2D simulations of Hall thrusters

By Eduardo Fernandez, Mark Cappelli, AND Krishnan Mahesh

1. Motivation and objectives

Closed-Drift (Hall) thrusters constitute an important electric propulsion technology for certain applications requiring low thrust levels, e.g. satellite station keeping and orbit transfer (Gulczinski and Spores, 1996). The thrust in Hall thrusters is generated by ions being accelerated through an annular plasma by the electric field set up between an anode and a cathode. This electric field is strongly coupled to an externally applied radial magnetic field which typically localizes the electric field near the channel exit. The ions are generated through electron-impact ionization of Xenon neutrals. Due to their large inertia, the ions are not magnetized, and stream out of the device without experiencing very many collisions. The electrons, on the other hand, collide with the background neutrals as they migrate to the anode across the magnetic field. The cathode is located a few centimeters downstream of the channel exit and provides enough electrons to supply much of the discharge current, ionize the incoming neutrals, and neutralize the beam of exiting ions.

While the overall operational characteristics of Hall thrusters are understood, some key issues remain to be resolved. In particular, the relationship between the various types of fluctuations in these devices and the overall engine efficiency needs to be determined. Electron conductivity is critical in the operation of Hall thrusters since it impacts the ionization of neutrals and the potential drop which accelerates the resulting ions. However, it's expected (classical) value, arising from electron-neutral and Coulomb collisions, is far too low to account for the measured electron current (Morozov *et al.* 1972). Two mechanisms have been proposed to account for the enhanced electron transport (often termed 'anomalous' in the plasma physics literature): electron wall interactions and azimuthal fluctuations in electron density (Morozov *et al.* 1972).

It is important that electron diffusion be accurately modeled if the essential physics of the thruster are to be represented by one- or two-dimensional computations. State of the art approaches (e.g. Fife *et al.* 1997) assume that the electron mobility is given by the 'Bohm model':

$$\mu = \frac{1}{16B} \tag{1}$$

where μ is the electron mobility and B is the magnetic field. It is known from experimental work that this coefficient is only approximate. 'Bohm diffusion' is commonly used to refer to diffusion which *scales* as $\frac{1}{B}$ where the scaling coefficient is arbitrary within a factor of 2 or 3. Early experimental (Janes and Lowder, 1960) and theoretical (Yoshikawa and Rose, 1962) work has shown that this diffusion arises from the correlation between fluctuations in the azimuthal electric potential

and electron density. Given a certain phase between potential and density, the diffusion coefficient can be cast in terms of the ratio of the rms level of electron density to the mean electron density. One of the objectives of this work are to evaluate the modeling of electron diffusion using Eq. (1) and develop alternative approaches.

Another issue of interest is the experimentally observed emergence of a virulent ionization instability in the current saturation part of the I-V curve for these engines (Meezan *et al.*, 1998). Unlike the azimuthal drift wave fluctuations associated with axial electron transport, the ionization instability is believed to be deleterious to engine performance. The more one pushes the engine into the saturation region, the larger the amplitudes of these modes (rms levels on the order of the mean are not uncommon). This instability appears to be caused by the non-uniform ionization of the neutral Xenon atoms. These atoms, emerging from the back of the thruster, enter a region (ionization zone) in which they are ionized upon colliding with the electrons. Once ionized, the resulting ions are quickly accelerated, thus creating a void of both neutral and charged particles. As neutrals replenish the empty region, ionization takes place and the sequence repeats itself. Since the ion velocity is so much greater than that of the neutrals, the relevant timescale of the instability should be proportional to the velocity of the incoming neutral atoms. However, theoretical work (Fife *et al.*, 1997) suggests that the mode frequency is, in fact, proportional to the geometric mean of the neutral and ion velocity. That theory implies that as the ion velocity increases, it actually dominates the mode frequency. The heuristic picture above, however, suggests that as the ion velocity increases it progressively decouples from the mode frequency. One of the objectives of this work is to test the scaling of mode frequency with neutral velocity.

2. Accomplishments

2.1 Approach

Our first step has been to perform two-dimensional hybrid simulations where the electron mobility is modeled assuming Bohm diffusion (Fife 1995). The governing equations are those used by Fife, however, details such as the computational grid, integration scheme, treatment of nonlinear terms, and tracking of heavy PIC particles in non-uniform grids are different.

The thruster modeled in our work is the SPT 100 Russian thruster for which a lot of experimental data exists. The computational geometry is as that used by Fife: it covers the channel and part of the plume.

Figure 1 shows a schematic of the computational geometry. A non-uniform orthogonal grid is used to span the thruster annular channel and part of the plume. The externally imposed magnetic field is obtained by solving a Laplace equation for the magnetic potential, having specified the geometry of the magnetic poles and assumed infinite permeability for the pole pieces. The algorithm is a mixture of particle and fluid approaches. The electrons are treated as a fluid since their effective mean-free-path is their gyro-radius. The ions, however, typically leave the channel without colliding with other particles. Also, experiments show that the ion velocity
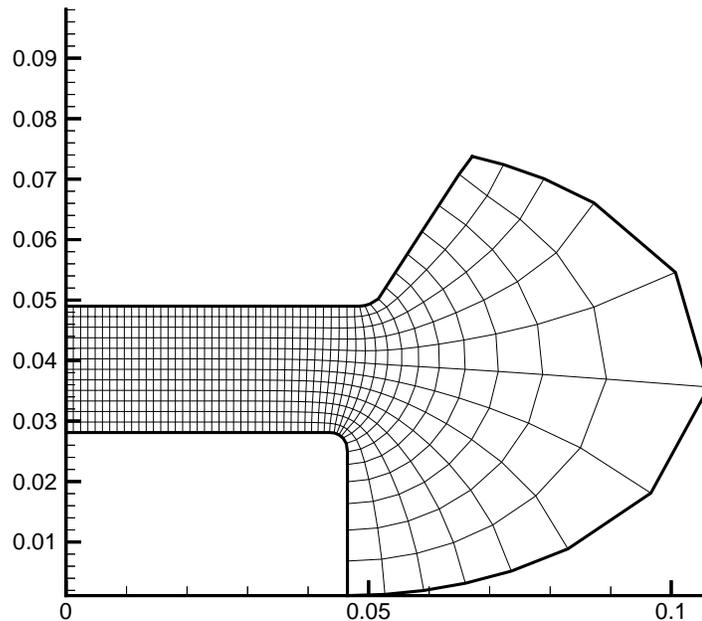
FIGURE 1.   Schematic of the computational geometry.

distributions vary markedly from Maxwellian. Therefore, the ions are treated as particles. The neutrals are also treated as particles since they, like the ions, have mean-free-paths longer than the length of the thruster. The details of the governing equations are given in Fife (1995); only a short summary is provided here.

The electron momentum equation perpendicular to the magnetic field (mostly in the axial direction since B is mostly in the radial direction) is given in terms of a drift-diffusion equation, representing a balance between electric, pressure, and drag forces. This form neglects inertial terms which are small in the Hall thruster. The equation can be cast in terms of an electron mobility and diffusion coefficient, representing the electron momentum response to electric and pressure forces applied in the axial direction in the presence of collisions; i.e.,

$$n_e(u_{e\hat{n}} - u_{i\hat{n}}) = -n_e\mu(E_{\hat{n}} + \frac{kT_e}{en_e}\frac{\partial n_e}{\partial \hat{n}} + \frac{k}{e}\frac{\partial T_e}{\partial \hat{n}}) \tag{2}$$

However, there is an added contribution to the axial flux which does not arise from axial electric or pressure forces, but rather from an $E \times B$ drift (azimuthal electric field crossed with radial magnetic field). This component is thus given by nE_θ/B. Although there is no mean E_θ, this term is not zero if there are correlated n and E_θ fluctuations. A two-dimensional approach has no azimuthal direction and, therefore, cannot capture this component. The effect of this term is, therefore, included through an effective mobility and diffusion. So far, a $1/B$ diffusion coefficient has

been used. Work is underway to incorporate a more realistic, axially dependent coefficient. Parallel to the magnetic field, the electric and pressure forces balance each other, yielding a Boltzman relation.

$$\phi = \frac{kT_e}{e}ln(n_e) + \phi^* \qquad (3)$$

with the electric field given by $E = -\nabla\phi$.

An electron temperature equation is also solved. Ionization of neutrals is a strong function of electron temperature, which makes the electron temperature equation an important part of the model. The equation is given by:

$$\frac{\partial}{\partial t}(\frac{3}{2}n_e kT_e) + \frac{\partial}{\partial \hat{n}}\left(\frac{5}{2}n_e u_{e\hat{n}} kT_e - K\frac{\partial T_e}{\partial \hat{n}}\right) = -n_e n_n \zeta(T_e)\varphi(T_e)E_i + j_{e\hat{n}}E_{\hat{n}} \qquad (4)$$

where $\zeta(T_e)$, $\varphi(T_e)$, and K are the ionization rate parameter, the ion production cost, and the thermal diffusivity respectively. The functions $\zeta(T_e)$ and $\varphi(T_e)$ have exponential terms and thus are highly nonlinear. These terms are evolved in their original forms and are not linearized, unlike Fife (1995). As in the momentum equation, the diffusivity can be expected to be anomalous. Presently, a simple (constant) value is taken as described by Lentz (1992). The terms on the right-hand side of Eq. (4) are the ohmic heating source term and the ionization-induced sink term. Since thermal conductivity parallel along field lines is much higher than across field lines, magnetic streamlines are assumed to be isothermal. We use this assumption and solve the temperature equation along magnetic stream lines as done by Fife (1995).

Electron continuity is enforced via a total discharge current conservation constraint. Since the plasma is assumed to be quasineutral, charge cannot build up anywhere in the device. Combining electron and ion continuity equations and integrating (along magnetic field lines) yields an equation stating discharge current conservation. This is given by:

$$I_a = \int_A n_e(u_{i\hat{n}} - u_{e\hat{n}})dS \qquad (5)$$

A fourth order Runge-Kutta scheme is used to time advance the temperature equation – other variables such as electron velocity and electric potential are given by algebraic expressions once the temperature has been obtained. The boundary conditions on the temperature are Dirichlet at the cathode and Neumann at the anode. For the electric potential Dirichlet boundary conditions are specified at the cathode and anode.

The boundary condition at the anode can pose problems. Several magnetic streamlines intersect the anode. The issue arises as to what streamline to choose to impose the boundary condition. As far as the physics is concerned, this region is clearly not being accurately modeled since the sheath is not being resolved. The

model equations already imply this: Poisson's equation is not solved, but rather quasineutrality is imposed. The concern is, therefore, not whether the the region close to the anode is accurately modeled, but whether the boundary condition causes numerical problems. The situation is most problematic in the transient period of the simulation when the particles and fields have not yet established an equilibrium. Large oscillations at the anode in temperature and potential, which turn into large oscillations in ion velocity and plasma density, develop and are convected downstream. If severe enough, these oscillations can terminate the simulation. These anode oscillations are also very problematic when one changes the diffusion coefficient.

As stated before, the ions and neutrals are evolved with a PIC approach. Since quasineutrality is enforced, determining the ion density by interpolating to the grid points also determines the electron density. Ion-neutral elastic, and charge exchange collisions are ignored since their cross sections are small. Therefore, a neutral changes its velocity only when it encounters a wall, in which case it is repelled back in a random direction. The only force acting on the ions is the electric field (the magnetic force is ignored due to the large ion inertia). When an ion strikes the wall, it recombines to form a neutral which is injected back in the domain at a random angle. Since the thruster has regions with sharp contrast of plasma and neutral density, the ion or neutral superparticle masses are not identical. Also, the algorithm uses a fractional time-advance step method. The fast time scale in the system is tied to electron dynamics whereas neutral and ion motion is much slower. In view of this fact, the neutrals and ions are time advanced every so many electron time steps. A standard leap frog scheme is used for the time advancement in the PIC approach.

2.2 Results

2.2.1 Evaluation of Bohm diffusion

The Bohm diffusion model for electron transport (Fife 1995) is evaluated in this section. Our simulations show that for the SPT configuration, the use of Bohm diffusion, as given by Eq. (1), yields electron density and ionization profiles that peaks too far upstream as compared to experimental data, while the temperature and electric field actually peaks downstream of the channel exit. Experimentally it is observed that all the above profiles tend to peak roughly at the same axial location, at the exhaust of the channel (Bishaev and Kim 1972).

This discrepancy is interpreted as being due to a diffusion coefficient which is too large: the larger electron current produces too much ionization upstream, thereby quickly depleting the neutral density. This results in ionization-induced temperature losses downstream that are low while the temperature source term is still high through the large electric field. Consequently, the electron temperature peaks too far downstream.

It is instructive to note that that a 1D version of the numerical model which used a $\frac{1}{16B}$ diffusion coefficient was previously used by Lentz (1992) to simulate a short
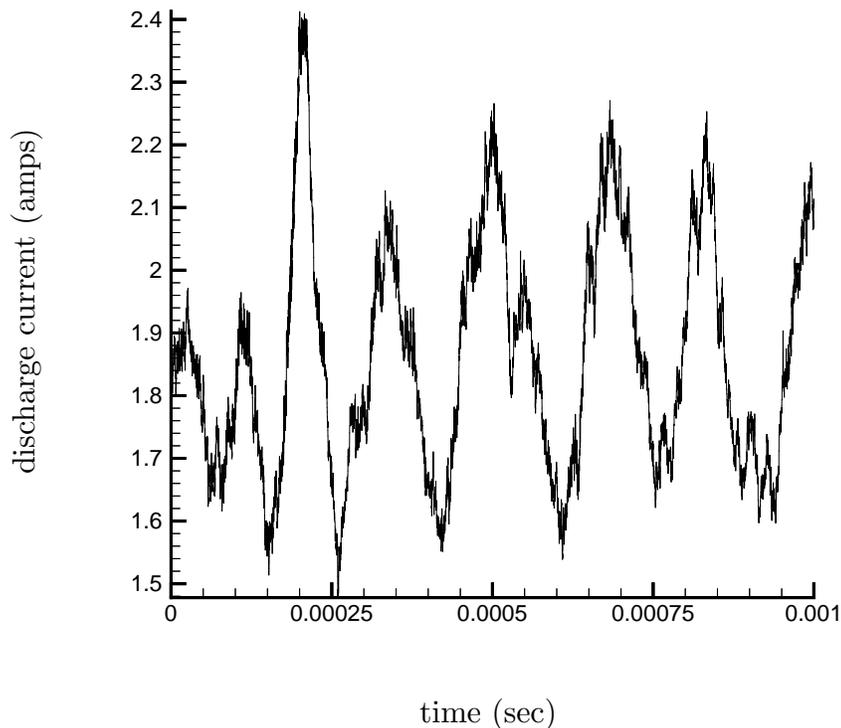
FIGURE 2.   Time history of the discharge current in a simulation with discharge voltage of 200 volts and mass flow rate of 3mg/s.

channel, high magnetic field Japanese thruster. In that case, the simulated and experimental profiles were in better agreement. This suggests that different thrusters are likely to have different fluctuation characteristics and electron transport, and that the often quoted $\frac{1}{16B}$ is not likely to apply to every thruster. In fact, a recent study aimed at modeling an American-designed thruster showed that the model reproduces the overall characteristics with a mobility of $\frac{.25}{16B}$ (Szabo, *et al.* 1998). Even if it applies to a given thruster, it's not likely that a fixed $\frac{1}{16B}$ value would be successful as one varied the operational parameters since the fluctuations themselves are a strong function of the current-voltage operational point. The relative importance of fluctuation-induced electron transport versus collisional transport depends on the relative value of the gyro-frequency and collisional frequency. While in the Japanese thruster the electron gyro-frequency was much larger than the collision frequency throughout the channel (due to the broad, large magnetic field), in the SPT these frequencies are not disparate close to the anode. However, at the channel exit the electron gyro-frequency is more than two orders of magnitude larger than the collision frequency. Therefore, one expects the diffusion to be a function of axial position.
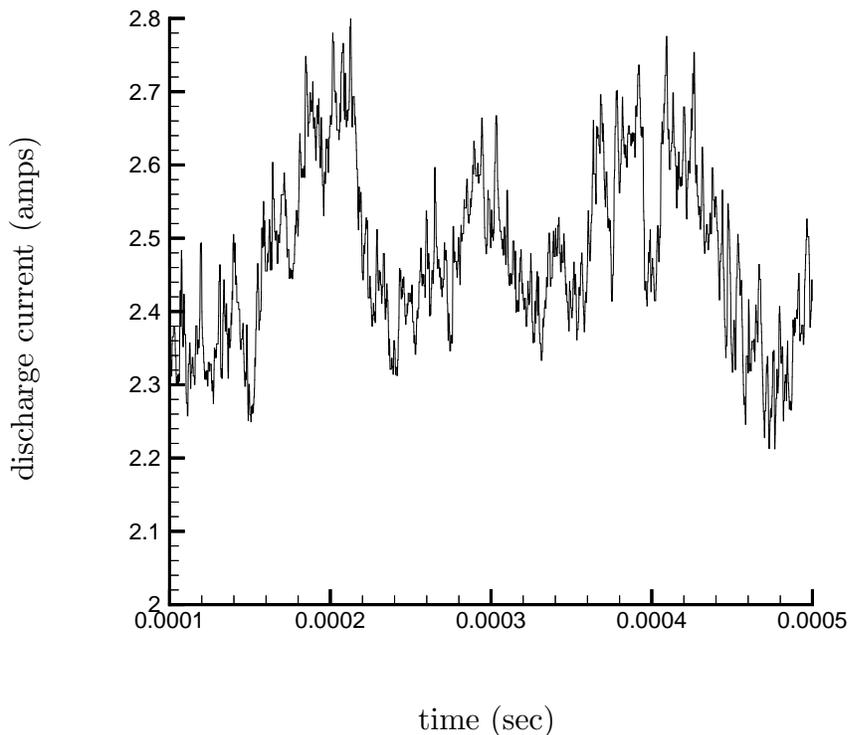
FIGURE 3.   Time history of the discharge current in a simulation with discharge voltage of 200 volts and mass flow rate of 3mg/s. The difference from Fig. 2 is the wall ion-neutral recombination mechanism.

2.2.2 Importance of heavy particle-wall interactions

The importance of electron-wall interactions has been discussed in the literature. Our simulations indicate that wall interactions involving heavy particle collisions also have a large influence on overall discharge dynamics. When ions strike the walls, they recombine to form neutrals. If the recombined neutrals have large kinetic energy they have a greater chance of leaving the channel before being ionized. This will tend to lower the discharge current for a given discharge voltage. The converse hold as well.

Figure 3 shows the time history of the discharge current for the same parameters as those of Fig. 2, but with different heavy particle-wall interaction. In Fig. 2, ions that strike the walls recombine to form neutrals which emerge at random direction with the ion's velocity. In Fig. 3, the recombined neutrals are injected back with random directions with the inflow neutral velocity. We see that the effect on discharge current is significant. The average discharge current is 2.5 amps as compared to the experimentally measured 3.1 amps. Given the strong sensitivity of the results to the chosen diffusion coefficient, it is not that meaningful at this point to comment on the quantitative difference between simulation and experimental results. The effect of the heavy particle wall interaction is also seen on plasma
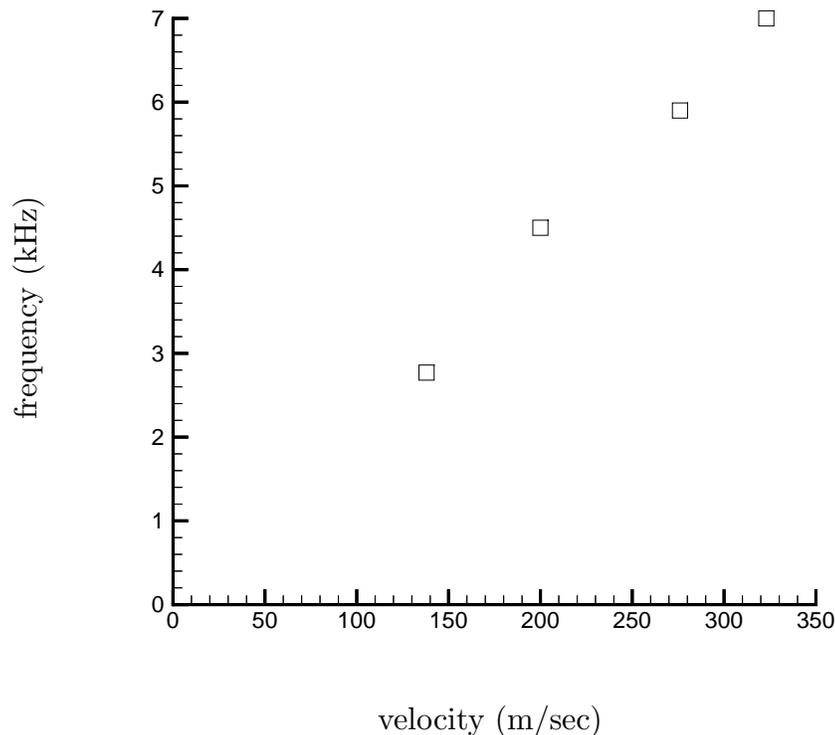
FIGURE 4.   Scaling of mode frequency with neutral velocity.

density: Fig. 3 corresponds to a higher plasma density than Fig. 2. The reason for the rise in discharge current (or plasma density) in the second case is that the neutrals coming off collisions with the walls, having lower speed, have a larger chance of being ionized. In reality, the speed of the neutrals after the collision will depend on the wall temperature as well.

2.2.3 Scaling of low frequency oscillations

Recall that characterizing the strong, ionization instability is of special interest for Hall thrusters as it sets limits to the operation of these engines. Figure 2 shows the time history of the discharge current for a simulation with discharge voltage of 200 volts, mass flow rate of 3mg/s, and peak magnetic field of 180 gauss. One observes that the steady state achieved is non-stationary, with large 7 kHz oscillations. This mode corresponds to the ionization instability described above. Superimposed on the dominant mode lie higher frequency components. The discharge current oscillations are about 20 per cent of their mean values. A simulation run with a flow rate of 5mg/s but with otherwise same parameters (including the same neutral velocity at the inflow) gives oscillations of about 60 per cent. Similarly, as one increases the voltage in the current saturation portion of the current-voltage curve, the rms oscillations are expected to increase. While high mass flow rates and high voltages are desirable, the ionization instability becomes very virulent and can extinguish the discharge, thus setting a limit on the operation of these engines.

Fife's *et al.* (1997) analysis of the ionization instability shows that the mode frequency scales as the geometric mean on the neutral and ion velocities. However, the theory is based on a small perturbation analysis which may not apply given the large rms levels of the instability. Intuitively, one rather expects the frequency to scale linearly with the neutral velocity, especially as the ion velocity tends to be much larger than the neutral velocity. The linear relationship is, in fact, observed in Fig. 4. The parameters for the simulation are the same as those in Fig. 2. We only changed the velocity of the neutrals at the inflow, while keeping the mass flow rate constant. The inverse of the slope of the linear fit through the data points yields a length of 4.3 cm. This length should reflect the extent of the ionization region. In fact, this region in the simulation is probably only 3 cm. The characterization of the mode frequency as the neutral velocity divided by the ionization region thus appears to be only qualitatively valid in view of the simulation results. Similar simulations with other voltages and mass flow rates should be performed to better characterize this mode. In particular, the notion that this mode is non-propagating (in the axial direction) and thus acts more as a standing wave needs to be reexamined. Present simulations suggest that the peak of the ionization rate profile moves back and forth about .75 centimeters axially, which would imply that the mode in fact propagates. Further simulations are warranted in view of the fact that previous simulations have not reported or seen this effect.

3. Conclusions and future plans

Two-dimensional simulations of the Hall thruster have been performed as a first step in an ongoing computational effort at CTR. The simulations reproduce some of the overall features observed in experiments such as the strong ionization instability. A linear relationship between the frequency of the instability and the neutral velocity at the inflow is predicted.

Modeling electron transport assuming Bohm diffusion is found to be problematic. The model results strongly depend upon the Bohm diffusion coefficient and can disagree strongly with experiments if appropriate values are not chosen. The thruster dynamics are also strongly influenced by heavy particle-wall interactions. We believe that improving the form of the diffusion coefficient will greatly improve the results. The theory of Yoshikawa and Rose serves as an excellent starting point in this task: the diffusion coefficient will no longer be a fixed value, but will rather depend on fluctuation amplitudes, becoming an axially-dependent 'eddy diffusivity'. Alternatively, work is underway to extract a diffusion coefficient by fixing the location of the ionization region, which can also be obtained from the magnetic field profiles.

Preliminary numerical experiments in which a magnetic field perturbation has been superposed to the equilibrium were performed. The results (not shown) suggest that the strong discharge current oscillation can be affected by applying a perturbation of the same frequency as the natural mode. By adjusting the phase of the applied perturbation, we were able to suppress the mode for about one and one half periods before it developed again. We must caution, however, that the form of the diffusion coefficient could greatly affect these results.

Another numerical experiment to be attempted in the near future consists of injecting a small amount of xenon neutrals from the exit computational boundary. This is meant to model the experimental situation in the laboratory in which a vacuum is never completely achieved. The neutrals can potentially affect ionization rates, electron temperature, and maybe even more importantly, the ionization instability. A quantitative prediction on the effect of these neutrals on engine performance would prove to be quite useful.

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