Topology change of vortices: weak and strong solutions

N. Kevlahan
Department of Mathematics & Statistics
McMaster University, CANADA

Topology change of vortices (e.g. vortex merging in two dimensions, or vortex reconnection in three dimensions) is a fascinating area of fluid dynamics, but one which is still poorly understood theoretically. Although numerical simulations have been used to explore the processes of vortex reconnection and vortex merging, no mathematical theory is able to fully describe it. Essentially inviscid theories of vortex interaction exist (e.g. Klein et al. 1995), but these theories break down when the separation distance between the vortex centres is of the same order as the vortex core radius. Thus, although these approaches present evidence for topology change, they can never actually demonstrate it. Methods that include diffusion enforce strong constraints on the dynamics: e.g. Ting & Klein (1991) approximate the long-time state of merging two-dimensional vortices, but must constrain the individual vortex centres to always lie on a circle. On the other hand, Agullo & Verga (1997) give an unconstrained stochastic weak solution of the two-dimensional vorticity equations which shows topology change, although the details of the interaction are clearly incorrect. None of the above approaches is able to properly describe all stages of vortex merger, even qualitatively.

In this talk I show precisely how the weak solution of Agullo & Verga (1997) differs from the exact solution, and how it may be improved. I also propose a new description of the reconnection of vorticity filaments in three dimensions based on combining the asymptotic theory of Klein et al. (1997) with the stochastic approach of Agullo & Verga (1997).

REFERENCES

Agullo, O. and Verga, A.D. 1997, "Exact two vortices solution of Navier-Stokes equations", Phys. Rev. Lett. (78) 2361-2364.

Klein, R., Majda, A.J. and K. Damodaran 1995 "Simplified equations for the interactions of nearly parallel vortex filaments", J. Fluid Mech. (288), 201-248.

Ting, L. and Klein, R. 1991 "Viscous vortical flows". Springer.