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the effect of the ratio of SPs on overall performance. We have conducted a set of experiments to examine this point and our results are reported in Figure 5. The interesting result we observe is that as far as the DPs are concerened, when we have 15% or 20% of SPs we have achieved the major gain in delay reduction. Going beyond those values does not significantly reduce the delay for DPs.

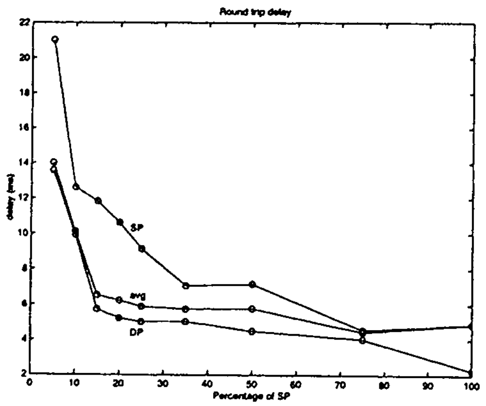

Figure 5: Average round-trip delay for smart (top) and dumb (bottom) packets, and average delay for all packets (center) as a function of the percentage of smart packets. These experimental results were obtained while links cpn10-cpn2, cpn2-cpn4 and cpn4-cpn5 were loaded with obstructing traffic.

6 Conclusions

We have summarized the basic principles of CPN. Then we have derived analytical results for best and worst case performance of one particular class of packets: the smart packets. We then describe in some detail the design and implementation of a test-bed network. Finally we have provided measurement data on the test-bed to illustrate the capacity of the network to adapt to changes in traffic load and to failures of links. Measurements have also been reported to evaluate the impact of the ratio of smart packets on the end-to-end delay experienced by all of the packets.

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Mesh Stability of Helicopters

Karl Hedrick, Aniruddha Pant, Pete Seiler[1][2]

1 Introduction

Intuitively mesh stability is the property of damping disturbances as they travel away from the source in an interconnected system [8]. In this paper we will give a general methodology for designing mesh stable controllers for interconnected systems where the subsystems have linear dynamics. In particular, the procedure will be applied to a linear helicopter model [9].

Section 2 outlines some background material and definitions we will use. Section 3 explains the linear transfer function model for the helicopter which is obtained by performing system identification experiments on a Yamaha R-50 crop dusting model helicopter [9]. We then describe a controller which stabilizes the linear helicopter dynamics. Section 4 proposes a mesh controller structure. Some advantages of the proposed structure over the previous mesh controller design models [6] are (a) it is applicable to a relatively general error propagation dynamics and (b) it leads to an inherently mesh stable cluster controllers. An algorithm for the design is presented. At last, we show the simulations with three helicopters flying as a string.

2 Background

In this paper we consider a string of helicopters. For notational details when the helicopters are in two dimensional formation refer to [8, 10].

Connective stability in one dimension is called string stability and has been studied [4], [6], and [7]. For string stability, we would like the maximum spacing error to decrease as it propagates down the chain. We will use the following norm definitions: $\|f(\cdot)\|_\infty = \sup_{t\geq 0}|f(t)|$ and $\|f(\cdot)\|_1 = \int_0^\infty |f(\tau)|d\tau$. If ϵ_i and ϵ_{i+1} are the errors at the i^{th} and $i+1^{th}$ vehicle in the chain, then we need $\|\epsilon_{i+1}\|_\infty \leq \|\epsilon_i\|_\infty$ for string stability.

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From linear system theory [2], if $y = h * u$, then we have the following relationship:

$$\|y(t)\|_\infty \leq \|h(t)\|_1 \; \|u(t)\|_\infty \tag{1}$$

Using a sliding control law, Hedrick and Swaroop [4] found an LTI convolution kernel, $h(t)$, which relates the errors in a vehicle following chain by: $\epsilon_{i+1} = h * \epsilon_i$. Thus string stability of the chain of vehicles can be determined by analyzing the one-norm of the error propagation impulse response, $h(t)$. Since this norm represents the maximum amplification of any error as it propagates down the chain, it provides a useful metric for string stability. If this norm is less than one, then all input errors will be attenuated in the ∞-norm sense as they propagate down the chain. If this norm is greater than one, then the system is string unstable and there exists an input error which will be amplified as it propagates.

If $h(t)$ does not change sign, the string stability condition, $\|h(t)\|_1 \leq 1$, is equivalent to the following frequency domain condition: The magnitude of the associated transfer function, $H(j\omega)$, should be less than one at all frequencies, i.e. $\|H(j\omega)\|_\infty \leq 1$. In [4], they found that the sliding control law resulted in a string stable system if reference vehicle information was used.

The SISO input/output norm results are easily generalized to the MIMO case. Let $f : \mathbb{R}_+ \rightarrow \mathbb{R}^n$ and define $\|f(\cdot)\|_\infty = \max_i \sup_{t\geq 0}|f_i(t)|$. If $h(t)$ is the convolution kernel for an n-input, n-output MIMO system, and $y = h * u$, then the input-output relationship is given by [2]:

$$\|y(t)\|_\infty \leq \left(\max_i \sum_{j=1}^{n} \|h_{ij}(t)\|_1 \right) \cdot \|u(t)\|_\infty \tag{2}$$

This can also be related to an equivalent frequency domain condition if none of the entries of the convolution kernel changes sign. Let $H(j\omega)$ be the $n \times n$ transfer function matrix for the LTI system given by $h(t)$. If none of the $h_{ij}(t)$ change sign, then:

$$\|y(t)\|_\infty \leq \left(\max_i \sum_{j=1}^{n} \|H_{ij}(j\omega)\|_\infty \right) \cdot \|u(t)\|_\infty \tag{3}$$

For the problem under consideration, $H(s) \in \mathbb{C}^{4\times4}$. The inputs are the desired Cartesian positions and yaw of the helicopter. The outputs are the realized Cartesian positions and yaw of the helicopter

3 Regulated Helicopter Model

In this section we briefly describe a linear model for a Yamaha R-50 agricultural helicopter [9]. We then describe the linear controller which is used to track desired position trajectories. The linear model is given by:

$$\dot{\bar{x}} = A\bar{x} + B\bar{u}$$

where the state and input vectors are given by:

$$\begin{aligned} \bar{x} &= [u\ v\ p\ q\ \Phi\ \Theta\ a_{1s}\ b_{1s}\ w\ r\ r_{fb}]^T \\ \bar{u} &= [u_{a_{1s}}\ u_{b_{1s}}\ u_{\Theta_M}\ u_{\Theta_T}]^T \end{aligned}$$

u,v,w are the x,y,z body-fixed velocities, respectively. Φ, Θ are the roll and pitch of the helicopter while p, q are the roll and pitch rate. r is the yaw rate. a_{1s}, b_{1s}, and r_{fb} are actuator states. The first two inputs, $u_{a_{1s}}$ and $u_{a_{1s}}$, control the flapping coefficients of the helicopter. These inputs primarily control the pitch and roll of the helicopter. The final two inputs, u_{Θ_M} and u_{Θ_T}, control the main rotor thrust and tail rotor thrust. A predictor-error method was used to obtain the parameters in the A,B matrices from experimental input-output data. We refer the interested reader to [9] for additional modeling details. We note that if the helicopter remains close to hovering, we can treat the x,y,z body-fixed velocities as global x,y,z velocities. Integration of these variables will then yield global position. Similarly, integration of the body-fixed yaw rate will yield the global heading if the helicopter is close to hovering.

A control law was designed by Shim [9] to stabilize the helicopter dynamics and steer the vehicle along a desired trajectory. The desired trajectory is given by:

$$r_d = [x_d\ y_d\ z_d\ \Psi_d]^T \tag{4}$$

Thus the goal is to force $y_1 = [x\ y\ z\ \Psi]^T = C_1\bar{x}$ to track r_d. In words, we are trying to steer the helicopter along a desired position and heading trajectory. This goal is accomplished using 4 proportional derivative controllers. Specifically, $u_{a_{1s}}$ is strongly coupled to the global y position. Similarly, $u_{b_{1s}}$, u_{Θ_M}, u_{Θ_T} are strongly coupled to global x, global z, and global heading, respectively. The four proportional derivative controllers were designed to

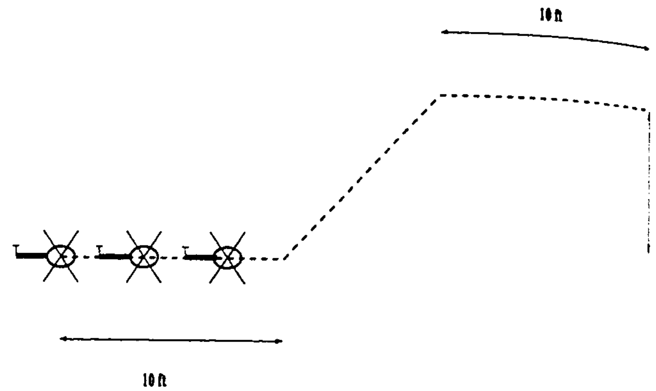

Figure 1: Desired Trajectory

control each loop. The final control law had the following state space form:

$$\bar{u} = K_1C_2\bar{x} + K_2r_d \tag{5}$$

The closed loop model is then given by:

$$\begin{aligned} \dot{\bar{x}} &= (A - BK_1C_2)\bar{x} + BK_2r_d \\ y_1 &= C_1\bar{x} \end{aligned}$$

This closed loop system has 4 inputs and 4 outputs. It can be represented in transfer function form as:

$$Y_1(s) = H(s)R_d(s) \tag{6}$$

Ideally this transfer function would be diagonal, but in reality there is coupling between the four modes which may degrade performance.

4 Proposed Structure for Mesh Controller

In this section we propose a structure for designing mesh controllers for the helicopters. The lowercase letters represent variables which are functions of time. The corresponding uppercase letters represent Laplace transforms of the time functions, e.g. $X(s) = \mathcal{L}(x(t))$. The procedure we describe produces inherently mesh stable controllers. This is an improvement on the previous design techniques where the controllers are designed and then checked for mesh stability. We consider three helicopters following the previous one, see figure 1. The mesh controller design structure is shown in the figure 2. The block *Regulated Helicopter* represents the stabilized dynamics of a helicopter. The tracking controller for an individual helicopter is designed as explained in Section 3. We assume that the regulated helicopter follows the desired trajectory faithfully. In the previous section we designed the

following MIMO transfer function relation,

$$X_i = H(s)X_{id} \tag{7}$$

Note that the notation has been changed slightly, in the previous section this equation was represented as, $Y_1(s) = H(s)R_d(s)$. Here $X_i \in \mathbb{C}^4$ are the regulated outputs of the helicopter i.e. longitudinal, lateral, vertical positions and yaw. In addition we assume that $H(0) = I_{4\times4}$; in other words, there is no steady state error between the desired and actual position. Define:

$$\begin{aligned} x^p_{id} &:= x_{i-1} - \delta & x^l_{id} &:= x_l - i\cdot\delta \\ e^p_i &:= x^p_{id} - x_i & e^l_i &:= x^l_{id} - X_i \end{aligned}$$

Here x^p_{id}, x^l_{id} are the desired positions of i^{th} vehicle with respect to the preceding and the leader vehicle respectively. e^p_i, e^l_i are the corresponding errors. For a safe formation flight e^p_i are the critical errors to protect against a crash. However it has been shown [8] that if we design controllers based only the preceding error information, then we get error amplification as we go down the chain. So taking a cue from that work we implement X_{id} as, see figure 2.

$$X_{id} = p\left(K(s)E^p_i + X^p_{id}\right) + (1-p)\left(K(s)E^l_i + X^l_{id}\right) \tag{8}$$

If we assume that $K(s) = 0_{4\times4}$ then the above expression is easy to interpret. It says the desired position of a helicopter is convex combination of its desired position with respect to leader and preceding helicopters. Here, p represents the coupling to the preceding vehicle. As stated above we would like p as large as possible for safety. Using the above definitions we get following relations which will be used in further simplification.

$$\begin{aligned} e^l_{i-1} - e^l_i &= -e_i \\ X^p_{i-1,d} - X^p_{id} &= X_{i-2} - X_{i-1} \\ X^l_{i-1,d} - X^l_{id} &= \delta \end{aligned}$$

The error satisfies,

$$E^p_i = H\left(X_{i-1,d} - X_{i,d} - \delta\right)$$

Note that here we have made use of the fact $H(s)\delta = \delta$ as δ is assumed to be constant and $H(0) = I$. Substituting the desired positions from equation 8 and simplifying using the identities above we get,

$$\begin{aligned} E^p_i = H[p\left(K\left(E_{i-1} - E_i\right) + \left(X_{i-2} - X_{i-1}\right)\right) + \\ (1-p)\left(-KE_i + \delta\right) - \delta] \end{aligned}$$

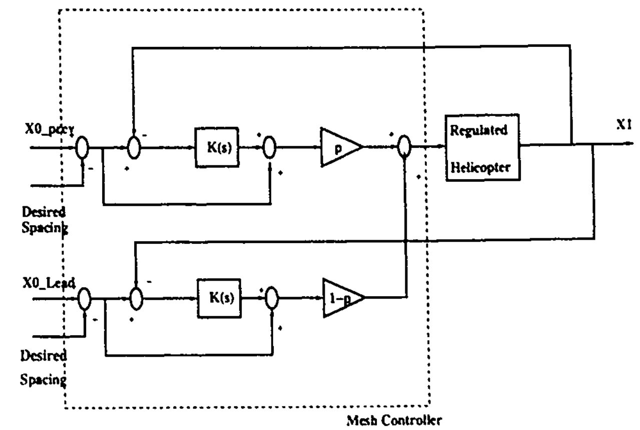

Figure 2: Mesh Controller Structure

Simplifying further one gets:

$$\frac{E^p_i}{E^p_{i-1}} = p[I + HK]^{-1}[H + HK] \tag{9}$$

Define:

$$G(s) := [I + HK]^{-1}[H + HK] \tag{10}$$

Note that $G(s) \in \mathbb{C}^{4\times4}$, $\forall s \in \mathbb{C}$. Now suppose $g(t) : \mathbb{R}_+ \to \mathbb{R}^{4\times4}$ is the impulse response of the individual transfer function entries of $G(s)$. From input-output relation in section 2 we know that $p\,\|g(t)\|_1 \leq 1$ implies that $\|e_i\|_\infty \leq \|e_{i-1}\|_\infty$, i.e. we have string stability. Using this information, we can see that we have two degrees of freedom for design of mesh controller, $K(s)$ and p.

5 Robustness to Disturbances

The analysis of the previous section assumes a linear model for the regulated helicopter as well as perfect tracking of the desired position command at steady state. In reality, the helicopter dynamics are nonlinear and the assumption of linearity is only justified by a small operating range or a feedback linearizing controller at the regulation layer. However, the regulation layer cannot achieve perfect tracking of the desired profile generated by the mesh controller. Furthermore, external disturbances acting on the UAVs, such as wind gusts, will cause additional errors. In this section, we justify the two-layer control structure. We will show that a mesh stable controller leads to the additional property that the effect of any such disturbances will be damped out as they propagate. Consider the regulated helicopter model with a disturbance reflected at the output:

$$X_i = H(s)X_{id} + D_i(s) \quad (11)$$

The helicopter position consists of the desired part tracked by the regulated helicopter plus a term, d_i, representing external disturbances and imperfect tracking of the regulation layer. We use the same mesh controller given in Equation 8.

Using analysis entirely analogous to the preceding section, we obtain the following relation:

$$E_i(s) = pG(s)E_{i-1}(s) + \bar{G}(s)(D_{i-1}(s) - D_i(s)) \quad (12)$$

where $pG(s)$ is given in Equation 10 and $\bar{G}(s) = [I + HK]^{-1}$. Equation 12 shows that the i^{th} error consists of a term which is propagated, via $pG(s)$, from other errors in the string. The i^{th} error also contains a term due to variations in the disturbances acting on the helicopters of the string. Note that if there is a disturbance which acts uniformly on the string, $d_i \approx d \; \forall i$, then the second term of Equation 12 is small.

It is reasonable to expect large disturbances will occasionally act on one portion of the string. The relation given above implicitly shows that the effect of disturbances on other members of the mesh is also propagated via $pG(s)$. Suppose that there is a large wind gust acting on the second ($i = 2$) helicopter and other disturbances are negligible. The disturbance affects the error, E_2, through the transfer function $\bar{G}$: $E_2 = pGE_1 + \bar{G}D_2$. However, it propagates to to other errors though pG. It is easy to show that for $i > 2$, $E_i = (pG)^{i-2}E_2 + (pG)^{i-3}\bar{G}D_3$.

Without being too rigorous, we note that $\|pG\|_\infty < 1$ when the mesh stable controller is used. This causes the magnitude of $(pG)^{i-3}$ to geometrically decay with i at each frequency. Thus disturbances acting on the string will decay as they propagate. If a mesh unstable controller is used, then $\|pG\|_\infty > 1$. This mesh unstable controller causes the magnitude of $(pG)^{i-3}$ to geometrically grow with i at some frequency. In this simple example, if the disturbance acting on the second helicopter has the right frequency, it's amplitude will grow geometrically down the string. For a large string, the result could be catastrophic.

In summary, the results above can be generalized to random disturbances propagating in the string. The key point is that we designed the mesh controller under the assumption of a linearized regulated helicopter with perfect steady state tracking.

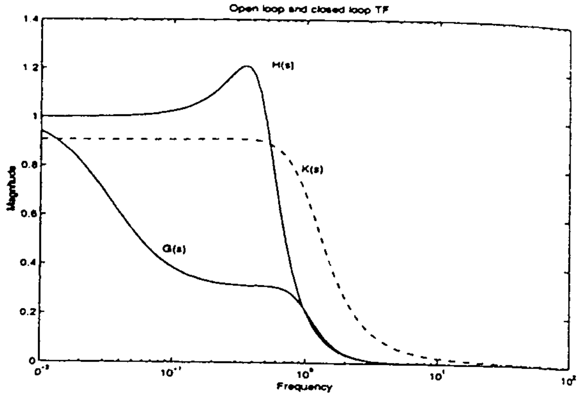

Figure 3: Mesh controller design

In reality, the regulated helicopter is not linear. Furthermore, external disturbances may cause additional errors. However, this analysis shows that errors caused by these effects are damped out as they propagate away from the source. If we use a mesh unstable controller, it is possible for disturbances to act at the proper frequency and amplify as they propagate.

6 Design Procedure

To design mesh controllers following is the procedure,

- Design $K(s)$ so that $\|G(j\omega)\|_\infty$ is minimum and $G(s)$ is stable.
- Denote the minimal value obtain in step 1 as v i.e $v := \|g(t)\|_1$.
- Select $p \leq \frac{1}{v}$

The design for the x-component of the transfer function matrix $H(s)$ is shown in the figure 3. Generally it was observed that after the $\|H(j\omega)\|_\infty$ design, the closed loop transfer function $G(s)$ had overdamped characteristics so $\|H(j\omega)\|_\infty \approx \|g(t)\|_1$. If $K(s) = 0$ then $G(s) = H(s)$. Note that $H(s)$ has a peak magnitude of 1.2 at 0.5 rad/s. So we need $p < \frac{1}{1.2}$ to attenuate all disturbances. On the other hand if we use the design procedure given above then $\|G(j\omega)\|_\infty \approx 1$. The disturbances will be attenuated if $p < 1$. As a result we have obtained mesh stability with a higher coupling to the preceding vehicle.

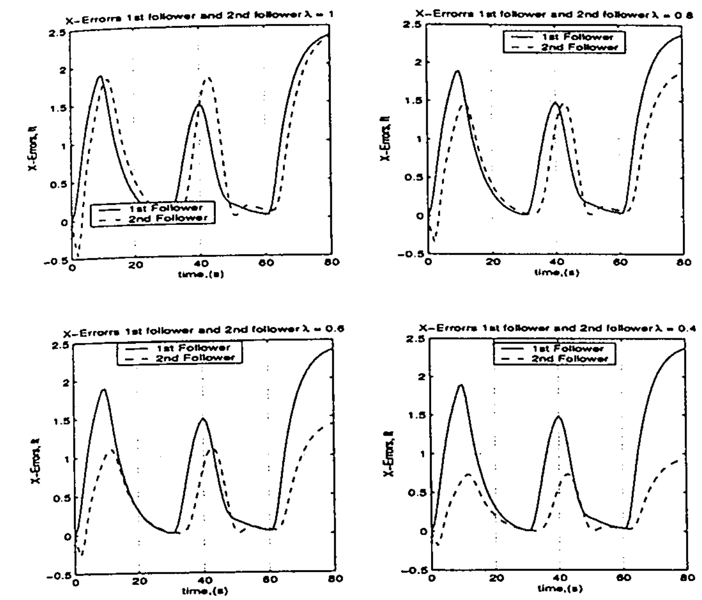

Figure 4: Error Propagation

7 Results and Simulations

In this section we will present results of simulations performed using the theory presented in previous sections. As a first cut $K(s)$ was assumed to be zero. The error propagation characteristics for different values of p are shown in figure 4. We can see that for $p = 1$, the maximum of the error for the second vehicle e_2 is greater greater than the maximum of the error for the first follower e_1. This is consistent with our theoretical result which says that the vehicle following based on only preceding vehicle information implies string instability. On the other hand for the other subplots in the figure we can see that the second error is lower than the first error and thus we have the error damping characteristic. It should be noted however that for formation safety one would want the controller as strongly connected to the closest vehicles as possible i.e. p should be as high as possible as long as we are string stable.

8 Conclusions

We proposed a general structure for designing mesh controllers for formation flying. The mesh stability property of the structure was demonstrated using linear transfer function theory as well as a simple simulation experiment. In future, we expect to perform experimental testing of the ideas presented in this paper.

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