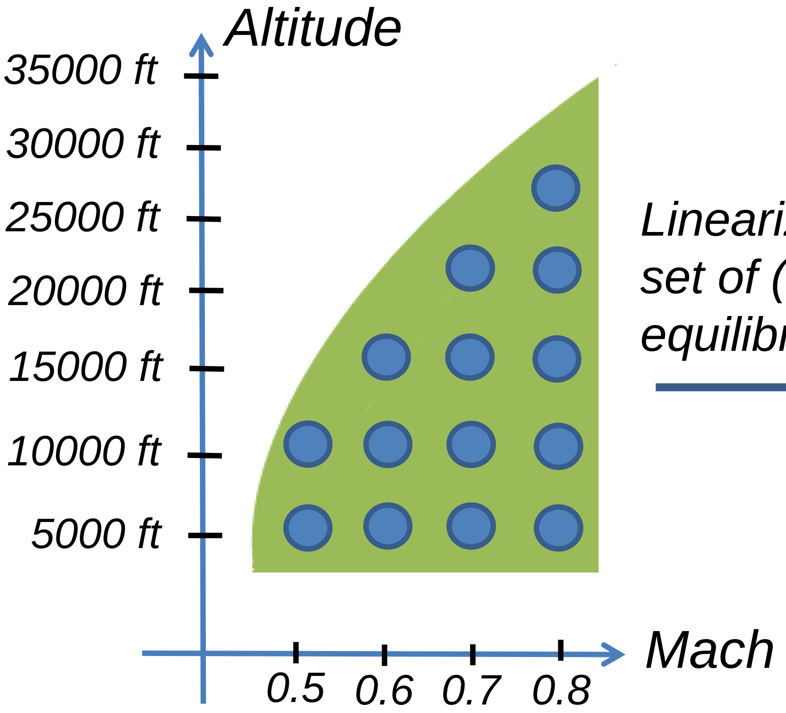
where

$$\delta_x(t) := x(t) - \bar{x}$$

Use for linear control design

Modeling for Aircraft Control

Rockwell B-1 Lancer (Photo: US Air Force)



Flight Envelope

Linearize near set of (fixed) equilibria

Parameterized LTI

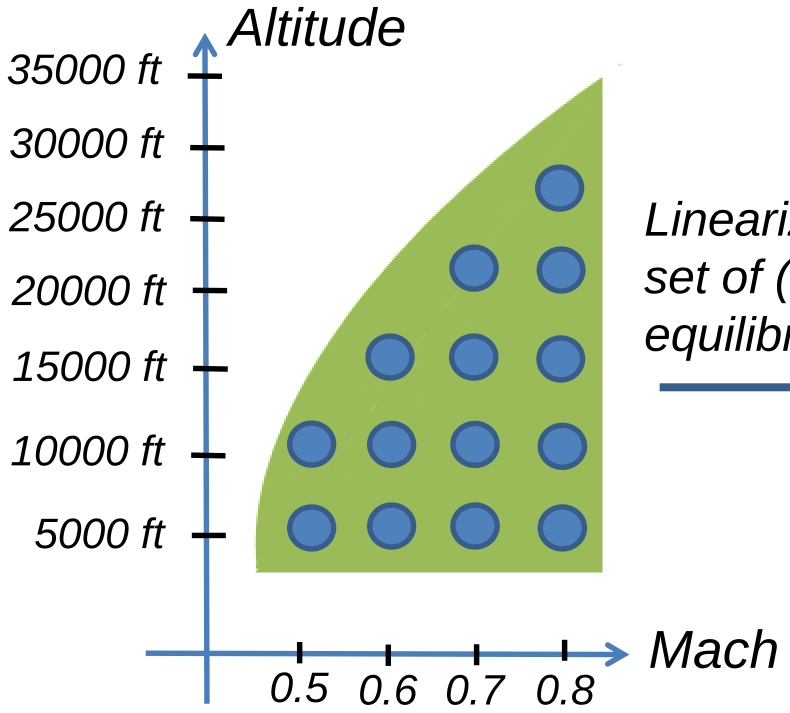
$$\dot{\delta}_x(t) = A(\rho)\ \delta_x(t) + B(\rho)\ \delta_u(t)$$

$$\delta_y(t) = C(\rho)\ \delta_x(t) + D(\rho)\ \delta_u(t)$$

where

$$\delta_x(t) := x(t) - \bar{x}(\rho)$$

Modeling for Aircraft Control



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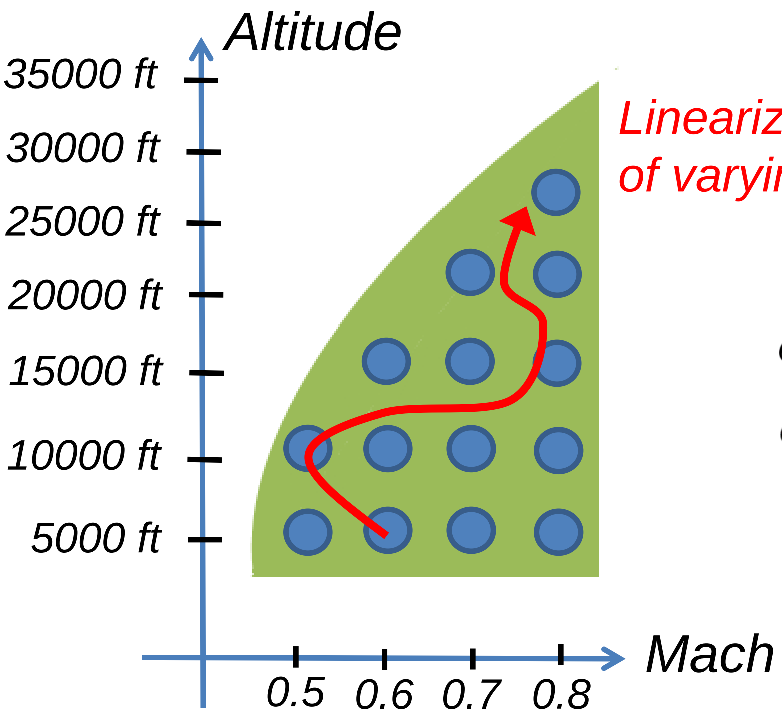
$$\delta_y(t) = C(\rho)\ \delta_x(t) + D(\rho)\ \delta_u(t)$$

Gain-Scheduling

Design controllers at many flight conditions and “stitch” together.

Modeling for Aircraft Control



Flight Envelope

Linearize around set of varying equilibria

Linear Parameter Varying (LPV)

$$\dot{\delta}_x(t) = A(\rho(t))\ \delta_x(t) + B(\rho(t))\ \delta_u(t) - \dot{\bar{x}}(\rho(t))$$
$$\delta_y(t) = C(\rho(t))\ \delta_x(t) + D(\rho(t))\ \delta_u(t)$$

where

$$\delta_x(t) := x(t) - \bar{x}(\rho(t))$$

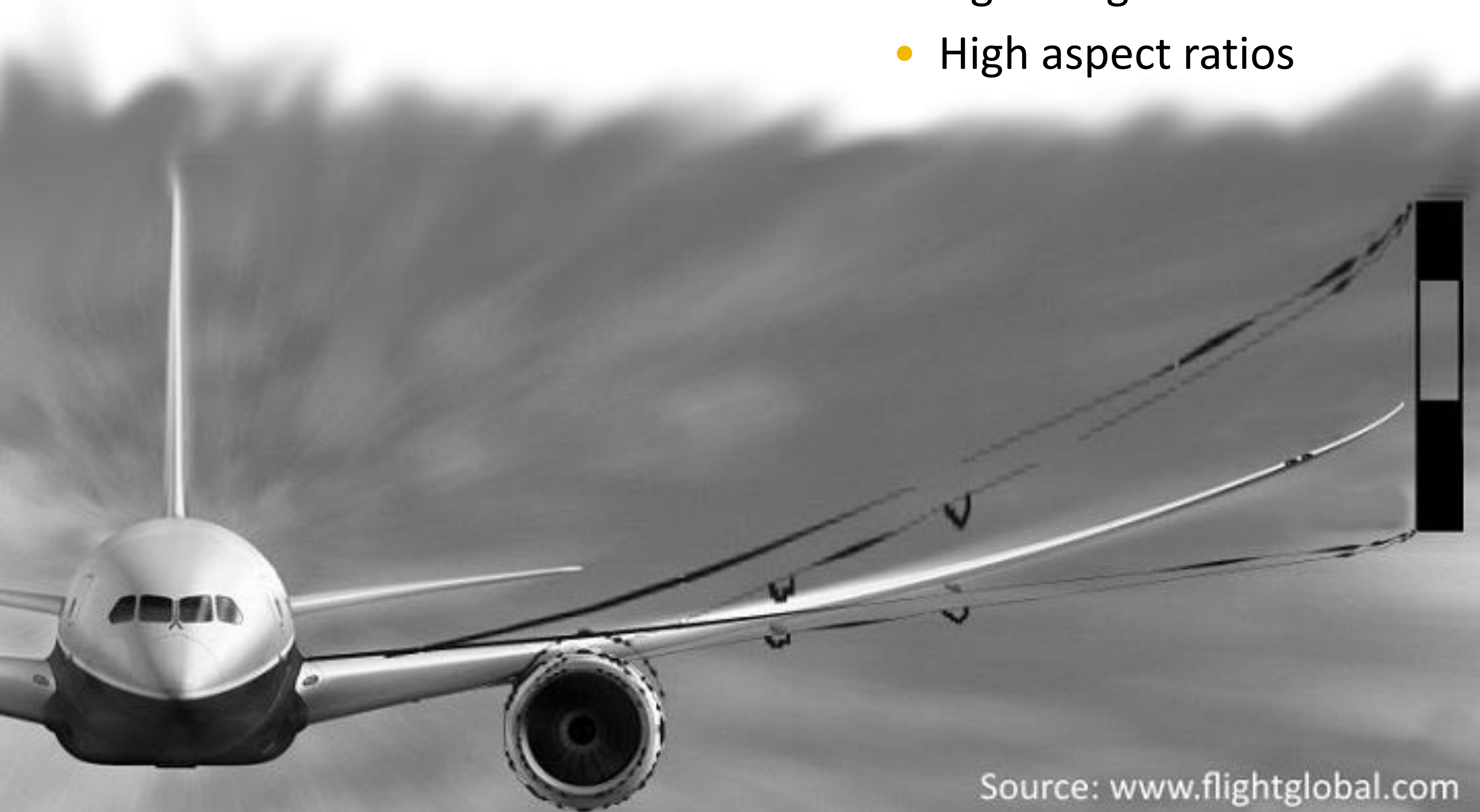
Outline

- Linear Parameter Varying (LPV) Systems
- **Applications**
  - **Flexible Aircraft**
  - **Wind Farms**
- Theory for LPV Systems
  - Robustness Analysis
  - Model Reduction

Aeroservoelasticity (ASE)

Efficient aircraft design

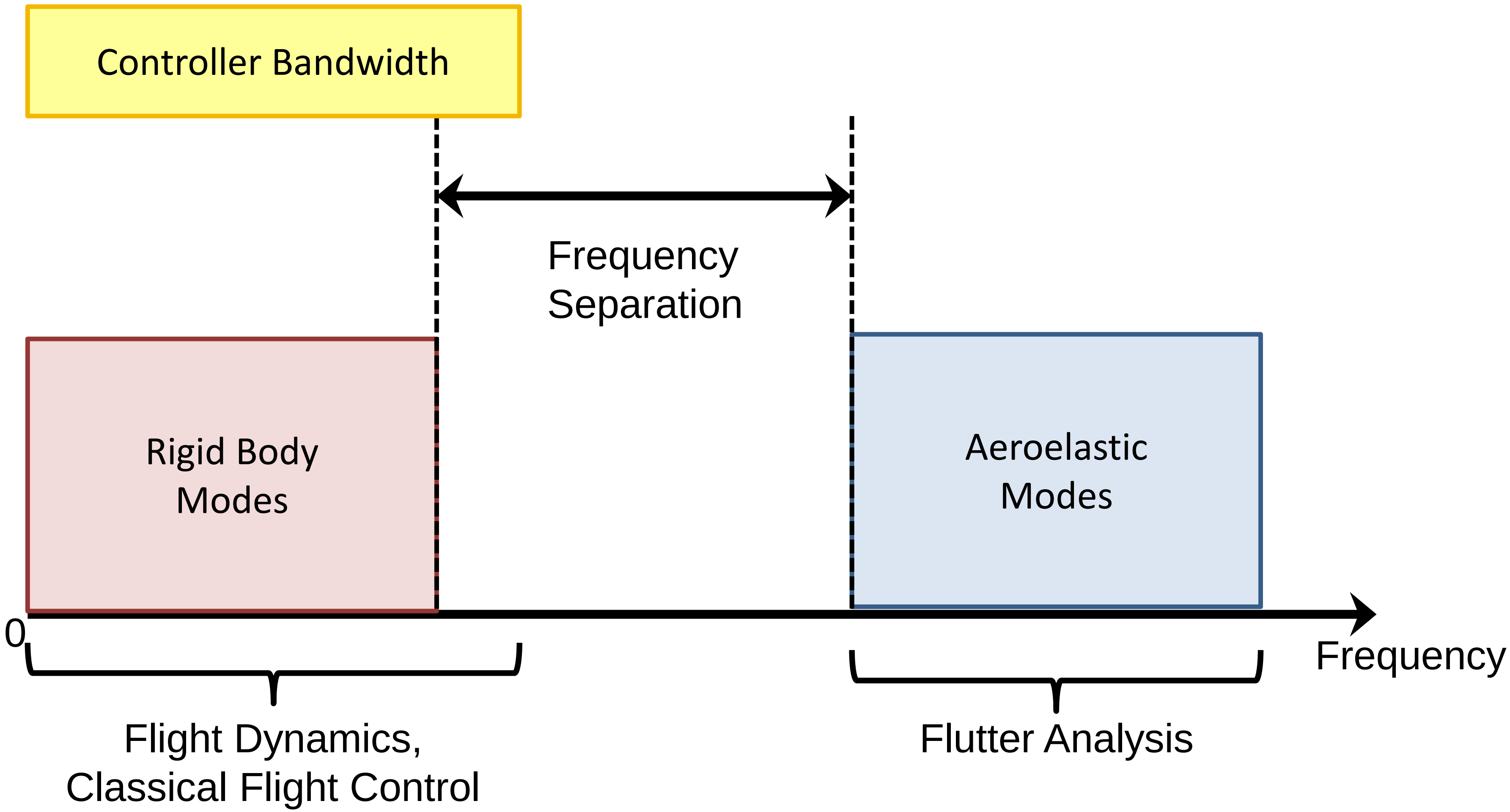
- Lightweight structures
- High aspect ratios



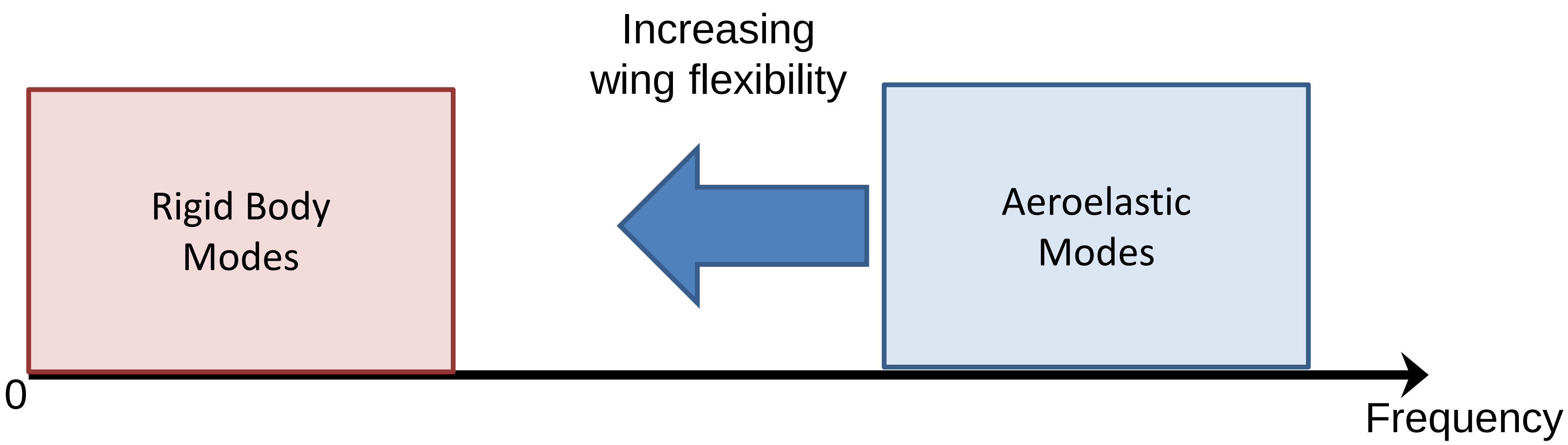
Flutter

Source: NASA Dryden Flight Research

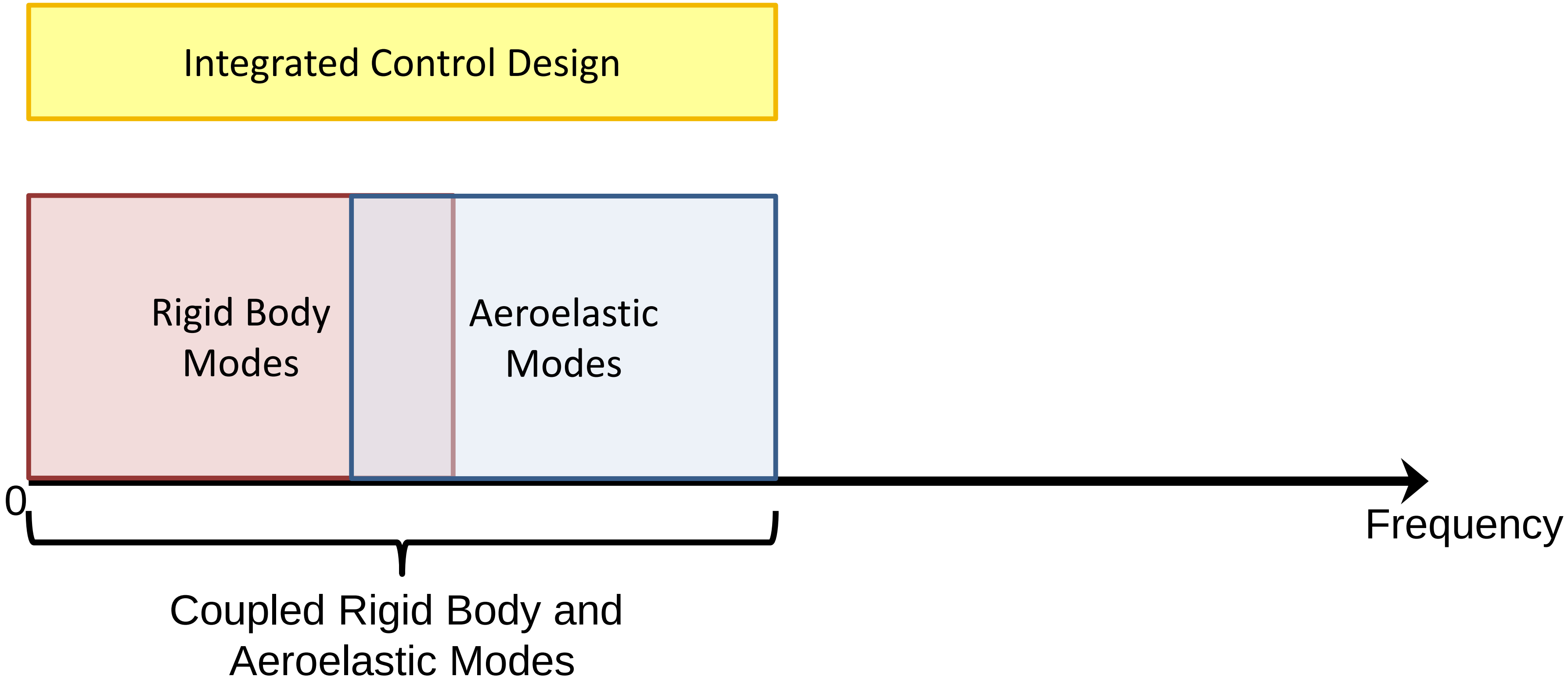
Classical Approach



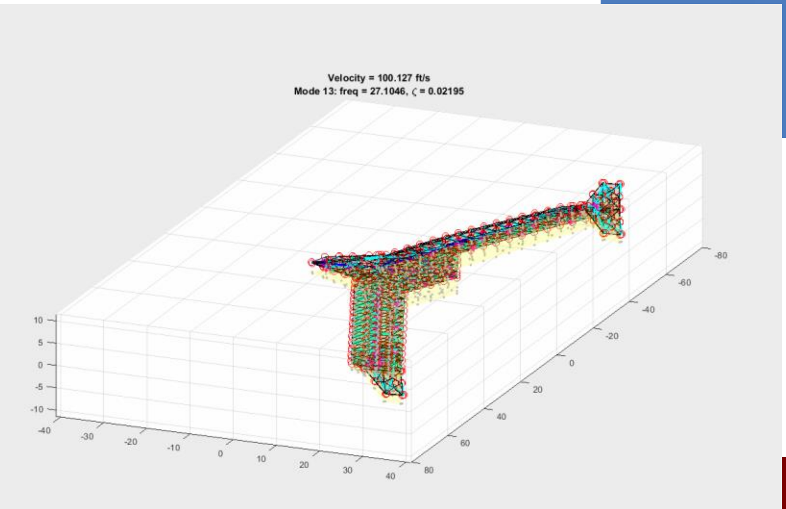
Flexible Aircraft Challenges



Flexible Aircraft Challenges

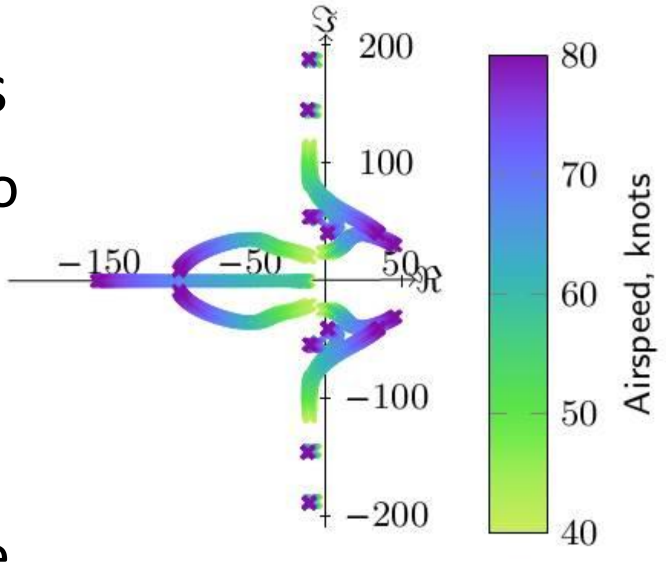


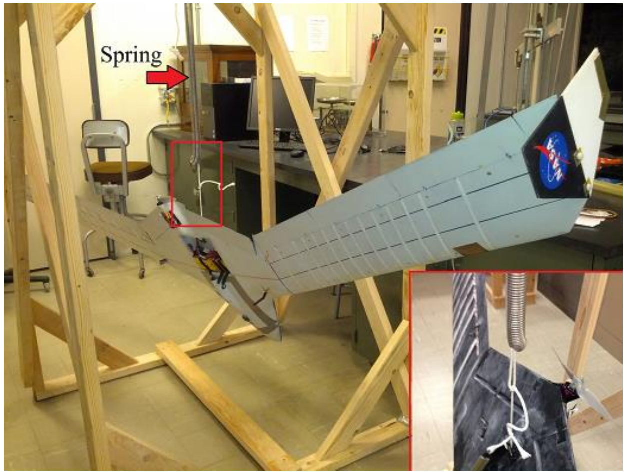
Body Freedom Flutter



Modeling and Control for Flex Aircraft

1. Parameter Dependent Dynamics
   - Models depend on airspeed due to structural/aero interactions
   - LPV is a natural framework.
2. Model Reduction
   - High fidelity CFD/CSD models have many (millions) of states.
3. Model Uncertainty
   - Use of simplified low order models OR reduced high fidelity models
   - Unsteady aero, mass/inertia & structural parameters



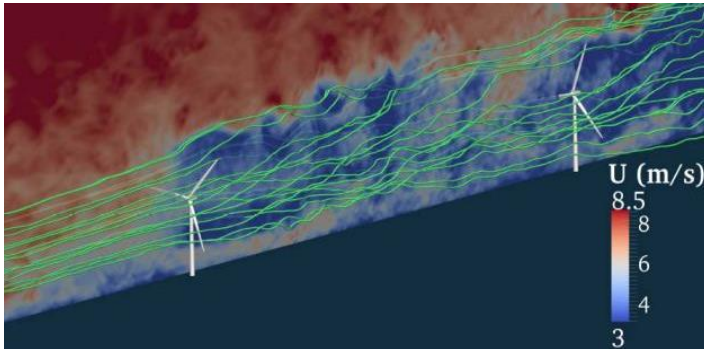


Modeling and Control for Wind Farms

1. Parameter Dependent Dynamics
   - Models depend on windspeed due to structural/aero interactions
   - LPV is a natural framework.
2. Model Reduction
   - High fidelity CFD/CSD models have many (millions) of states.
3. Model Uncertainty
   - Use of simplified low order models OR reduced high fidelity models

Eolos: http://www.eolos.umn.edu/

Saint Anthony Falls: http://www.safl.umn.edu/

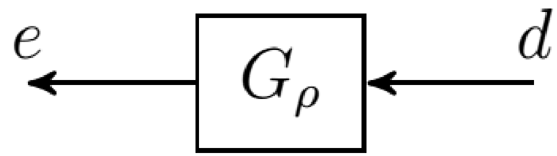


Simulator for Wind Farm Applications, Churchfield & Lee http://wind.nrel.gov/designcodes/simulators/SOWFA

Outline

- Linear Parameter Varying (LPV) Systems
- Applications
  - Flexible Aircraft
  - Wind Farms
- **Theory for LPV Systems**
  - **Robustness Analysis (Pfifer, Wang, Hu, Lacerda, Venkataraman)**
  - Model Reduction

LPV Analysis



Gridded LPV System

$$\dot{x}(t) = A(\rho(t))\ x(t) + B(\rho(t))\ d(t)$$
$$e(t) = C(\rho(t))\ x(t) + D(\rho(t))\ d(t)$$

$$\rho \in \mathcal{A} := \text{Set of allowable trajectories}$$

Induced L_2 Gain

$$\sup_{\rho \in \mathcal{A}} \|G_\rho\|_{2\to 2} = \sup_{\rho \in \mathcal{A}} \sup_{0 \neq d \in L_2} \frac{\|e\|_2}{\|d\|_2}$$

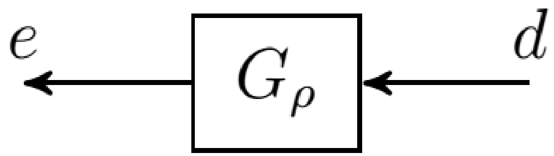
(Standard) Dissipation Inequality Condition

Theorem (Wu, 1995)

If there exists $V(x, \rho) \geq 0$ such that

$$\dot{V} + e^T e \leq \gamma^2 d^T d$$

then $\sup_{\rho \in \mathcal{A}} \|G_\rho\|_{2\to 2} \leq \gamma$.



Proof: Integrate the dissipation inequality

$$\underbrace{V(x(T))}_{\geq 0} + \underbrace{V(x(0))}_{=0} + \int_0^T e(t)^T e(t) dt \leq \gamma^2 \int_0^T d(t)^T d(t) dt \quad \blacksquare$$

Comments

- Dissipation inequality can be expressed/solved using LMIs.
  - Finite dimensional LMIs for LFT/Polytopic LPV systems
  - Parameterized LMIs for Gridded LPV (requires basis functions, gridding, etc)
- **Condition is IFF for LTI systems but only sufficient for LPV**

Uncertainty Modeling

- **Goal:** Assess the impact of model uncertainty/nonlinearities
- **Approach:** Separate nominal dynamics from perturbations
  - Pert. can be parametric, LTI dynamic, and/or nonlinearities (e.g. saturation).

$$\dot{x} = (a + \Delta a)x + f(x) + d$$

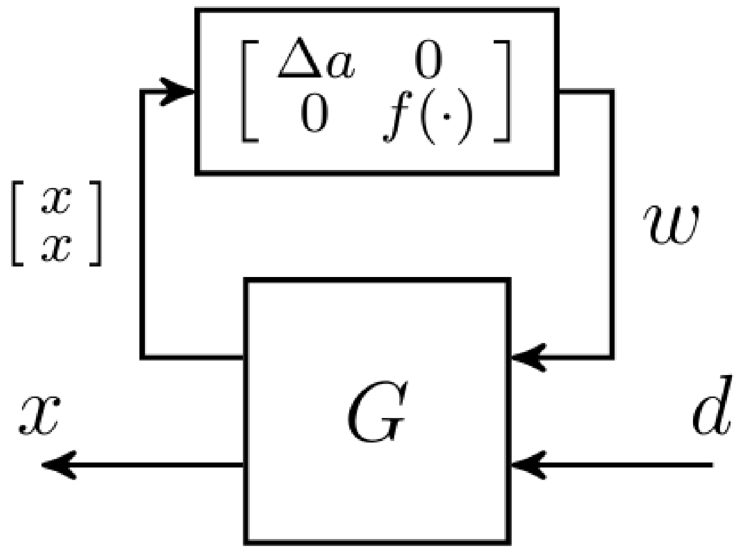
Nominal LTI, G

$$\dot{x} = ax + w_1 + w_2 + d$$

$$w_1 = \Delta a \cdot x$$

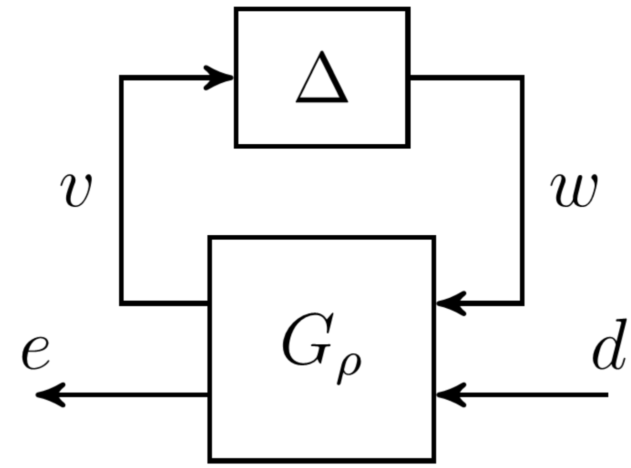
$$w_2 = f(x)$$

Perturbation, Δ



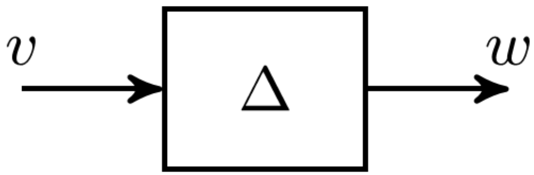
Robustness Analysis for LPV Systems

- **Goal:** Extend analysis tools to LPV



- **Approach:**
  - Use Integral Quadratic Constraints to model input/output behavior (Megretski & Rantzer, TAC 1997).
  - Extend dissipation inequality approach for robustness analysis
- **Results for Gridded Nominal system**
  - Parallels earlier results for LFT nominal system by Scherer, Veenman, Köse, Köroğlu.

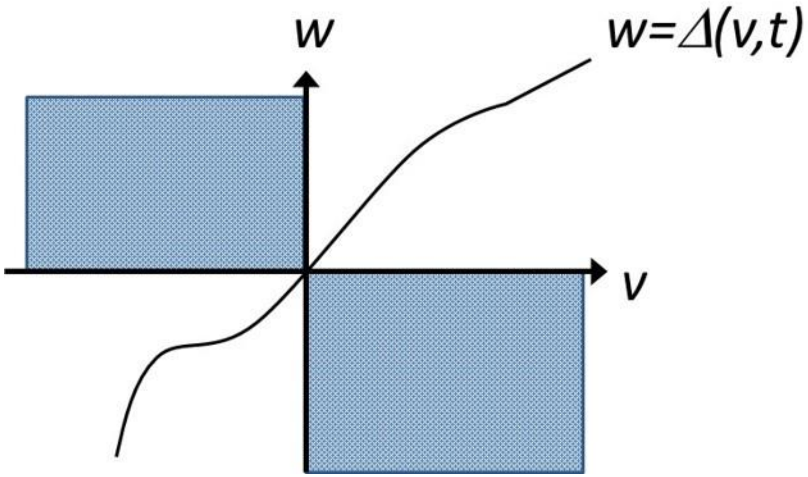
IQC Example: Passive System



$w = \Delta(v, t)$ is a passive system (pointwise in time).

$$2v(t)^T w(t) \geq 0 \; \forall t$$

$$\begin{bmatrix} v(t) \\ w(t) \end{bmatrix}^T \begin{bmatrix} 0 & I \\ I & 0 \end{bmatrix} \begin{bmatrix} v(t) \\ w(t) \end{bmatrix} \geq 0 \, \forall t$$



Pointwise Quadratic Constraint

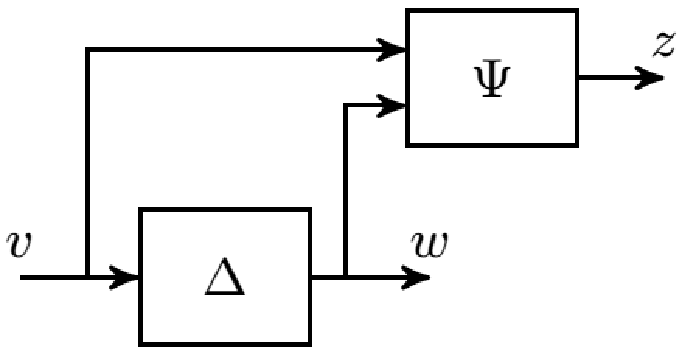
General (Time Domain) IQCs

General IQC Definition:

Let Ψ be a stable, LTI system and M a constant matrix.
Δ satisfies IQC defined by Ψ and M if

$$\int_0^T z(t)^T M z(t) dt \geq 0$$

$\forall v \in L_2[0, \infty)$, $w = \Delta(v)$, and $T \geq 0$.



Comments:

- Megretski & Rantzer ('97 TAC) has a library of IQCs for various components.
- IQCs can be equivalently specified in the freq. domain with a multiplier Π
- A non-unique factorization connects $\Pi = \Psi^* M \Psi$.
- Multiple IQCs can be used to specify behavior of Δ.

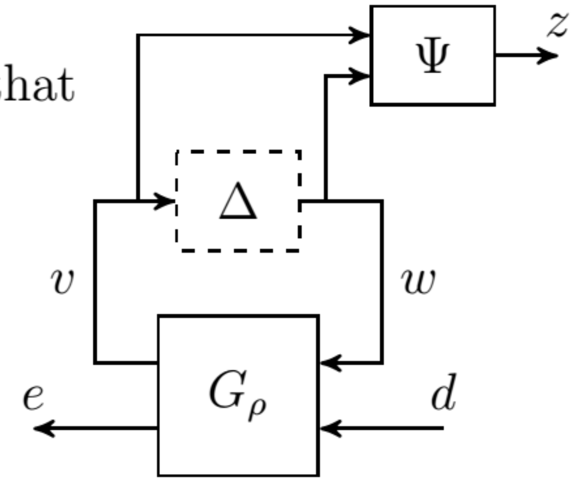
IQC Dissipation Inequality Condition

Theorem

If $\Delta \in IQC(\Psi, M)$ and there exists $V(x,\rho) \geq 0$ such that

$$\dot{V} + z^T M z + e^T e \leq \gamma^2 d^T d$$

then $\sup_{\rho \in \mathcal{A}} \|G_\rho\|_{2\to 2} \leq \gamma$.



Proof: Integrate the dissipation inequality

$$\underbrace{V(x(T))}_{\geq 0} + \underbrace{V(x(0))}_{=0} + \underbrace{\int_0^T z(t)^T M z(t) dt}_{\geq 0} + \int_0^T e(t)^T e(t) dt \leq \gamma^2 \int_0^T d(t)^T d(t) dt$$

∎

Comment

- Dissipation inequality can be expressed/solved as LMIs.
- Extends standard D/G scaling but requires selection of basis functions for IQC.

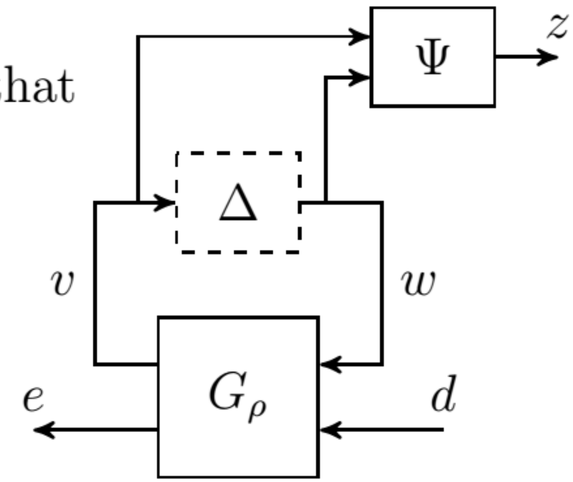
Less Conservative IQC Result

Theorem

If $\Delta \in IQC(\Psi, M)$ and there exists $V(x,\rho) \geq 0$ such that

$$\dot{V} + z^T M z + e^T e \leq \gamma^2 d^T d$$

then $\sup_{\rho \in \mathcal{A}} \|G_\rho\|_{2\to 2} \leq \gamma$.



Technical Result

- Positive semidefinite constraint on *V* and time domain IQC constraint can be dropped.
- These are replaced by a freq. domain requirement on $\Pi=\Psi^* M \Psi$.
- Some energy is "hidden" in the IQC.

Refs:

P. Seiler, Stability Analysis with Dissipation Inequalities and Integral Quadratic Constraints, IEEE TAC, 2015.

H. Pfifer & P. Seiler, Less Conservative Robustness Analysis of Linear Parameter Varying Systems Using Integral Quadratic Constraints, submitted to IJRNC, 2015.

Less Conservative IQC Result

Theorem

If $\Delta \in IQC(\Psi, M)$ and there exists $V(x,\rho) \geq 0$ such that

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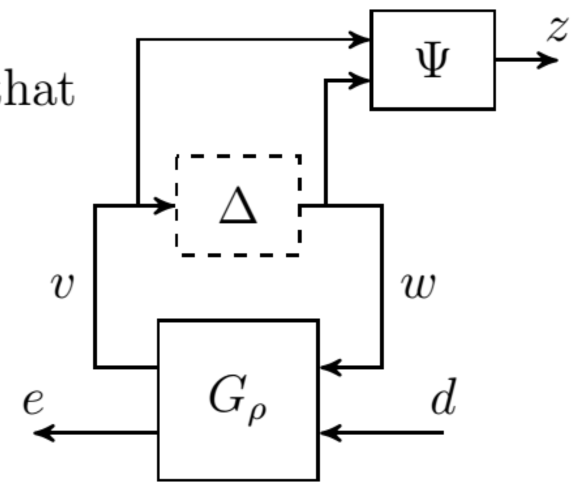

Key Idea:

$$\underbrace{V(x(T))}_{\geq 0} + \underbrace{V(x(0))}_{=0} + \underbrace{\int_0^T z(t)^T M z(t) dt}_{\geq 0} + \int_0^T e(t)^T e(t) dt \leq \gamma^2 \int_0^T d(t)^T d(t) dt$$

We only need the sum of the boxed terms to be ≥ 0, i.e. each term individually need not be ≥ 0.

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H. Pfifer & P. Seiler, Less Conservative Robustness Analysis of Linear Parameter Varying Systems Using Integral Quadratic Constraints, submitted to IJRNC, 2015.

Time-Domain Dissipation Inequality Analysis

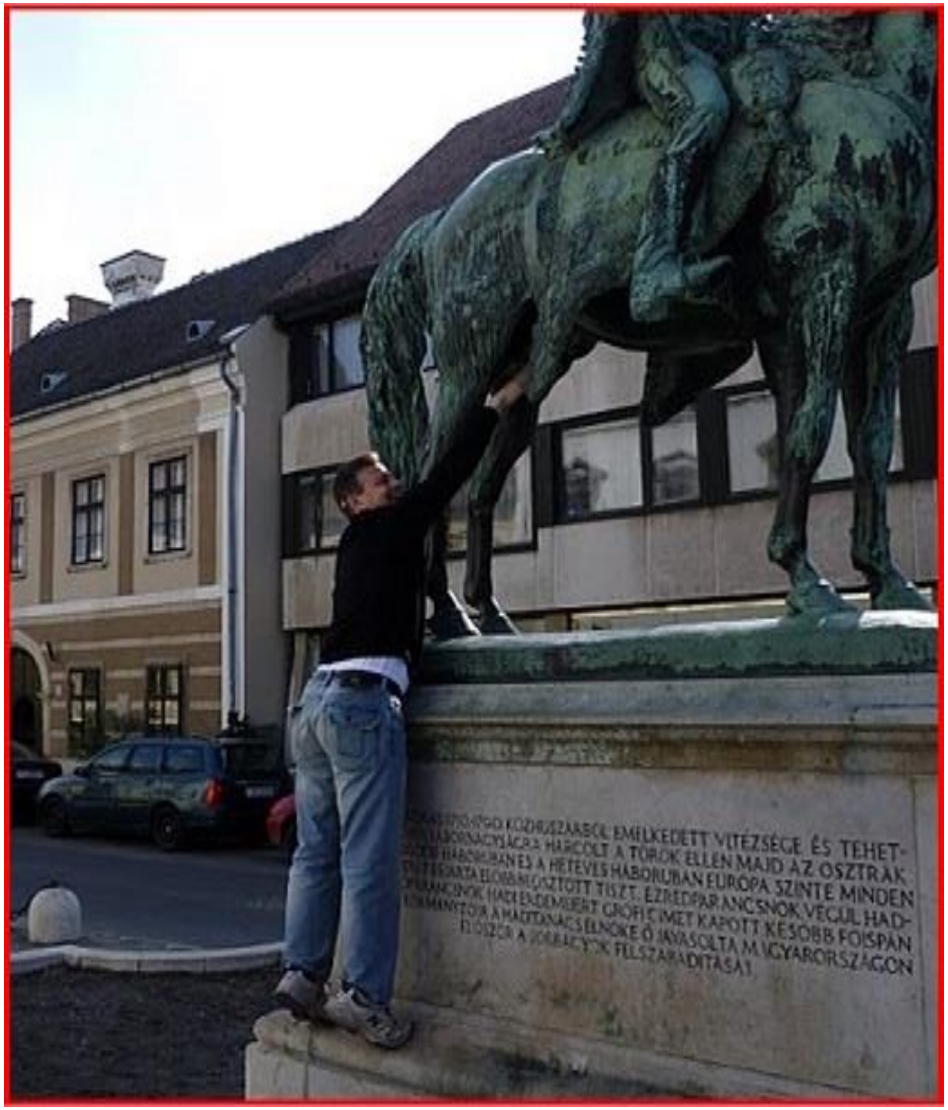
Summary: Under some technical conditions, the frequency-domain conditions in (M/R, '97 TAC) are equivalent to the time-domain dissipation inequality conditions.

Applications:

1. LPV robustness analysis (Pfifer, Seiler, IJRNC)
2. General LPV robust synthesis (Wang, Pfifer, Seiler, accepted to Aut)
3. LPV robust filtering/feedforward (Venkataraman, Seiler, in prep)
   - Robust filtering typically uses a duality argument. Extensions to the time domain?
4. Exponential rates of convergence (Hu,Seiler, accepted to TAC)
   - Motivated by optimization analysis with ρ-hard IQCs (Lessard, Recht, & Packard)
5. Nonlinear analysis using SOS techniques
6. Discrete-time IQC analysis (Hu, Lacerda, Seiler, submitted to IJRNC)

Item 1 has been implemented in LPVTools. Items 2 & 3 parallel results by (Scherer, Köse, and Veenman) for LFT-type LPV systems.

Outline



- Linear Parameter Varying (LPV) Systems
- Applications
  - Flexible Aircraft
  - Wind Farms
- **Theory for LPV Systems**
  - Robustness Analysis
  - **Model Reduction (Annoni, Theis, Singh)**

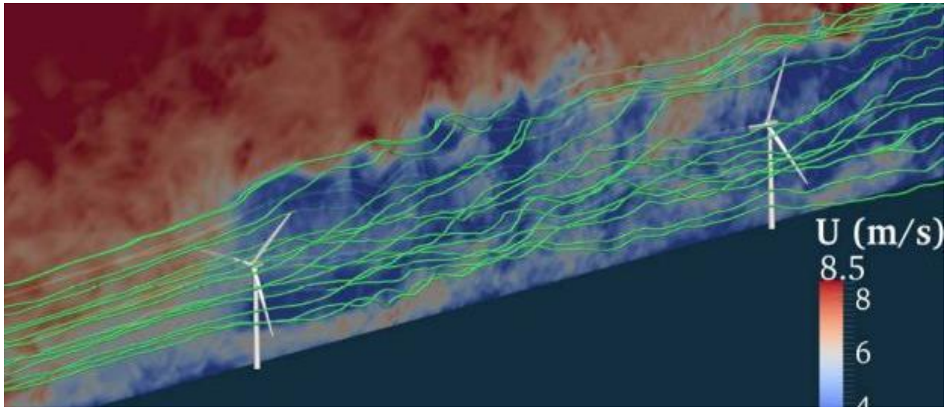
LPV Model Reduction

- Both flexible aircraft and wind farms can be modeled with high fidelity fluid/structural models.
- LPV models can be obtained via Jacobian linearization:

$$\dot{x}(t) = A(\rho(t))\ x(t) + B(\rho(t))\ d(t)$$

$$e(t) = C(\rho(t))\ x(t) + D(\rho(t))\ d(t)$$

- State dimension can be extremely large ($>10^6$)
- LPV analysis and synthesis is restricted to ≈50 states.
- Model reduction is required.



High Order Model Reduction

Large literature with recent results for LPV and Param. LTI

- Antoulas, Amsallem, Carlberg , Gugercin, Farhat, Kutz, Loeve, Mezic, Poussot-Vassal, Rowley, Schmid, Willcox, …

Two new results for LPV:

1. Input-Output Reduced Order Models (Annoni)
   - Combine subspace ID with techniques from fluids (POD/DMD).
   - No need for adjoint models. Can reconstruct full-order state.
2. Parameter-Varying Oblique Projection (Theis)
   - Petrov-Galerkin approximation with constant projection space and parameter-varying test space.
   - Constant projection maintains state consistency avoids rate dependence.

References

1A. Annoni & Seiler, A method to construct reduced-order parameter varying models, submitted to IJRNC, 2015.

1B. Annoni, Nichols, & Seiler, "Wind farm modeling and control using dynamic mode decomposition." AIAA, 2016.

2. Theis, Seiler, & Werner, Model Order Reduction by Parameter-Varying Oblique Projection, submitted to 2016 ACC.

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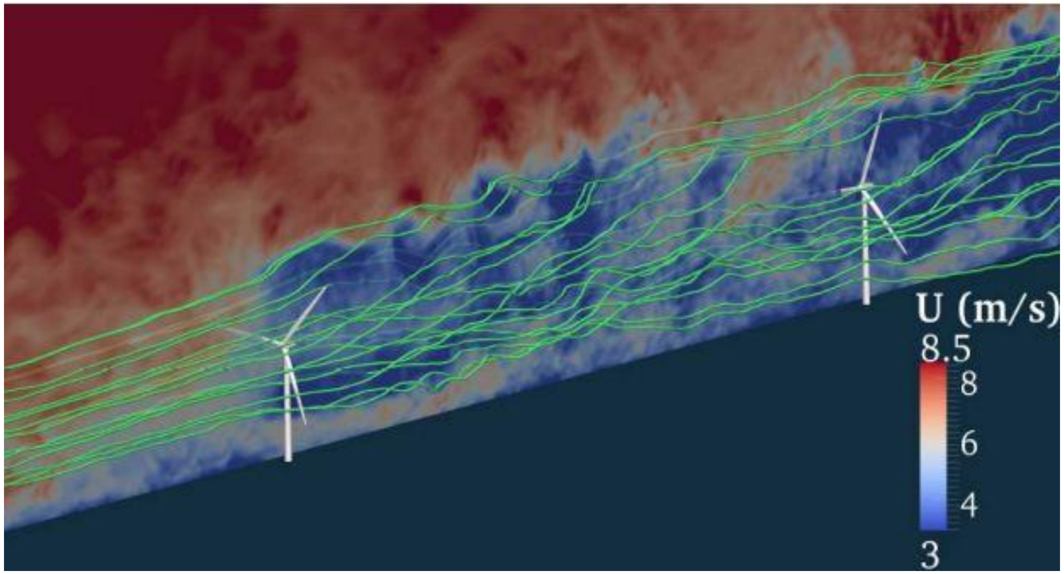
Application: Large Eddy Simulation (LES)

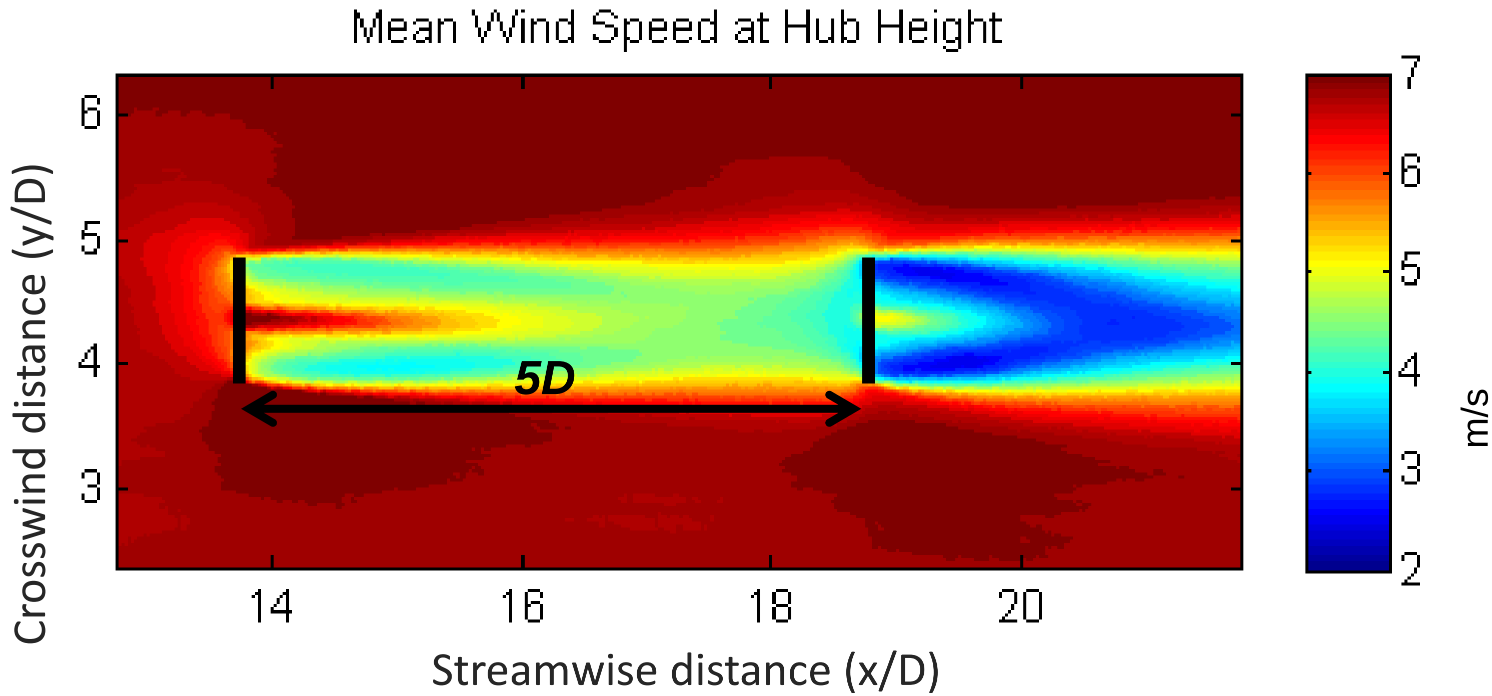
- Simulator fOr Wind Farm Applications (SOWFA)
- 3D unsteady spatially filtered Navier-Stokes equations
- Simulation time (wall clock): 48 hours



Churchfield, Lee
https://nwtc.nrel.gov/SOWFA

Wind Turbine Array Setup

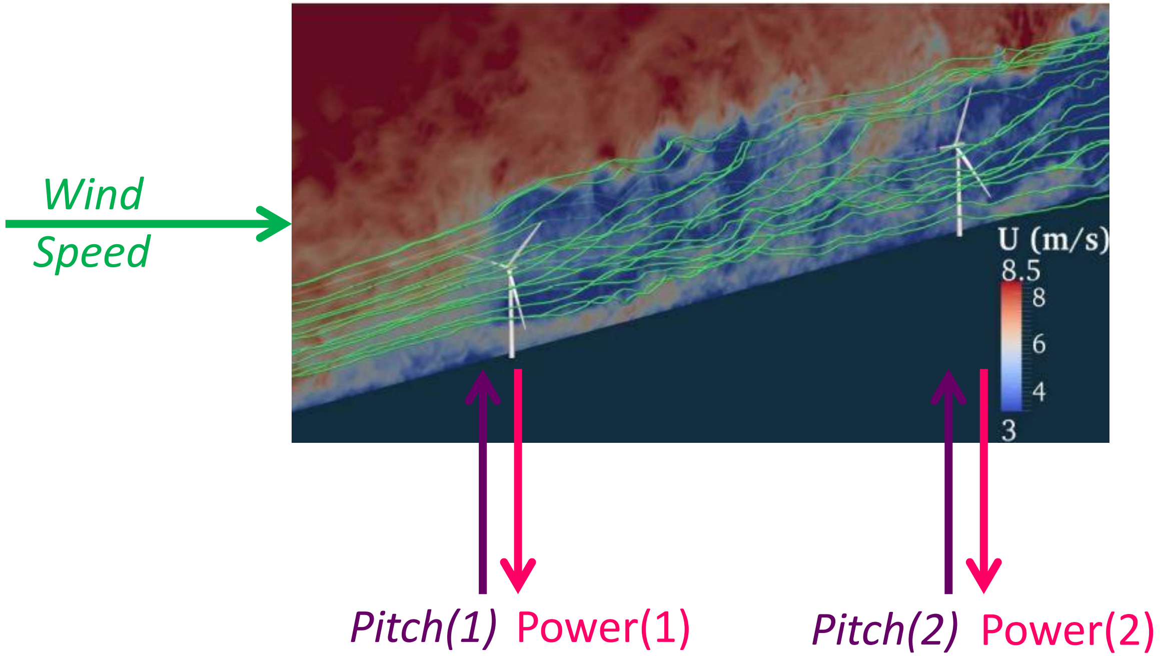
- Two turbine setup (NREL 5 MW turbines)



- Turbine Diameter D=126m
- Approximately 1.2 million grid points
- 3 velocity components → **3.6 million states**

Wind Turbine Array Setup

- Two turbine setup (NREL 5 MW turbines)



- Control inputs: Blade pitch angle
- Control outputs: Power at each turbine
- Exogenous Disturbance: Mean wind speed

Discrete-Time Direct Subspace ID (Viberg, 95)

- Gather snapshots of inputs, outputs, and state

$$X_0 = [x_1, x_2, \ldots, x_{m-1}] \qquad U_0 = [u_1, u_2, \ldots, u_{m-1}]$$

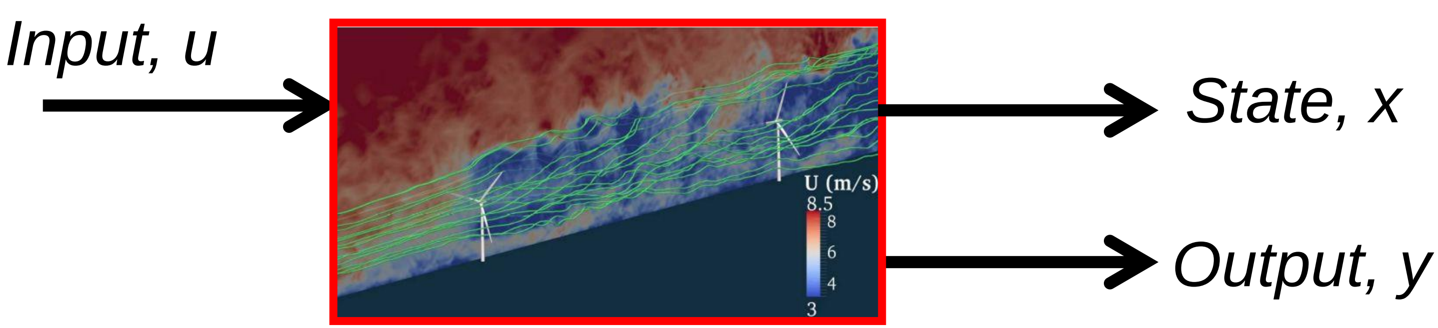
$$X_1 = [x_2, x_3, \ldots, x_m] \qquad Y_0 = [y_1, y_2, \ldots, y_{m-1}]$$

- Fit a linear state-space model to the data

$$X_1 = AX_0 + BU_0$$
$$Y_0 = CX_0 + DU_0$$

$$\longrightarrow \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} X_1 \\ Y_0 \end{bmatrix} \begin{bmatrix} X_0 \\ U_0 \end{bmatrix}^+$$

Computationally Intractable for large systems

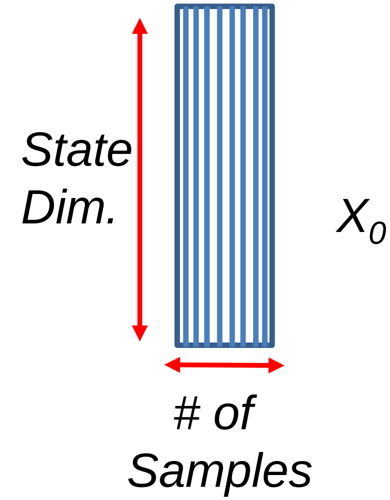


Reduced Order Model

- Compute SVD of state snapshot data:

$$X_0 = U\Sigma V^T$$

POD modes



- Project state data onto the POD modes:

$$Z_0 = U^* X_0 \qquad Z_1 = U^* X_1$$

- Fit a linear state-space model to the reduced data:

$$\begin{aligned} Z_1 &= AZ_0 + BU_0 \\ Y_0 &= CZ_0 + DU_0 \end{aligned} \longrightarrow \begin{bmatrix} \tilde{A} & \tilde{B} \\ \tilde{C} & D \end{bmatrix} = \begin{bmatrix} Z_1 \\ Y_0 \end{bmatrix} \begin{bmatrix} Z_0 \\ U_0 \end{bmatrix}^+$$

- Comments:
  - SVD can be done on laptop in a few hours with Tall QR methods.
  - This is a variation of DMDc by Proctor, et al, 2014.
  - We can approximate the full state as $x_k = Uz_k$

Summary

$$X_0 = [x_1, x_2, \ldots, x_{m-1}]$$
$$X_1 = [x_2, x_3, \ldots, x_m]$$
$$U_0 = [u_1, u_2, \ldots, u_{m-1}]$$
$$Y_0 = [y_1, y_2, \ldots, y_{m-1}]$$

Excite system & collect data

$$X_0 = \boxed{U}\Sigma V^T$$

Compute spatial modes (POD)

$$Z_0 = U^* X_0$$
$$Z_1 = U^* X_1$$

Project states onto (low-order) modes

$$\begin{bmatrix} \tilde{A} & \tilde{B} \\ \tilde{C} & D \end{bmatrix} = \begin{bmatrix} Z_1 \\ Y_0 \end{bmatrix} \begin{bmatrix} Z_0 \\ U_0 \end{bmatrix}^+$$

Estimate low-order state matrices via least-squares

$$z_{k+1} = \tilde{A} z_k + \tilde{B} u_k$$
$$y_k = \tilde{C} z_k + D u_k$$

Reduced-order model

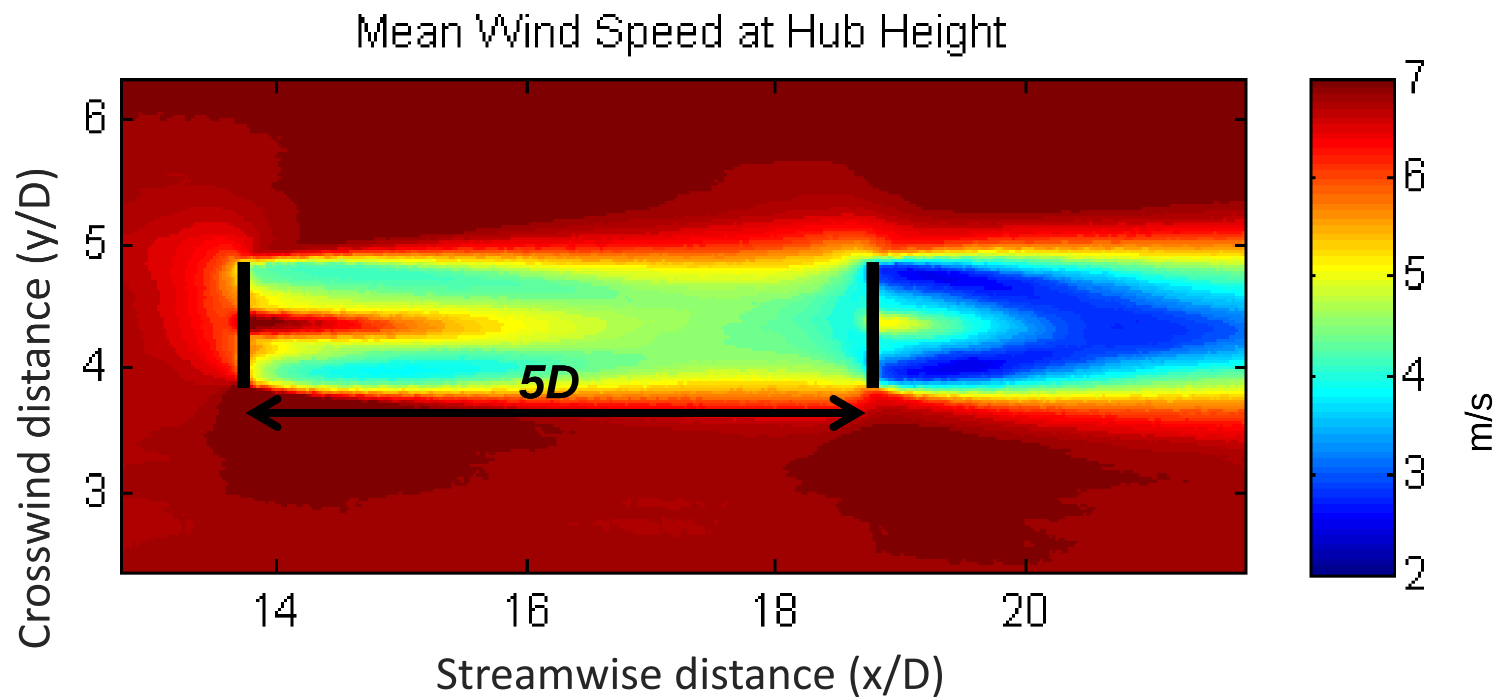
- SVD can be done on laptop in a few hours with Tall QR methods.
- This combines techniques from system ID and fluids (POD/DMD)
  - The approach is a variation of DMDc by Proctor, et al, 2014.
  - The method does not require adjoints or solution of Lyapunov Eqns.
- We can approximate the full state from the reduced state:

$$x_k \approx U z_k$$

- The state consistency can be used to extend the approach to LPV model reduction.
  - Annoni, Seiler, submitted to IJRNC, ‘16

Wind Turbine Array Setup

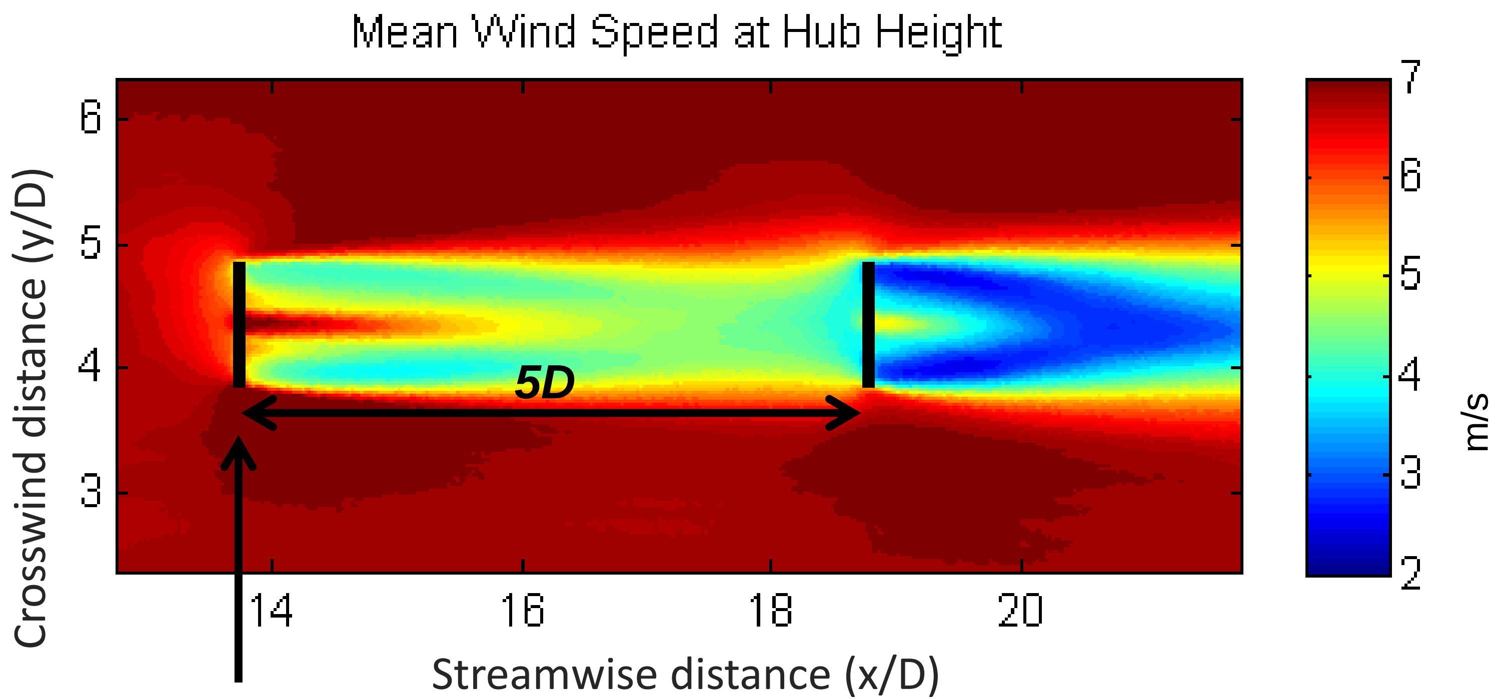
- Two turbine setup (NREL 5 MW turbines)

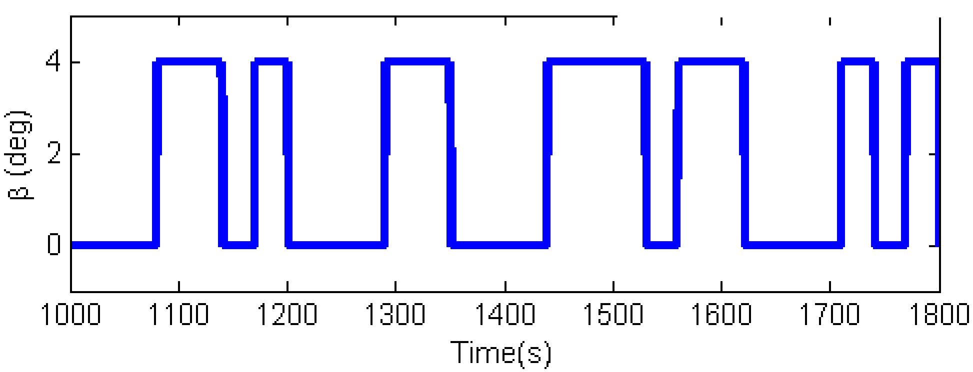


- D = turbine diameter (126 m)
- Neutral boundary layer
- 7 m/s with 10% turbulence

IOROM with SOWFA

- Two turbine setup (NREL 5 MW turbines)

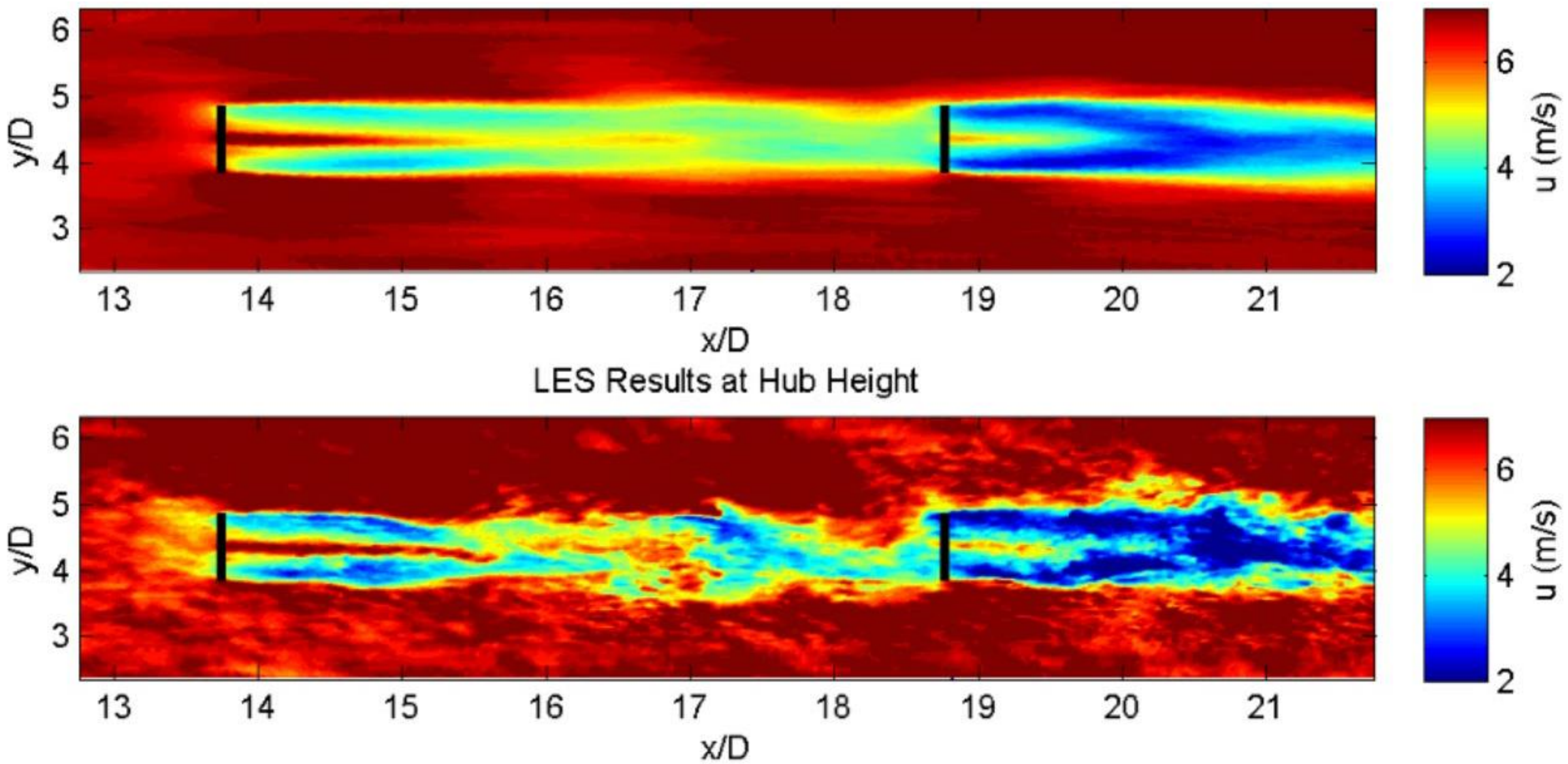




Blade pitch angle changes from 0^o to 4^o

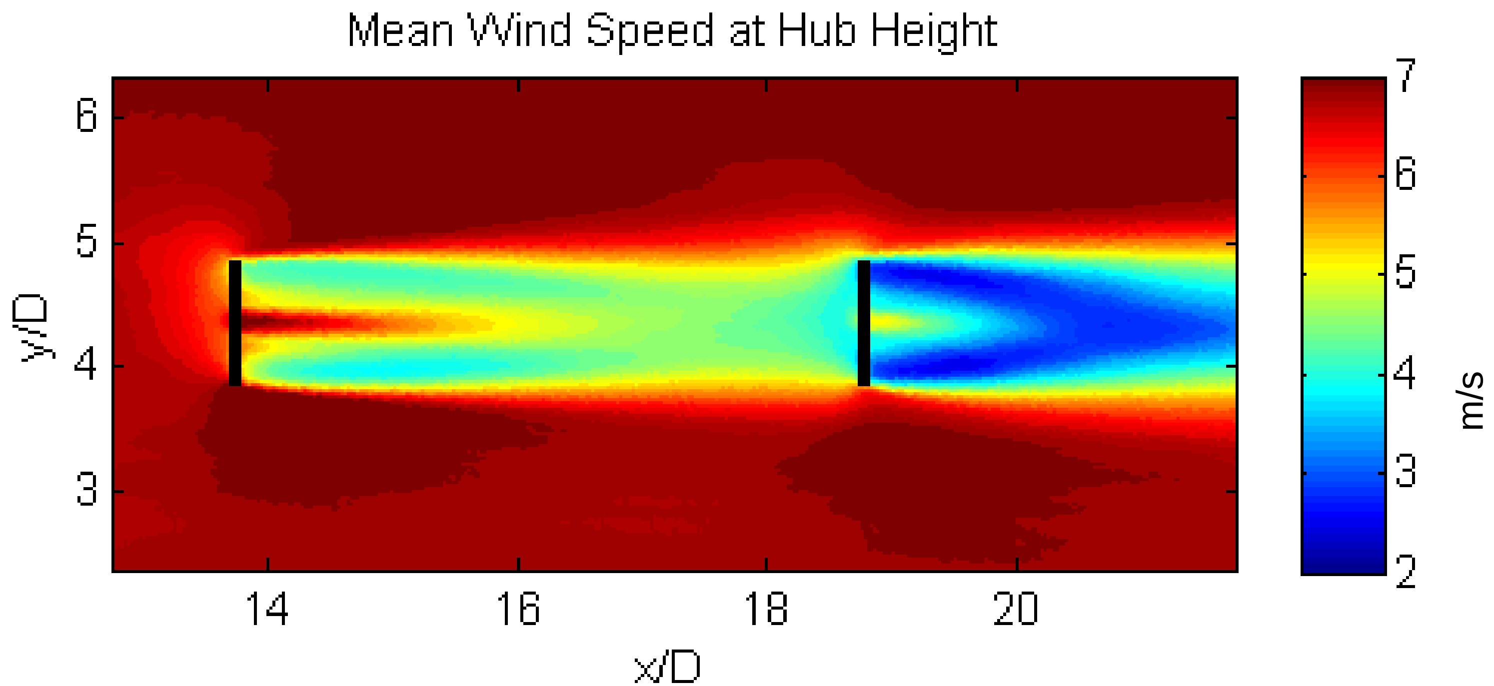
Reconstructed Flow

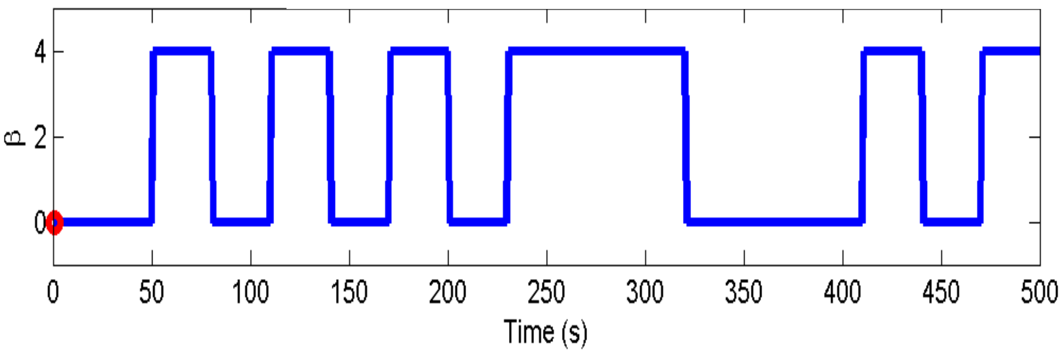
- Model constructed using 20 modes



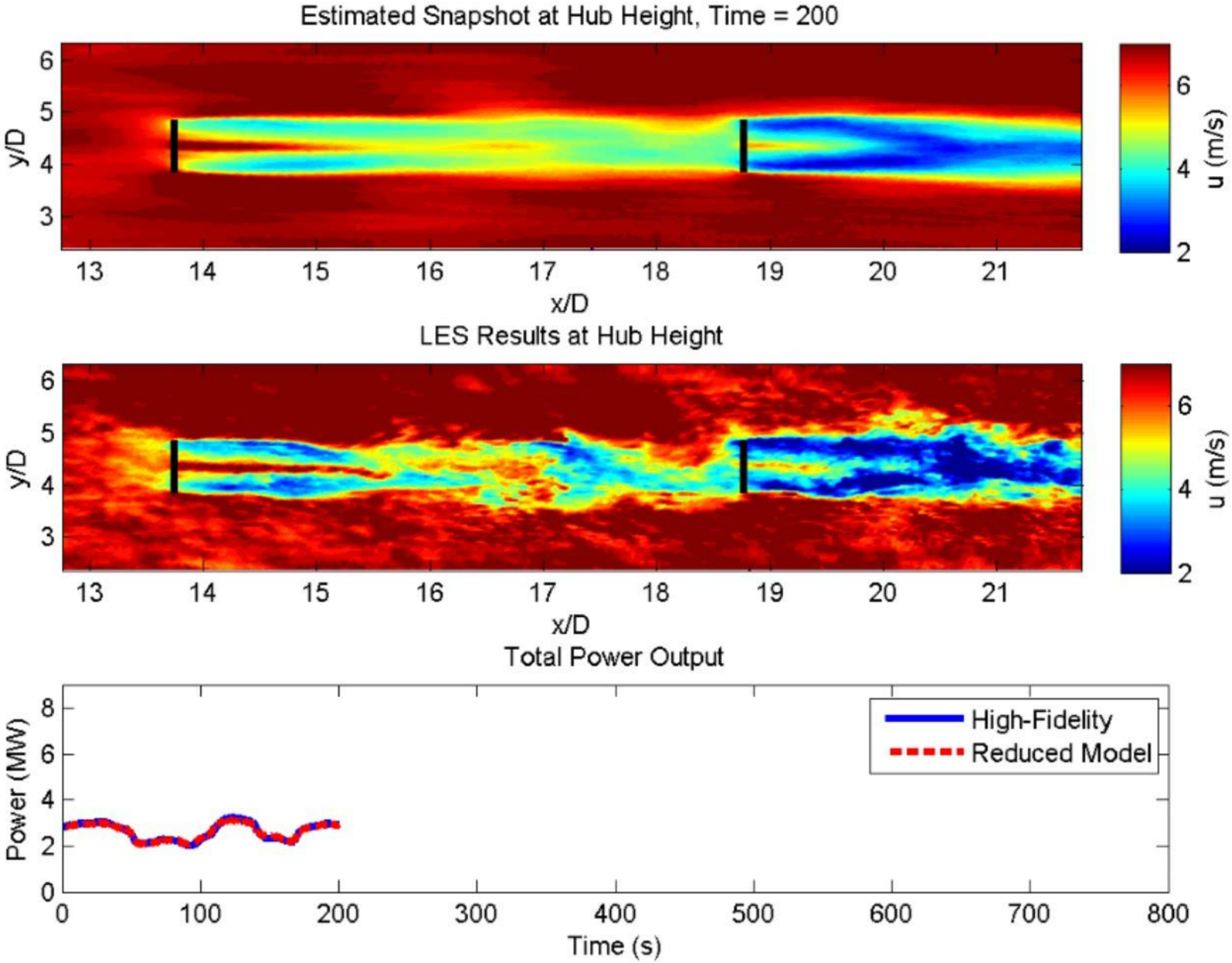
Model applied to Validation Data

- Validation case – same setup with a different input





Blade pitch angle changes from 0^o to 4^o


Estimated Snapshot at Hub Height, Time = 200
y/D
x/D
u (m/s)
LES Results at Hub Height
y/D
x/D
u (m/s)
Total Power Output
Power (MW)
Time (s)
High-Fidelity
Reduced Model

Acknowledgements

- US National Science Foundation
  - Grant No. NSF-CMMI-1254129: “CAREER: Probabilistic Tools for High Reliability Monitoring and Control of Wind Farms.” Prog. Manager: J. Berg.
  - Grant No. NSF/CNS-1329390: “CPS: Breakthrough: Collaborative Research: Managing Uncertainty in the Design of Safety-Critical Aviation Systems”. Prog. Manager: D. Corman.
- NASA
  - NRA NNX14AL36A: "Lightweight Adaptive Aeroelastic Wing for Enhanced Performance Across the Flight Envelope," Tech. Monitor: J. Bosworth.
  - NRA NNX12AM55A: “Analytical Validation Tools for Safety Critical Systems Under Loss-of-Control Conditions.” Tech. Monitor: C. Belcastro.
  - SBIR contract #NNX12CA14C: “Adaptive Linear Parameter-Varying Control for Aeroservoelastic Suppression.” Tech. Monitor. M. Brenner.
- Eolos Consortium and Saint Anthony Falls Laboratory
  - http://www.eolos.umn.edu/ & http://www.safl.umn.edu/

Conclusions

Main Contributions in LPV Theory:

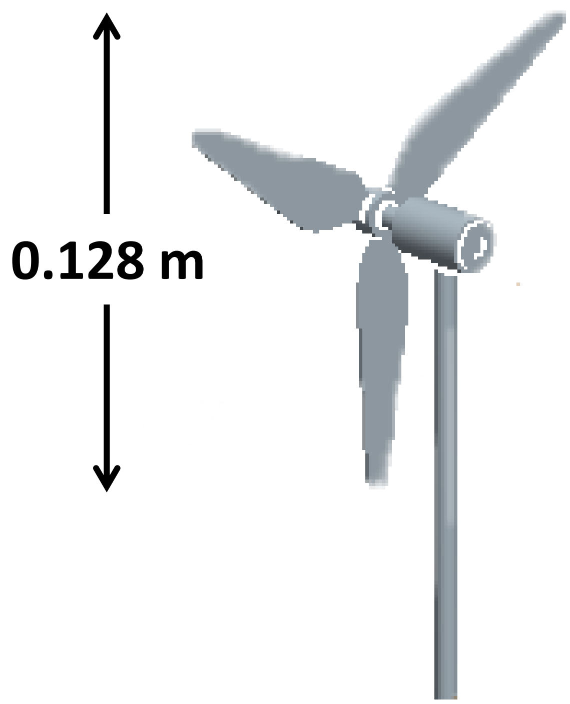
- Robustness analysis tools
- Model reduction methods

Applications to:

- Flexible and unmanned aircraft
- Wind energy
- Hard disk drives

http://www.aem.umn.edu/~SeilerControl/

Model Turbines



- Scale ⟶ 1:750
- 4.5 m/s
- 10% turbulence intensity

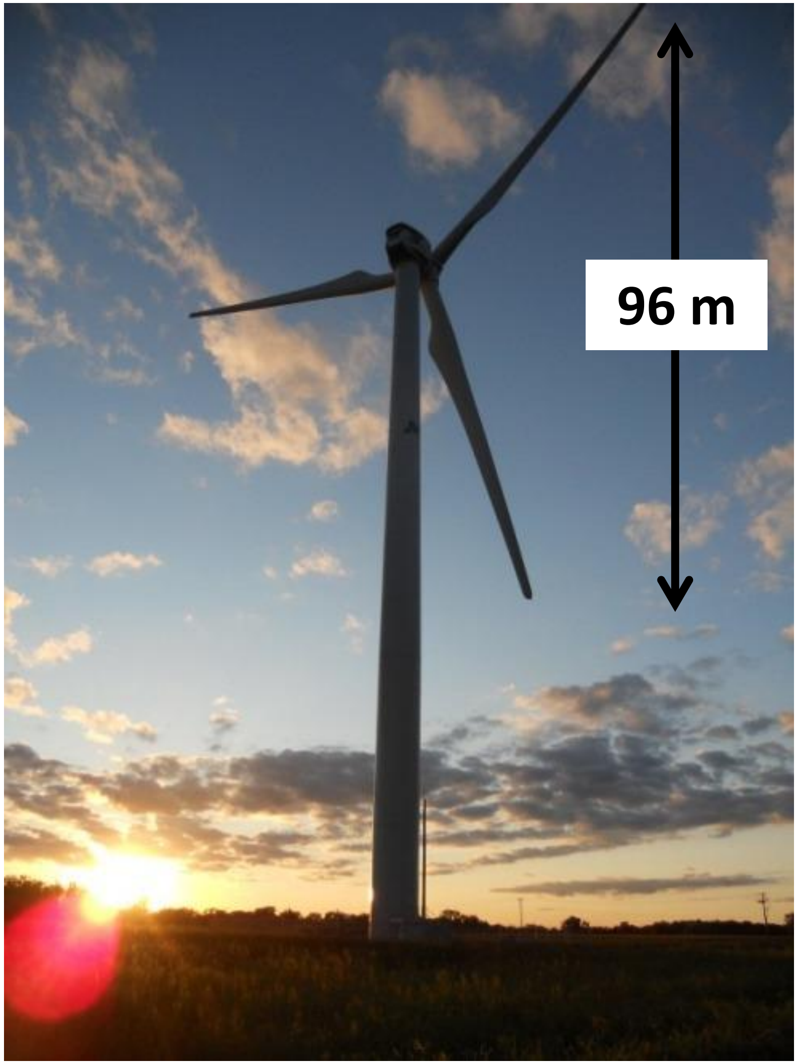
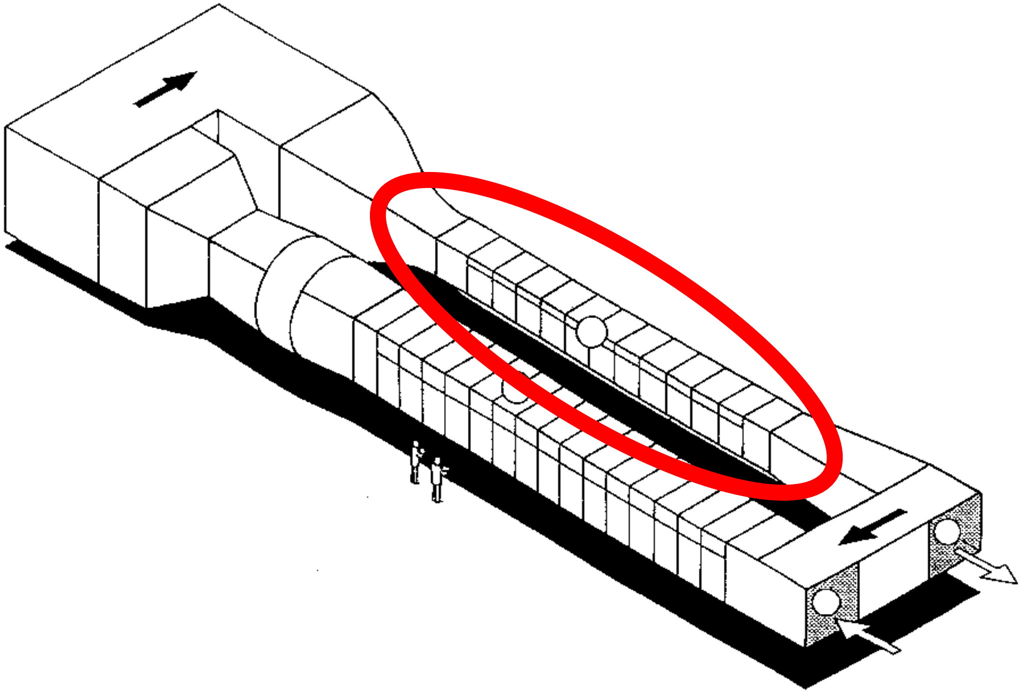


Photo credits: Kevin Howard

SAFL Wind Tunnel



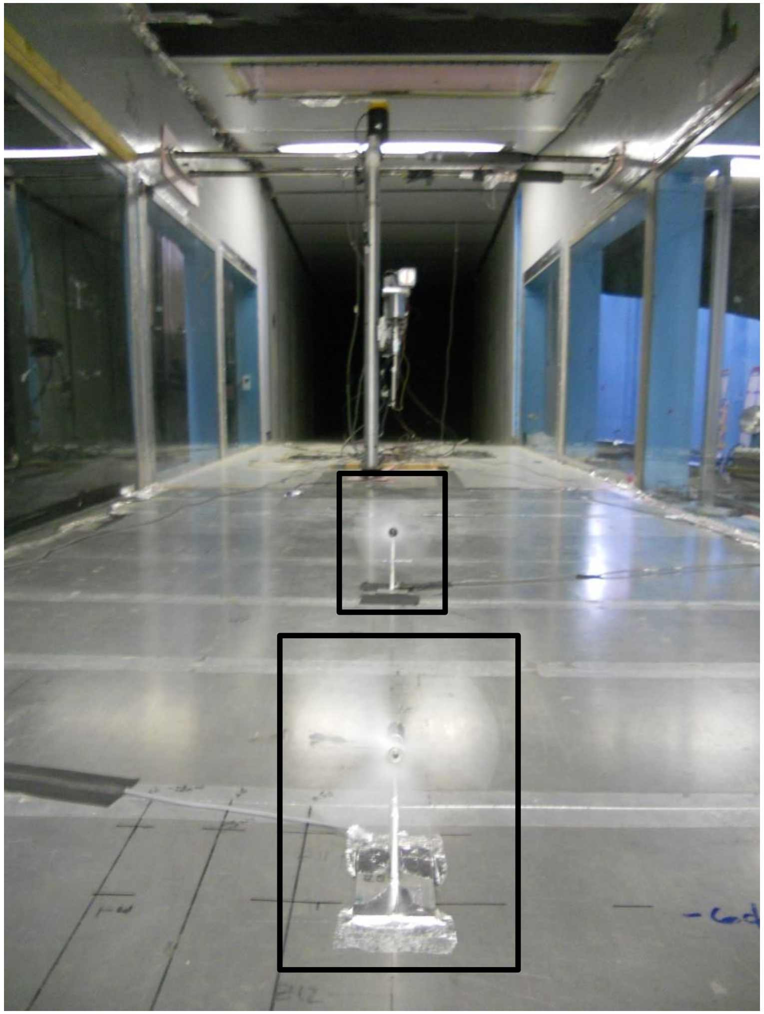
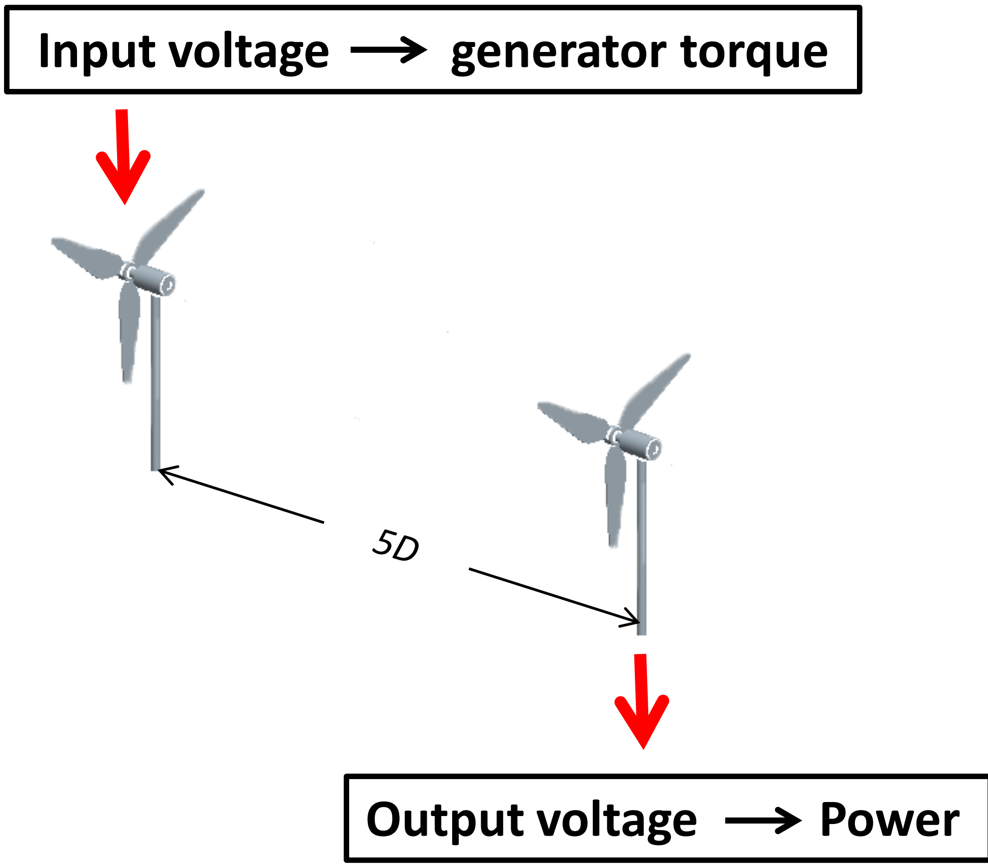
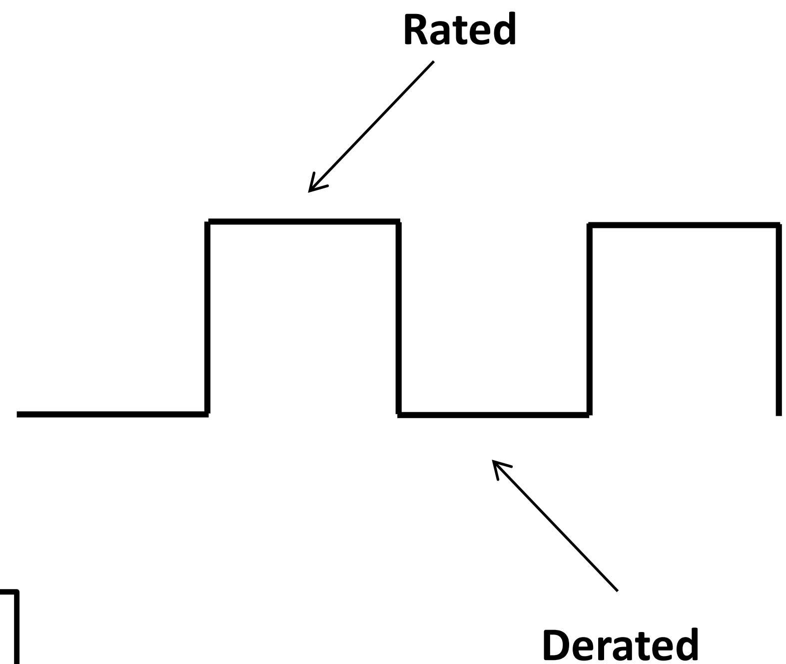


Photo credits: Kevin Howard

Voltage Measurements

- Understand the input/output dynamics

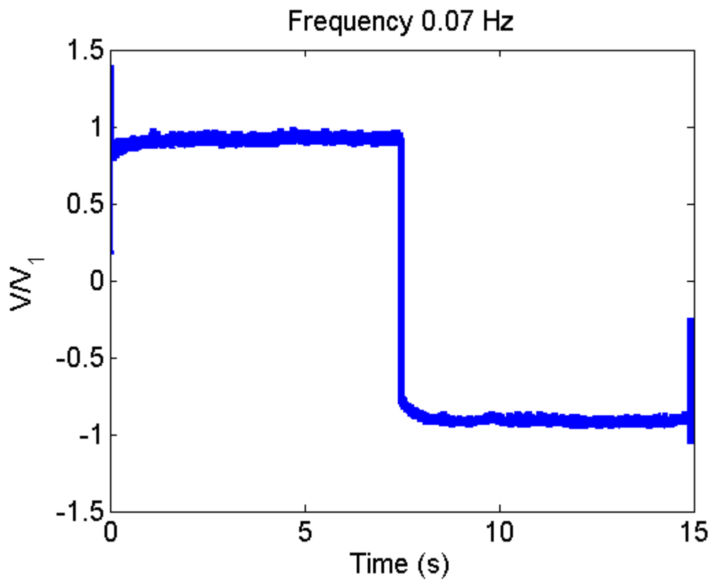




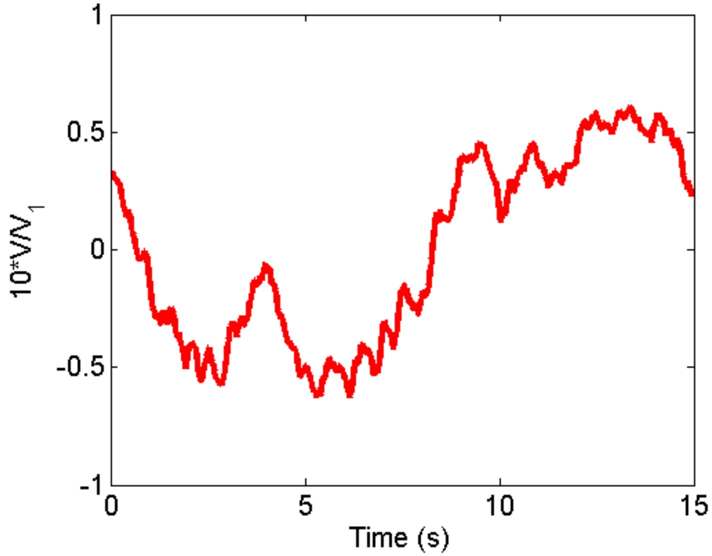
- Square waves with varying frequencies: 0.02Hz to 10Hz

Typical Result

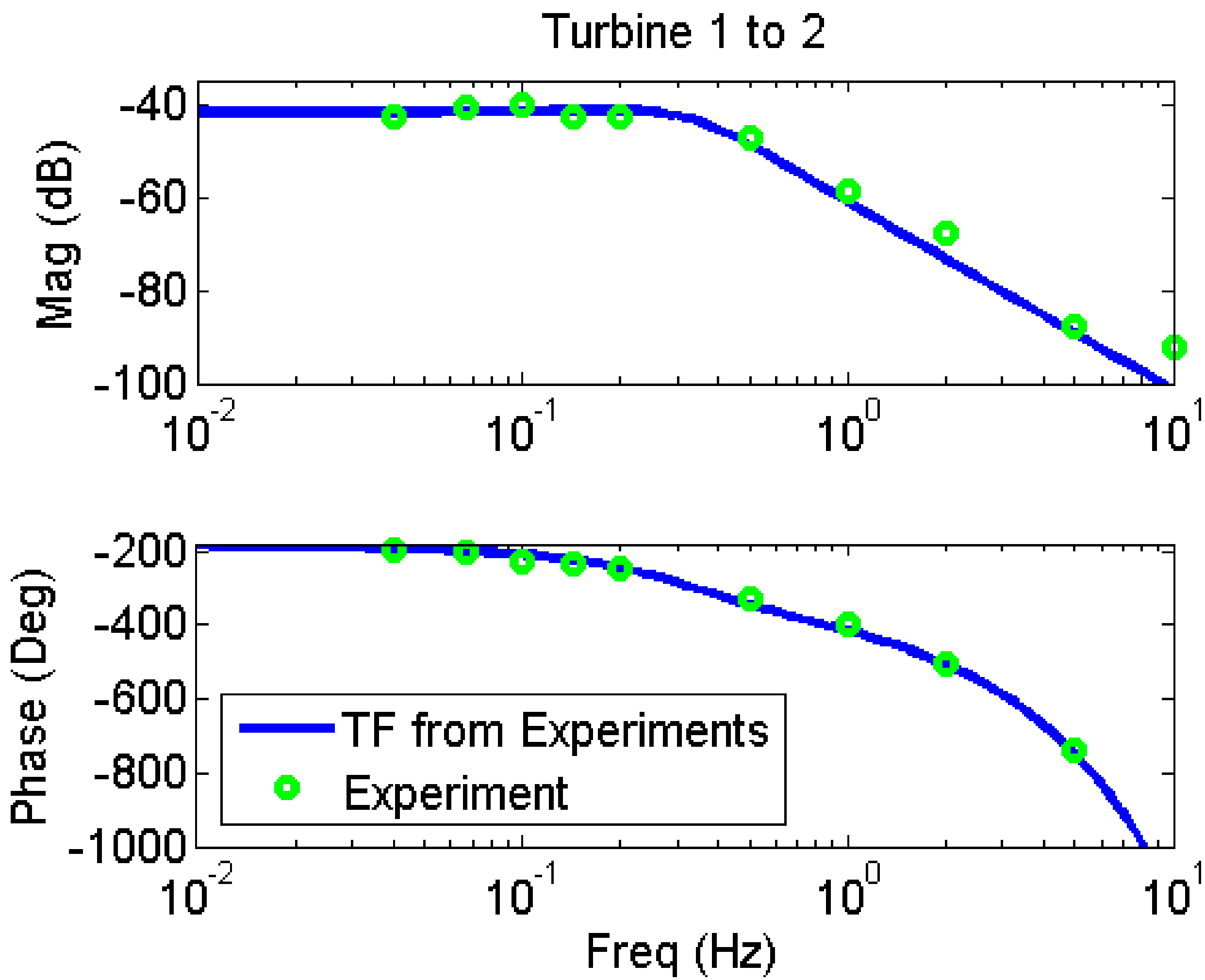
Turbine 1



Turbine 2



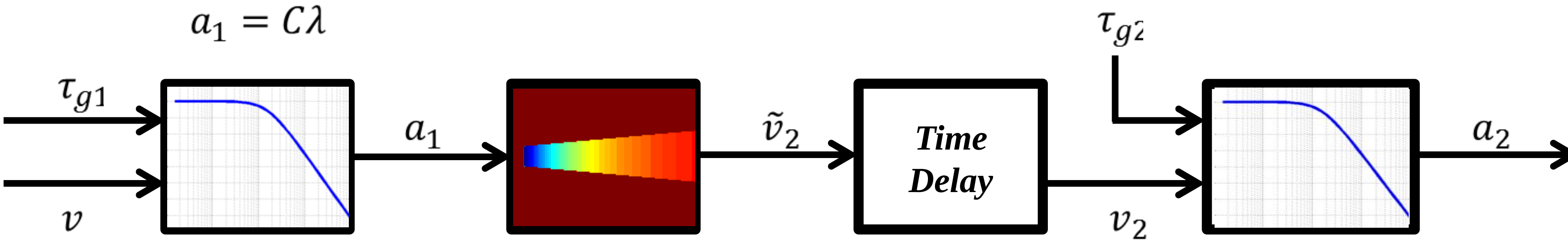
Dynamic Response



Dynamic Park Model

First Order Dynamics

$$\dot{\lambda} = A\lambda + B\tau_{g1}$$
$$a_1 = C\lambda$$



First Order Dynamics

$$\dot{\lambda} = A\lambda + B\tau_{g2}$$
$$a_2 = C\lambda$$

Dynamic Park Model