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A theory of magnetostriction oriented towards applications

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A simplified theory of magnetostriction, derived from micromagnetics, is presented. The theory is designed to provide a systematic, yet manageable, quantitative tool in the design of magnetostrictive actuators, by making the ''calculation'' of the overall response of magnetostrictive specimens subjected to applied magnetic fields and loads a feasible task. Indeed, for simple geometries, the computation of energy minimizing domain patterns is reduced to a finite dimensional problem. As an application of the theory, a mechanism explaining the anisotropic magnetostriction observed in Terfenol-D rods, based on computed energetically optimal domain arrangements, is proposed. © *1997 American Institute of Physics.* [S0021-8979(97)51108-X]

I. INTRODUCTION

The development of rare-earth–iron compounds capable of magnetostrictive strains of the order of some parts per mill[1] has fostered the search for commercially viable designs of magnetostrictive actuators.

The basic mechanisms by which macroscopic deformations involving, say, length changes of a magnetostrictive rod, can be induced by applied magnetic fields are well known. In the absence of external fields, the rod will be typically subdivided into fine domains, i.e., regions within which the material is uniformly magnetized and distorted. Applying some mechanical loads, e.g., a compressive stress along the rod axis, will bias the system towards domain patterns corresponding to the shortest end-to-end rod length. Let us assume that, for such domain arrangements, the magnetization is perpendicular to the rod axis. Then a strong enough magnetic field applied along the rod axis will cause the magnetization to align with the axis, and the rod to lengthen. For a practical device, however, it is not only desirable that the deformations accompanying the switch from initial to final configurations be the largest possible, but also that there exist low energy paths joining the two domain patterns in configuration space, so that the field strengths needed to drive the process be reasonably low.

Optimizing the design of a magnetostrictive actuator requires, then, comparing the performance of devices based on different materials (here, both crystallography and texture may play a decisive role), subjected to a huge variety of biasing loads and driving fields. The goal of the theory presented below is to provide a systematic, yet manageable tool to approach such questions in a quantitative fashion.

II. ENERGY MINIMIZING CONFIGURATIONS

The starting point of our model is micromagnetics,[2] and we adopt a geometrically linear description of the deformations experienced by a magnetostrictive material occupying the region of space Ω. Thus, at a point x of Ω, the state of the material is described by its magnetization $\mathbf{M}(x)$, a vector whose magnitude is the saturation magnetization M_s, and by the linear strain $\mathbf{E}(x)$, where

$$2E_{ij}(x)=2E_{ij}(x)\frac{\partial u_i(x)}{\partial x_j}+\frac{\partial u_j(x)}{\partial x_i},$$

and $\mathbf{u}(x)$ is the displacement vector at x. Stable equilibria correspond to pairs of magnetization and strain fields minimizing a free-energy of the form

$$W=W_{\text{exch}}+W_{\text{mst}}+W_{\text{ext}}+W_{\text{mel}}, \tag{1}$$

where W_{exch} is the exchange energy, W_{mst} is the magnetostatic (or stray-field) energy, W_{ext} is the energy associated with the external magneto-mechanical loads consisting of a uniform prestress $\mathbf{S}$ applied at the boundary of Ω, and of a uniform applied magnetic field $\mathbf{H}$,

$$W_{\text{ext}}=-\int_{\Omega}[\mathbf{S}\cdot\mathbf{E}(x)+\mathbf{H}\cdot\mathbf{M}(x)]dx, \tag{2}$$

and, finally,

$$W_{\text{mel}}=\int_{\Omega}w(\mathbf{M}(x),\mathbf{E}(x))dx. \tag{3}$$

Here the magnetoelastic energy density w accounts for the energy stored in the material due to deviations of its local state from one of those favored by crystalline anisotropy.

Since we are mainly interested in the macroscopic response of a specimen much larger than the typical domain size, we neglect the exchange energy (note that, together with the exchange constant, we are setting equal to zero the intrinsic length scale determining the size of the domains: some admissible magnetization distributions will thus be represented mathematically by domain patterns of vanishingly small size).[3] We further simplify the model by examining the structural properties of the magnetoelastic energy density. Indeed, for a given crystalline material, w will be minimized when evaluated on a pair consisting of a magnetization along an easy direction and of the corresponding stress-free strain. In view of crystallographic symmetry, there will be several, symmetry related, energy minimizing magnetization-deformation pairs. Assuming, without loss of generality, that the corresponding minimum value of w is zero, we can define the set of the ''energy wells'' of the material,

$$K=\{(\pm\mathbf{M}_1,\mathbf{E}_1),\ldots,(\pm\mathbf{M}_n,\mathbf{E}_n)\}, \tag{4}$$

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as the zero level set of w, which is then strictly positive outside of K. For example, for Terfenol-D, the material with the largest known room temperature magnetostriction, the energy wells comprising K are eight symmetry related ''variants,'' whose expressions in the cubic basis are given by

$$\pm\mathbf{M}_1=\pm\frac{M_s}{\sqrt{3}}(1,1,1),\qquad \mathbf{E}_1=\frac{\lambda_{111}}{2}\begin{bmatrix}0&1&1\\1&0&1\\1&1&0\end{bmatrix},$$

$$\pm\mathbf{M}_2=\pm\frac{M_s}{\sqrt{3}}(-1,1,1),\qquad \mathbf{E}_2=\frac{\lambda_{111}}{2}\begin{bmatrix}0&-1&-1\\-1&0&1\\-1&1&0\end{bmatrix},$$

$$\pm\mathbf{M}_3=\pm\frac{M_s}{\sqrt{3}}(1,-1,1),\qquad \mathbf{E}_3=\frac{\lambda_{111}}{2}\begin{bmatrix}0&-1&1\\-1&0&-1\\1&-1&0\end{bmatrix},$$

$$\pm\mathbf{M}_4=\pm\frac{M_s}{\sqrt{3}}(1,1,-1),\qquad \mathbf{E}_4=\frac{\lambda_{111}}{2}\begin{bmatrix}0&1&-1\\1&0&-1\\-1&-1&0\end{bmatrix}.$$

Here λ_{111} is the saturation magnetostrictive strain along $[111]$. Now, it turns out that the energy density w is rather steep near its energy wells. More precisely: For most of the possible combinations of applied fields and loads, and always when their strength is small with respect to the magnetoelastic moduli characterizing the slope of w, the energy minimizers will take on values in a small neighborhood of K. We will therefore restrict our attention to states of our system such that, ''everywhere'' in Ω, $(\mathbf{M}(x),\mathbf{E}(x))$ belongs to K and the magnetoelastic energy contribution W_{mel} vanishes (see Ref. 4 for a formal justification of this argument). With these assumptions, the state variables of our theory become the $2n$ scalar functions $\lambda_1^+(x)$, $\lambda_1^-(x)$, ..., $\lambda_n^-(x)$, which, at each x, represent the volume fraction of a vanishingly small neighborhood of x in which the material is in the state $(\mathbf{M}_1,\mathbf{E}_1)$, $(-\mathbf{M}_1,\mathbf{E}_1)$, ..., $(-\mathbf{M}_n,\mathbf{E}_n)$, respectively.

For specimens of ellipsoidal shape (including the degenerate cases of an infinite cylinder, or of an infinite slab), it is easily shown that the lowest magnetostatic energy corresponding to states with average magnetization

$$\langle\mathbf{M}\rangle=\sum_{i=1}^{n}(\langle\lambda_i^+\rangle-\langle\lambda_i^-\rangle)\mathbf{M}_i, \tag{5}$$

where

$$\langle\lambda_i^{\pm}\rangle=\frac{1}{|\Omega|}\int_\Omega\lambda_i^{\pm}(x)dx, \tag{6}$$

and $|\Omega|$ denotes the volume of Ω, is

$$\overline{W}_{\text{mst}}(\langle\mathbf{M}\rangle)=\tfrac{1}{2}|\Omega|\langle\mathbf{M}\rangle\cdot(\mathbf{D}\langle\mathbf{M}\rangle), \tag{7}$$

with $\mathbf{D}$ the demagnetizing tensor associated with the ellipsoid Ω. Moreover, the minimium value $W_{\text{mst}}(\langle\mathbf{M}\rangle)$ is achieved by an admissible distribution of magnetizations (and deformations) such that the functions $\lambda_i^{\pm}$ are in fact independent of x, and $\lambda_i^{\pm}\equiv\langle\lambda_i^{\pm}\rangle$.

The original infinite-dimensional minimization problem is thus reduced to a discrete, constrained optimization problem. In fact, setting

$$\langle\mathbf{E}\rangle=\sum_{i=1}^{n}(\langle\lambda_i^+\rangle+\langle\lambda_i^-\rangle)\mathbf{E}_i, \tag{8}$$

we obtain the quadratic programming problem

$$\tfrac{1}{2}\langle\mathbf{M}\rangle\cdot(\mathbf{D}\langle\mathbf{M}\rangle)-\mathbf{S}\cdot\langle\mathbf{E}\rangle-\mathbf{H}\cdot\langle\mathbf{M}\rangle\rightarrow\min!, \tag{9}$$

in the $2n$ unknown *scalars* $\langle\lambda_i^{\pm}\rangle$, $i=1,\ldots,n$, which are only subject to the natural constraints

$$\sum_{i=1}^{n}(\langle\lambda_i^+\rangle+\langle\lambda_i^-\rangle)=1, \tag{10}$$

$$\langle\lambda_i^{\pm}\rangle\in[0,1],\quad i=1,\ldots,n, \tag{11}$$

expressing the fact that the $\langle\lambda_i^{\pm}\rangle$ represent volume fractions of regions making up for the whole body. It is worth emphasizing that our whole argument rests on the following nontrivial observation:[4] For every admissible $\langle\mathbf{M}\rangle$, the minimal magnetostatic energy $W_{\text{mst}}(\langle\mathbf{M}\rangle)$ given in (7) *can* be achieved by an admissible state of the system. (The proof of this assertion requires the construction of pairs of magnetization and strain fields taking only values on the set of the energy wells K and satisfying the appropriate jump conditions ensuring both kinematic compatibility and absence of magnetic poles at the interfaces between domains. Such construction seems possible for most magnetostrictive materials.[4])

The discrete optimization problem (9)–(11), in which $\mathbf{S}$ and $\mathbf{H}$ appear as parameters, can be solved either analytically, or resorting to commercial software packages. This leads to the determination of the macroscopic average magnetization $\langle\mathbf{M}\rangle$ and deformation $\langle\mathbf{E}\rangle$ associated with energetically optimal domain patterns under the prescribed loads and fields, and to relevant information on the corresponding microgeometries.

III. APPLICATIONS TO TERFENOL-D

Teter *et al.* have compared the magnitude of the magnetostrictive strains along the $[112]$ and the $[111]$ directions, for a magnetostrictive rod of growth-twinned Terfenol-D subjected to a compressive uniaxial stress and an applied magnetic field, both along the $[112]$ rod axis.[5] A distinctive feature of such curves, shown in Figure 1, is that the steep rise of the strains along $[111]$ occurs for field strengths smaller than those causing large strains along $[112]$. Interestingly, while the domain patterns corresponding to the initial and the final states (fine domains with magnetizations orthogonal to the rod axis, and a saturated state with magnetization along $[112]$, respectively) can be easily identified, the same is not true for domain structures corresponding to the intermediate field strengths.

As an application of the theory presented in the previous section, we have computed the $[112]$ and $[111]$ magnetostriction curves for a Terfenol-D slab with midplane orthogonal to $[111]$, and subjected to fields and loads as in the experiments of Ref. 5, with the aim of simulating the behav-

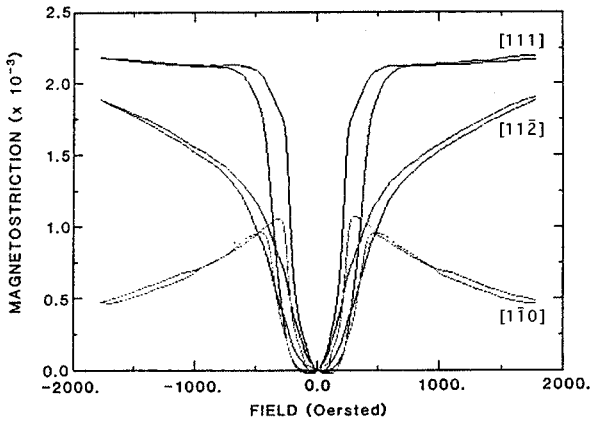

FIG. 1. Magnetostriction vs applied magnetic field for three orthogonal crystallographic directions in $Tb_{0.3}Dy_{0.7}Fe_{1.95}$. The [112] data are along the crystal growth direction and are positive in sign. The [111] data are negative in sign while the [110] are positive. Reprinted with permission from Teter *et al.*, Journal of Applied Physics **67**, 5005 (1990). © *1990 American Institute of Physics.* .

ior of one of the lamellae comprising a typical growth-twinned specimen. The computed curves are shown in Fig. 2 where, for consistency with Fig. 1, we have changed the sign of the [111] curve (the [110] curve, not shown, can be obtained as the ''difference'' of the displayed curves since all deformations involved are volume preserving). The computations leading to such curves of Fig. 2 show the existence of an intermediate regime between the initial and the final domain structures, for field strengths just exceeding 250 Oe, corresponding to which the magnetization in parts of the specimen goes out of the plane generated by the [112] and [111] directions. This seems to explain the observed behavior: The transition from the initial state $(\pm\mathbf{M}_4,\mathbf{E}_4)$ to, say, $(\mathbf{M}_3,\mathbf{E}_3)$ is accompanied by large strain changes along [111], and small strain changes along [112]. The further transition to $(\mathbf{M}_1,\mathbf{E}_1)$ (the closest we can get to the saturated state along [112] within our model), determines significant [112] strain changes, but no changes of length along [111]. Our proposed interpretation of the observed behavior is also consistent with calculations of Jiles and Thoelke,[7,8] which are based on a three-dimensional anisotropic rotation model (i.e., the specimen is idealized as an array of noninteracting single-domain particles, each exhibiting a Stoner–Wohlfarth-type behavior), rather than on global energy minimization.

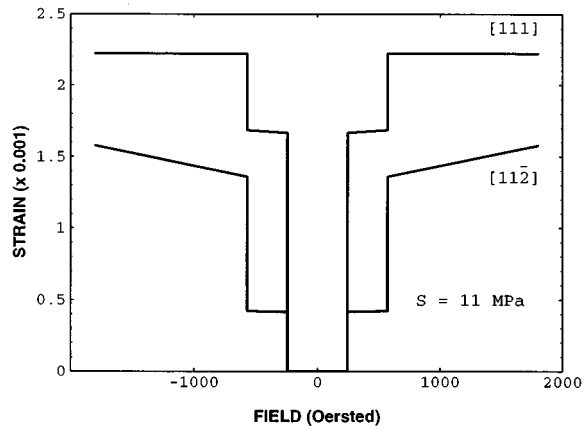

FIG. 2. Computed magnetostriction curves for a Terfenol-D slab orthogonal to [111] subjected to a magnetic field and a compressive uniaxial stress of 11 MPa, both along [112]. The values of the material parameters used in the calculations are $M_s = 800$ emu/cm^3 and $\lambda_{111} = 2.5 \times 10^{-3}$.

IV. DISCUSSION

We have presented a simplified model of magnetostriction, derived from micromagnetics, which for simple geometries reduces the minimization problem to a finite dimensional one. As an application, we have discussed a possible mechanism explaining anisotropic magnetostriction observed in Terfenol-D, based on computed energy minimizing domain patterns.

It seems worth reminding the reader of some obvious limitations of our model. First, we cannot describe system states close to saturation when the magnetic field is applied along directions far away from those of the easy axes. Moreover, we obtain paths in configuration space by computing global energy minimizers corresponding to given histories of magnetomechanical loads. The existence of hysteresis shows, however, that a system may rest on local, rather than global minimizers, and that only some of the available low energy pathways are actually accessed. In order to select realistic domain pattern evolutions, more detailed models, involving more than energetics, would be desirable.

The great simplicity of our model, which is responsible for the shortcomings described above, is however also its strength. Since a given material is described only through the set K of its energy wells, whose structure is largely dictated by material symmetry,[6] the model suffers little from uncertainties in the experimental determination of the constitutive parameters, but it delivers reliable quantitative predictions within the regime of small applied magnetic fields and loads.

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