Linear Parameter Varying Techniques Applied to Aeroservoelastic Aircraft: In Memory of Gary Balas

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Abstract:
This paper presents an overview of Dr. Gary Balas research activities in linear, parameter-varying (LPV) systems applied to aeroservoelastic (ASE) aircraft. More efficient aircraft can be designed by reducing weight and structure in the wings and fuselage. This makes the aircraft more flexible leading to increased ASE effects. Such ASE aircraft can be modeled as a linear parameter varying (LPV) system with an arbitrary, i.e. not necessarily rational, dependence on the scheduling parameters. The system involves coupling of the aircraft structural dynamics and aerodynamics thus resulting in large state dimension. This large dimension necessitates special approaches to modeling, order reduction and control design. The paper describes the process of designing an LPV controller for flutter suppression of a flexible unmanned aircraft starting from the nonlinear equation of motions for the vehicle.

Keywords: Aerospace, Diagnosis and Control of LPV Systems

1. INTRODUCTION

This paper is a tribute to Dr. Gary Balas' work in the field of linear parameter varying (LPV) systems. Dr. Balas made countless major contributions to the LPV community over the last 25 years. His main research interest was narrowing the gap between engineering requirements, real-time control implementation, and theoretical control analysis and design techniques. Dr. Balas successfully applied LPV methods to a variety of systems. These include fighter aircraft (Balas et al., 1997), turbofan engines (Balas, 2002), active vehicle suspension (Fialho and Balas, 2002), and supercavitating vehicle (Vanek et al., 2010; Escobar Sanabria et al., 2014). Dr. Balas saw a significant potential for LPV techniques to improve the control of aeroservoelastic (ASE) aircraft given that their dynamics are naturally parameter dependent. His interest in flexible structures dates back to his early research as a PhD student at Caltech (Balas, 1990). He continued this work in the late nineties when he participated on the development of the Benchmark Active Controls Technology wind tunnel model (Barker and Balas, 2000). Finally, he dedicated the last five years to develop a research and experimental platform for control of ASE vehicles. The purpose of this paper is to summarize his final contributions to LPV systems and ASE aircraft.

ASE research aids the design of lightweight, slender wings, whose flexibility can increase aircraft endurance and maneuverability. These modifications can achieve fuel efficiency gains and extended range due to reduced air resistance, in turn reducing the cost of operation. However, the high flexibility and significant deformation in flight exhibited by these aircraft increases the interaction between the aerodynamics and structural dynamics, resulting in adverse handling qualities and may even lead to dynamic instability. This instability, called flutter, can destroy the aircraft if left uncontrolled. Hence, an integrated active approach to flight control, flutter suppression and structural mode attenuation is required to make full use of the benefits of modern highly flexible aircraft.

Dr. Balas established a research infrastructure at University of Minnesota (UMN) within the Unmanned Aerial Vehicle (UAV) Laboratory to study ASE vehicles. Details on this infrastructure are given in Section 2. Prof. Balas heartedly embraced a “research through development” philosophy for ASE aircraft. He partnered with NASA to “bring back the spirit of learning by flying” envisioned for the future fixed wing research (Warwick, 2014). Through this partnership, Dr. Balas obtained a small flexible UAV and initiated the development of UMN's own research platform. Based on those two platforms, his research focused on the modeling and control of this type of systems. Specifically, LPV models have been developed for these platforms and he worked on LPV order reduction schemes which are suitable for large scale ASE systems, see Section 3. Moreover, Dr. Balas worked on the design of LPV controllers to address flutter suppression and mode attenuation (Section 4). Dr. Balas had ambitious goals for the coming years. An extensive flight testing campaign is planned and UMN, along with other project partners, will design, develop and fly a performance adaptive aeroelastic wing, see Section 5.

Dr. Balas also highly valued a philosophy of open source development and availability of its resources to the larger aerospace community. Therefore, all the research data as well as the software used to obtain said data for the ASE research can be downloaded from (`http://www.aem.umn.edu/~AeroServoElastic/`). The ASE research group hopes that this data becomes an important resource and a future benchmark for researchers in the field.

2. FLIGHT TEST INFRASTRUCTURE

Reliable analysis tools and versatile experimental platforms are essential for the transition of theoretical research into real world application. Therefore, to facilitate research in ASE controls and other fields, an extensive support infrastructure is developed at the UAV laboratory at UMN. UMN has two flight test platforms available to conduct ASE research, namely the body freedom flutter (BFF) aircraft (Burnett et al., 2010) and the mini-MUTT (Multi Utility Technology Testbed), shown in Fig. 1. The former was developed by Lockheed Martin and Air Force Research Laboratory and later on donated to UMN. It is a high aspect ratio flying wing with a span of 3 m. The basic aircraft configuration of the BFF vehicle with location of sensors and control surfaces is presented in Fig. 2. The aircraft has 8 control surfaces and 11 sensors available for control. Sensor measurements include gyros, accelerometers and hot-film sensors located at the leading-edge stagnation point (LESP) to estimate the lift distribution.

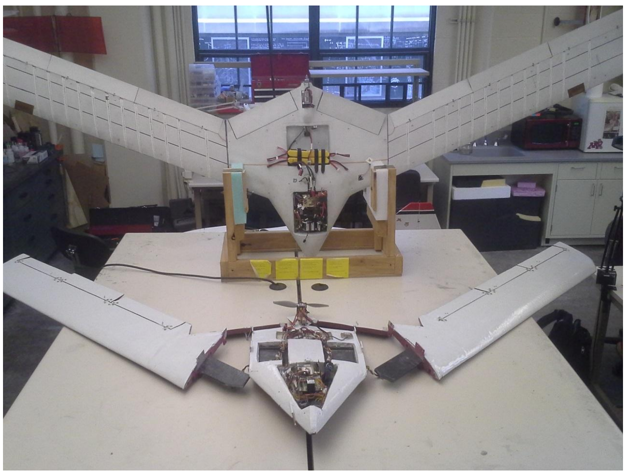


Fig. 1. BFF Vehicle (back) and Mini-MUTT (front)

The mini-MUTT has been completely designed and manufactured in-house at UMN based on the BFF aircraft's outer mold line. The key distinction between both aircraft is that the mini-MUTT follows a modular design philosophy similar to the X-56 MUTT aircraft. It features a common rigid center body and interchangeable wings of varying flexibility. This modular approach allows testing several wing configurations at a low cost.

Note that all the results presented in this paper are based on the BFF vehicle. In the future, it is contemplated to apply the same approach on the mini-MUTT.

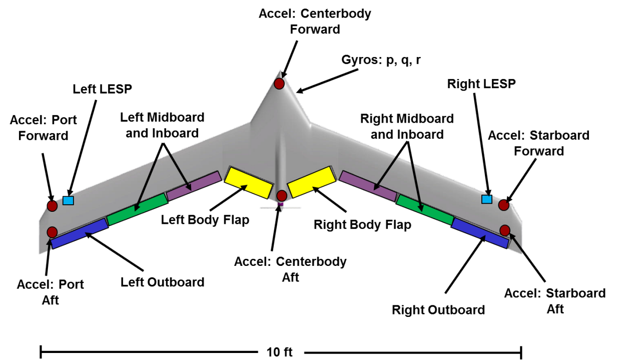


Fig. 2. Body Freedom Flutter Vehicle (Burnett et al., 2010)

3. LPV MODELING

This section briefly describes the method to obtain an LPV model for an ASE aircraft via Jacobian linearization. Consider a nonlinear system of the following form:

$$\begin{aligned} \dot{x}(t) &= f(x(t), u(t), \rho(t)) \\ y(t) &= h(x(t), u(t), \rho(t)) \end{aligned} \tag{1}$$

where f and h are differentiable, input $u(t) \in \mathbb{R}^{n_u}$, output $y(t) \in \mathbb{R}^{n_y}$, state variable $x(t) \in \mathbb{R}^{n_x}$ and $\rho(t) \in \mathbb{R}^{n_\rho}$ is a measurable exogenous parameter vector, called the scheduling parameter. The parameter vector ρ is assumed to be a continuously differentiable function of time and the admissible trajectories are restricted based on physical considerations to a known compact subset $\mathcal{P} \subset \mathbb{R}^{n_\rho}$. The rates of the parameter variation $\dot{\rho}$ are assumed to be bounded in some applications, i.e. $\dot{\rho} \in \dot{\mathcal{P}}$, where $\dot{\mathcal{P}} \subset \mathbb{R}^{n_\rho}$ is a compact subset. The set of admissible trajectories is defined as $\mathcal{A} := \{\rho : \mathbb{R}^+ \to \mathbb{R}^{n_\rho} \ : \ \rho(t) \in \mathcal{P},\ \dot{\rho}(t) \in \dot{\mathcal{P}}\ \forall t \geq 0\}$. Throughout the paper the explicit dependence on t is suppressed to shorten the notation.

Assumption 1. There is a family of equilibrium points $(\bar{x}(\rho), \bar{u}(\rho))$ such that

$$\left.\begin{aligned} f(\bar{x}(\rho), \bar{u}(\rho), \rho) &= 0 \\ \bar{y}(\rho) &= h(\bar{x}(\rho), \bar{u}(\rho), \rho) \end{aligned}\right. \quad \forall \rho \in \mathcal{A}. \tag{2}$$

The nonlinear system given by (1) can be linearized about the equilibrium points via Jacobian linearization based on Taylor series expansion. Define the deviation variables as

$$\delta_x := x - \bar{x}(\rho), \quad \delta_d := d - \bar{d}(\rho), \quad \delta_y := y - \bar{y}(\rho). \tag{3}$$

Differentiating the δ_x term of (3) results in

$$\dot{\delta}_x = \dot{x} - \dot{\bar{x}}(\rho) = f(x, u, \rho) - \dot{\bar{x}}(\rho). \tag{4}$$

The Taylor series expansion of f and h about the equilibrium point yields

$$\begin{aligned} \dot{\delta}_x &= \nabla_x f\big|_o \delta_x + \nabla_u f\big|_o \delta_u + \epsilon_f(\delta_x, \delta_u, \rho) - \dot{\bar{x}}(\rho) \\ \delta_y &= \nabla_x h\big|_o \delta_x + \nabla_u h\big|_o \delta_u + \epsilon_h(\delta_x, \delta_u, \rho), \end{aligned} \tag{5}$$

where the $|_o$ denotes the evaluation at the equilibrium point $(\bar{x}(\rho), \bar{u}(\rho), \rho)$, ϵ_f and ϵ_h represent the higher order terms of the Taylor series expansion. The term $\dot{\bar{x}}(\rho)$ arises due to the time variation in ρ. It is important to emphasize that this term is equal to zero considering a single operating condition (i.e., a constant value of ρ) and hence this term disappears in the normal linearization process. However, it must be retained here as we require our control design to apply to changing operating conditions (i.e., time varying $\rho(t)$). The linearization is performed with respect to (x, u) but the nonlinear dependence on ρ is retained. Define $L(\rho) := -\nabla \bar{x}(\rho)$. Then the linearization about the family of trim points (2) takes the form:

$$\begin{aligned} \dot{\delta}_x &= A(\rho)\delta_x + B(\rho)\delta_u + L(\rho)\dot{\rho} + \epsilon_f(\delta_x, \delta_d, \rho) \\ \delta_y &= C(\rho)\delta_x + D(\rho)\delta_u + \epsilon_h(\delta_x, \delta_u, \rho) \end{aligned} \tag{6}$$

The LPV system is commonly obtained by assuming that the higher order terms of the Taylor series are negligible, i.e. ϵ_f and $\epsilon_h \approx 0$. In addition, it is typically assumed that the parameter variation is sufficiently slow so that $L(\rho)\dot{\rho} \approx 0$. Under these assumptions, an LPV system is defined using x, u, y instead of the deviation variables as:

$$\begin{aligned} \dot{x} &= A(\rho)x + B(\rho)u \\ y &= C(\rho)x + D(\rho)u \end{aligned} \tag{7}$$

Note that other approaches in literature exist to obtain LPV models from a nonlinear system, e.g., function substitution (Tan, 1997). An overview of different techniques to obtain LPV models for aircraft is given in Marcos and Balas (2004). The most prevalent approach, however, is based on Jacobian linearization as presented here. This is also the approach pursuit in this paper to obtain LPV models of the ASE systems.

3.1 LPV Models of ASE Systems

Lockheed Martin constructed a collection of linearized models for the BFF aircraft on a grid of flight speeds. They provided this grid based LPV model along with the actual BFF aircraft to the UMN UAV lab. The gridded model consists of 21 linear models parameterized by equivalent airspeed from 40 to 80 knots in steps of 2 knots, see Burnett et al. (2010). The model represent straight level flight at 1000 ft altitude. The airframe model, excluding sensor and actuator models, has 148 states. The results presented in this paper are based on this Lockheed Martin model.

In parallel to this work, UMN is developing nonlinear models for flexible aircraft from first principles. Various techniques exist in literature to derive the equations of motion for a flexible aircraft. The main difference is in the coordinate system used in the derivation. The two predominant approaches in literature are based on a mean axis reference frame (Waszak and Schmidt, 1988; Schmidt, 2012) and a body fixed reference frame (Meirovitch, 1989; Meirovitch and Tuzcu, 2004). The former is a floating frame, i.e. not fixed to a material point on the aircraft. Its origin is chosen at the instantaneous center of mass. The latter is fixed to a material point of the undeformed aircraft.

Modeling ASE systems requires the integration of structural dynamics, aerodynamics, and flight dynamics. Each of theses models can be developed separately. The aerodynamics of the BFF aircraft is modeled using the doublet lattice method (DLM). DLM is a panel method that considers unsteady aerodynamic effects and solves for flow across a harmonically oscillating lifting surface (Albano and Rodden, 1969). A DLM code has been developed inhouse at UMN that is openly available on the group's website (Kotikalpudi et al., 2015). The structural dynamics model of the BFF aircraft is obtained from a finite element model constructed using linear Euler beams, see Moreno et al. (2014a); Gupta et al. (2015).

Independent of the chosen approach and the complexity of the subcomponents, the equations of motion of a flexible aircraft will have the general form of a nonlinear system as given in (1). The scheduling parameter ρ for typical flexible aircraft can include, e.g., airspeed, altitude, mass and center of gravity position. An LPV model of the aircraft dynamics can be obtained by applying the Jacobian linearization based approach described above.

3.2 LPV Model Order Reduction

The computational complexity associated with synthesizing LPV controllers strongly depends on the dynamic order of the models. Including structural dynamics and aeroelastic effects, as described in the previous section, results in high order LPV models that cannot be used for control design. Hence, reduced order LPV models of ASE systems are needed. This section summarizes the balanced LPV model reduction method derived by Wood et al. (1996) and a local approximation method termed "modal matching" (Theis et al., 2015b). A more comprehensive discussion about extensions of LTI model reduction to LPV systems is given in Moreno et al. (2014b).

Balanced LPV model reduction is based on the measures of controllability and observability of a state-space model. These measures are provided by symmetric positive definite matrices $P \succ 0$ and $Q \succ 0$ satisfying the Lyapunov inequalities

$$\begin{aligned} A(\rho)P + P\,A^T(\rho) + B(\rho)\,B^T(\rho) \prec 0 \\ A^T(\rho)\,Q + Q\,A(\rho) + C^T(\rho)\,C(\rho) \prec 0 \end{aligned} \quad \forall \rho \in \mathcal{A} \tag{8}$$

The matrices P and Q are called Gramians, which in general can be parameter varying. However, in this approach only constant Gramians are considered. Additionally, Gramians exist only for stable LPV systems. The approach is hence not suitable for ASE models that include unstable dynamics. To address this problem, a contractive right coprime factorization of an LPV system (7) is defined as

$$\begin{bmatrix} \dot{x} \\ y \\ u \end{bmatrix} = \begin{bmatrix} A(\rho) + B(\rho)F(\rho) & B(\rho)S^{-1/2}(\rho) \\ C(\rho) + D(\rho)F(\rho) & D(\rho)S^{-1/2}(\rho) \\ F(\rho) & S^{-1/2}(\rho) \end{bmatrix} \begin{bmatrix} x \\ q \end{bmatrix}, \tag{9}$$

where $q \in \mathcal{L}_2$ is introduced as an auxiliary signal, $S(\rho) = I + D^T(\rho)\,D(\rho)$, and $F(\rho) = -S(\rho)^{-1}(B^T(\rho)\,X + D^T(\rho)\,C(\rho))$. In the following, the dependence on ρ is omitted for brevity. The controllability Gramian P and observability Gramian Q of the realization (9) are

$$Q = X, \qquad P = (I + Y\,X)^{-1}Y, \tag{10}$$

where $\forall \rho \in \mathcal{A}$, the matrices $X \succ 0$ and $Y \succ 0$ must satisfy the generalized Riccati inequalities represented by the linear matrix inequalities (LMIs)

$$\begin{bmatrix} \Lambda_X - B\,S^{-1}B^T & X\,C^T \\ \star & -R \end{bmatrix} \prec 0, \tag{11a}$$

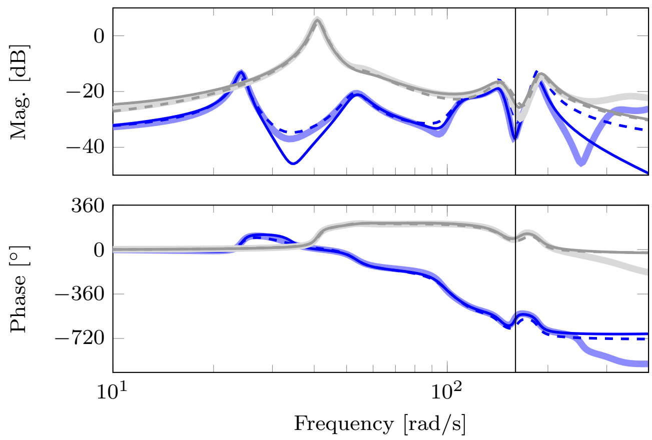
$$\begin{bmatrix} \Lambda_Y - C^T R^{-1} C & Y\,B \\ \star & -S \end{bmatrix} \prec 0, \tag{11b}$$

with $\Lambda_X = X\,(A - B\,S^{-1}\,D^T C)^T + (A - B\,S^{-1} D^T C)\,X$, $\Lambda_Y = Y\,(A - B\,D^T R^{-1} C) + (A - B\,D^T R^{-1} C)^T Y$, and $R = I + D\,D^T$.

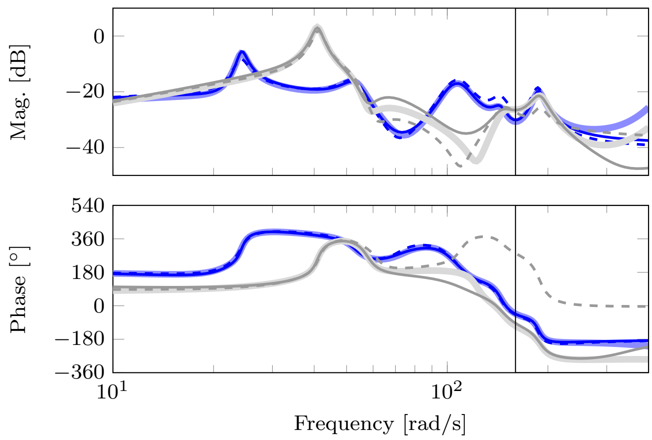
A significant problem with this approach is the growth of the LMI problem (11) with the dynamic order and the number of scheduling parameters of an LPV model. Higher order systems hence require a heuristic pre-processing to reduce the number of states before applying the balanced reduction.

In contrast to this global LPV model order reduction, Theis et al. (2015b) proposed to perform reduction at each grid point individually and to define a new LPV model from the resulting set of local LTI models. While this allows numerically stable and well-tractable computational methods to be applied, a new problem arises. Since individual reduction results in different state space bases for each reduced order model, a consistent representation needs to be constructed in order to define an LPV system. This is achieved by transforming the local models into a canonical modal form with approximately consistent state space bases for all models.

Both the global and the local order reduction approaches were successfully applied to the BFF vehicle in order to obtain a low-order model for control systems design (Moreno et al., 2014b; Theis et al., 2015b). With the global approach, a sequence of truncating low-frequency dynamics and residualizing high frequency dynamics was used to pre-process the LPV model before the balanced LPV reduction was applied. The result of this procedure is an LPV model with 26 states. Following the local approach, modal decomposition and balanced reduction were applied individually to the local models. An LPV model with 15 states is afterwards obtained by interpolating the canonical modal state space representations of the local reduced order systems. Similar accuracy for the frequency range of interest (10–160 rad/s) was observed for both reduced order models. Bode plots showing these results are presented in Fig. 3.



(a) From starboard body flap to starboard wing forward



(b) From port outboard flap to starboard wing forward

Fig. 3. Bode plots of full order model (148 states) at 43 knots ( ) and 77 knots ( ), locally reduced model with 15 states (/), and globally reduced model with 26 states (/). The lightened area denotes the frequency range beyond interest.

4. LPV CONTROL DESIGN

The induced $\mathcal{L}_2$-norm of an LPV system G_ρ from input d to output e is defined as

$$\|G_\rho\| = \sup_{d \in \mathcal{L}_2 \setminus \{0\},\ \rho \in \mathcal{A},\ x(0)=0} \frac{\|e\|_2}{\|d\|_2}\,, \tag{12}$$

i. e., the largest amplification of $\mathcal{L}_2$ input signals over all admissible trajectories. It can be used to specify performance for a feedback interconnection in terms of a generalized plant P_ρ, analog to $\mathcal{H}_\infty$-norm optimal design for LTI systems. An LPV controller K_ρ can be synthesized using the by now well-known bounded-real type LMI conditions developed by Wu (1995); Wu et al. (1996). The controller is guaranteed to internally stabilize the closed-loop interconnection given by the lower fractional transformation $\mathcal{F}_L(P_\rho, K_\rho)$ and to achieve a performance index γ that provides an upper bound on the induced $\mathcal{L}_2$-norm, i. e. $\|\mathcal{F}_L(P_\rho, K_\rho)\| < \gamma$.

Dr. Balas was a major contributor in the development of the Robust Control Toolbox for Matlab. This toolbox played a significant role in transitioning theoretical robust control results to industrial practice. In the last few years, Dr. Balas was involved in the development of another Matlab toolbox, LPVTools (Hjartarson et al., 2014). LPVTools contains a suite of functions for modeling, analysis, and control design of LPV systems. Among its various functions, LPVTools implements the infinite dimensional LPV controller synthesis conditions (Wu, 1995) as a finite dimensional approximation on a grid $\{p_k\}_1^{n_{\text{grid}}} \subset \mathcal{A}$ and allows to efficiently solve the resulting convex optimization problem of minimizing the performance index γ.

Researchers at UMN are using LPVTools for the design and analysis of control systems for ASE aircraft. One particular control design objective for the BFF vehicle is to stabilize the aircraft across the flight envelope and to increase damping of flexible modes up to 60 knots. Selection of adequate sensors and control effectors plays a critical role in the control design process. A wrong choice may put fundamental limitations on performance and robustness that cannot be overcome even by available advanced control design techniques. An approach based on the sensor selection via closed-loop objectives by Balas and Young (1999) has been extended to consider uncertainty in aeroservoelastic systems. The goal is to find the best configuration of actuators and sensors that provides sufficient robustness and the desired performance by accounting for model uncertainty (Moreno et al., 2015). Based on this analysis, the considered control inputs u for flutter suppression are deflections of the right and left body as well as outboard flaps. The output measurements y are the six accelerometers in the wings and center body.

A reduced order model of the BFF aircraft as described in the previous section is used to design a dedicated flatter suppression controller. The performance specifications are shown in Fig. 4. The generalized velocities of the (unstable) first symmetric bending, symmetric torsion and antisymmetric torsion modes are collected into the signal η that is weighted and forms the penalty e_1 (Theis et al., 2015a; Hanel, 2001). The performance weight used in the design is $W_\eta = \mathrm{diag}\{10, 10, 5\}$. Disturbances to the control surfaces are modeled by a signal d_1 and weights $W_d = 1$. Control authority is limited by weights $W_u = 1000(s + 165.2)/(s + 1.835 \times 10^5)$ to avoid excitation of unmodeled high-frequency dynamics outside of the control bandwidth. In addition, noise d_2 on the acceleration measurements is included with a weight $W_n = 0.2$. The controller K_ρ minimizes the induced $\mathcal{L}_2$-norm from $[d_1^T \ d_2^T]^T$ to $[e_1^T \ e_2^T]^T$ and hence seeks to attenuate structural vibration of the aircraft with limited control authority.

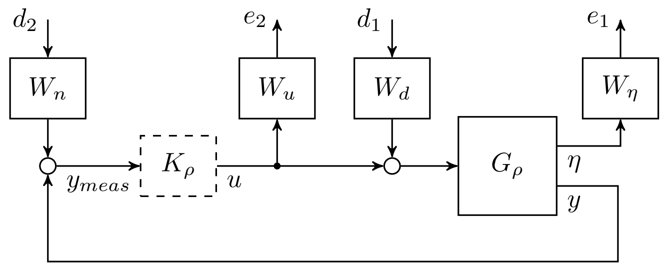
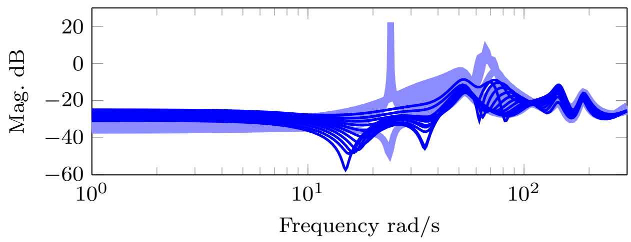


Fig. 4. Control interconnection for output feedback design

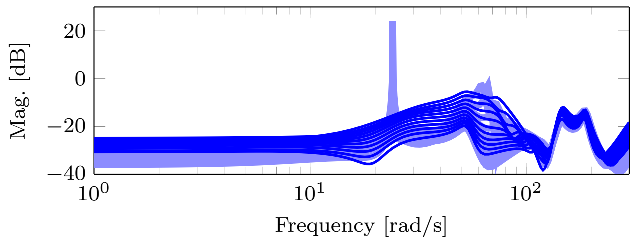
An LPV controller is synthesized for a grid that covers airspeed from 40 to 60 knots in increments of 2 knots. Fig. 5 shows the full-order (148 states) open-loop and closed-loop frequency responses of the system at the grid points. Without active flutter suppression, flying above the critical airspeed would inevitably result in immediate structural failure due to the fast growing oscillation indicated by the large peak at around 25 rad/s. The LPV controller completely compensates this peak and stabilizes the aircraft. Moreover, the magnitude of the closed-loop gain is decreased well below 0 dB and has a flattened characteristic compared to the open-loop response. This shows that disturbances are well attenuated evenly across frequency and for all considered airspeeds. For low frequencies and at some mid frequencies, visible e. g. in Fig. 5(b) for 30–50 rad/s, disturbance sensitivity is slightly increased.

5. FUTURE WORK

Future work will mature the presented LPV approach to ASE control in order to design and control a performance adaptive aeroelastic wing. Valuable knowledge gained from prototype flight experiments, see Fig. 6, will be exploited to drive advances in modeling, controls, optimization, effector/sensor selection and design processes. The performance adaptive aeroelastic wing will optimally include many distributed control effectors and a large distributed sensor network enabling innovative control solutions for flutter suppression and alleviation of gust and turbulence



(a) From starboard outboard flap to starboard wing aft



(b) From starboard body flap to starboard wing forward

Fig. 5. Frequency response of full order open-loop model ( ) and closed-loop ( ).

loading, further decreasing the structural weight. This will result in a wing that is significantly lighter than the current state of the art. This wing will suppress flutter, including body-freedom flutter, and gust loads while morphing its shape to minimize drag over a range of cruise conditions and produce high-lift for takeoff and landing. It will significantly reduce fuel burn, emissions, and noise while enabling operations at local, non-hub airports. These results will contribute towards an aviation infrastructure with a secure future that is environmentally friendly.

Fig. 6. Mini-MUTT Flight Test

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