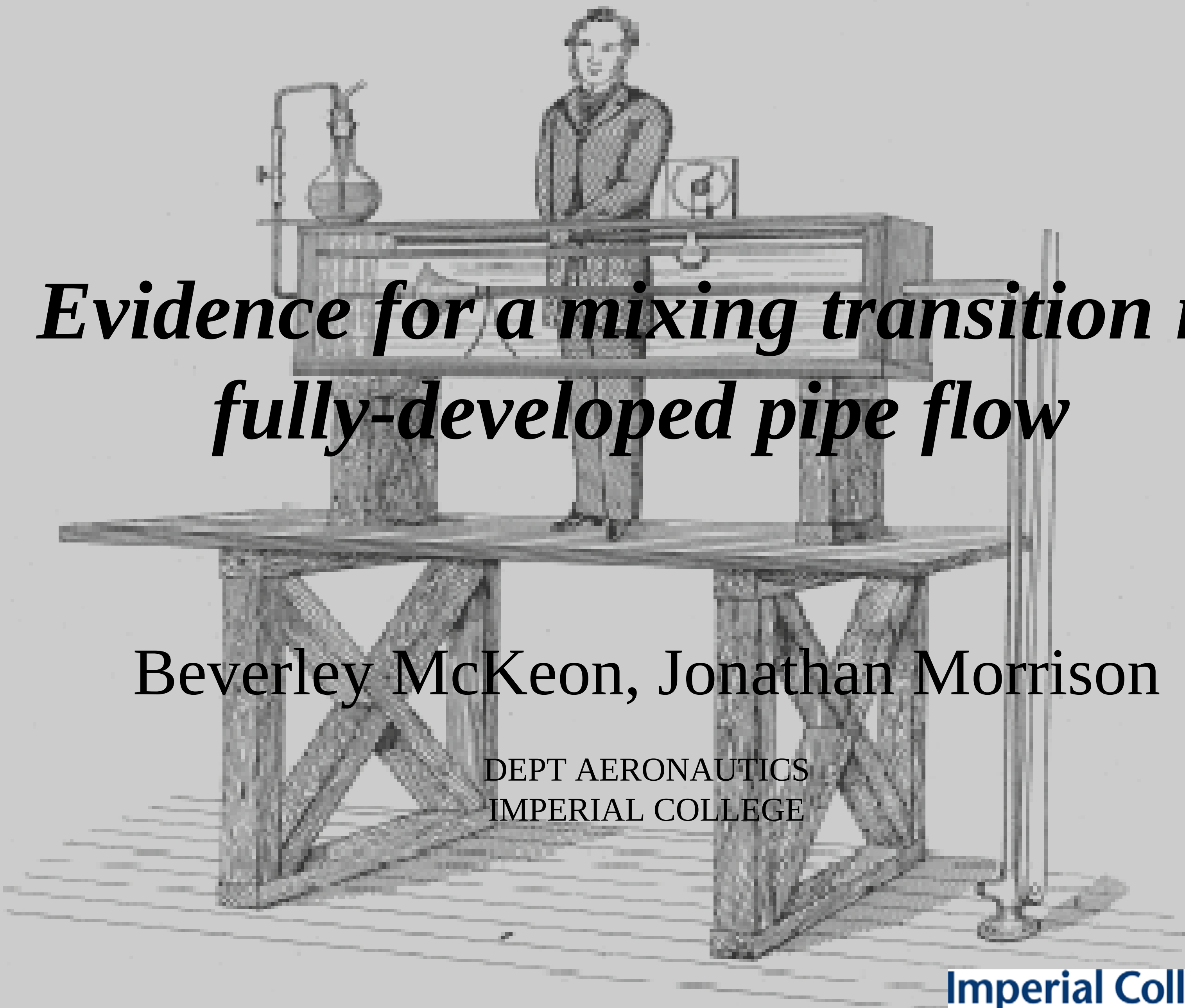

Evidence for a mixing transition in fully-developed pipe flow

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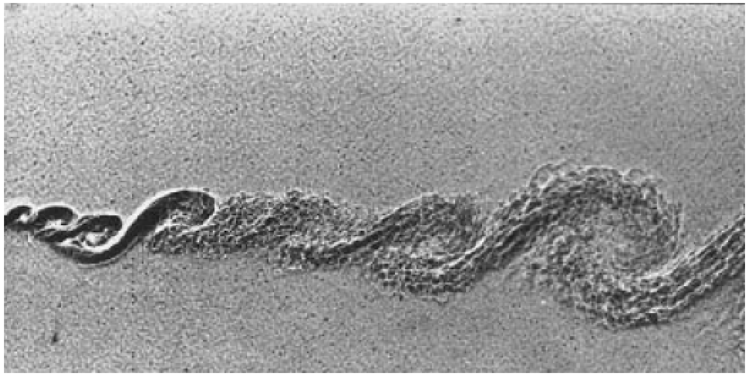
Summary

- Dimotakis’ “mixing transition”
- Mixing transition in wall-bounded flows
- Relationship to the inertial subrange
- Importance of the mixing transition to self-similarity
- Insufficient separation of scales: the mesolayer

Dimotakis' mixing transition

J. Fluid Mech. 409 (2000)

- Originally observed in free shear layers (e.g. Konrad, 1976)
- "Ability of the flow to sustain three-dimensional fluctuations" in Konrad's turbulent shear layer



- Dimotakis details existence in jets, boundary layers, bluff-body flows, grid turbulence etc

Dimotakis' mixing transition

J. Fluid Mech. 409 (2000)

- Universal phenomenon of turbulence/criterion for fully-developed turbulence
- Decoupling of viscous and large-scale effects
- Usually associated with inertial subrange
- Transition: $Re_{\delta*} \sim 10^4$ or $R_\lambda \sim 100 – 140$

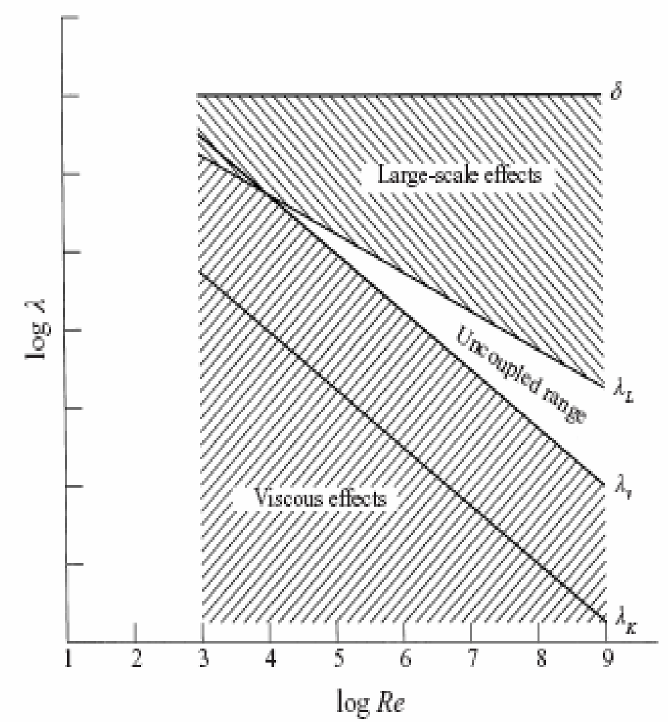
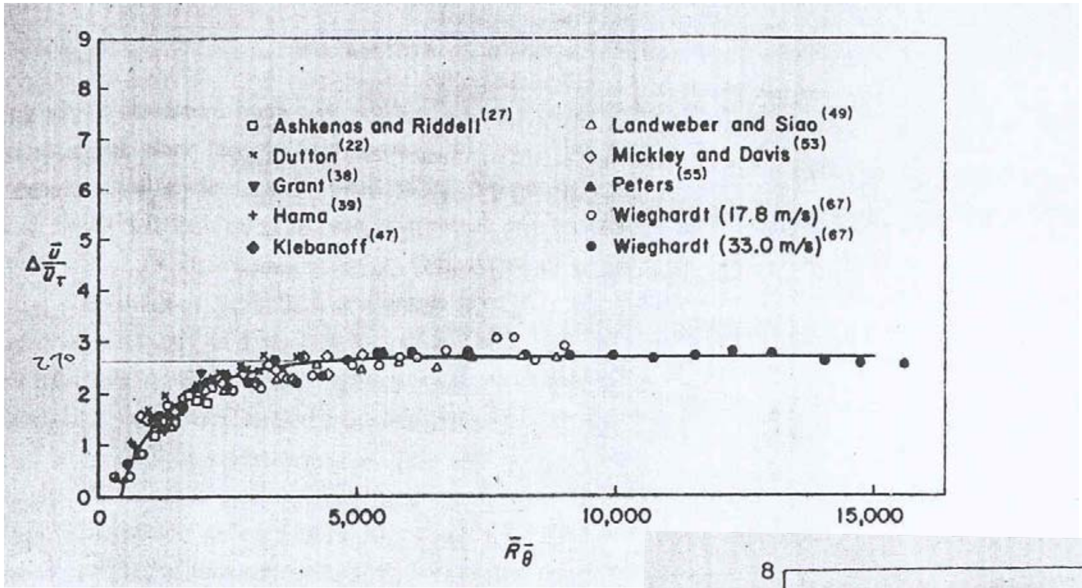
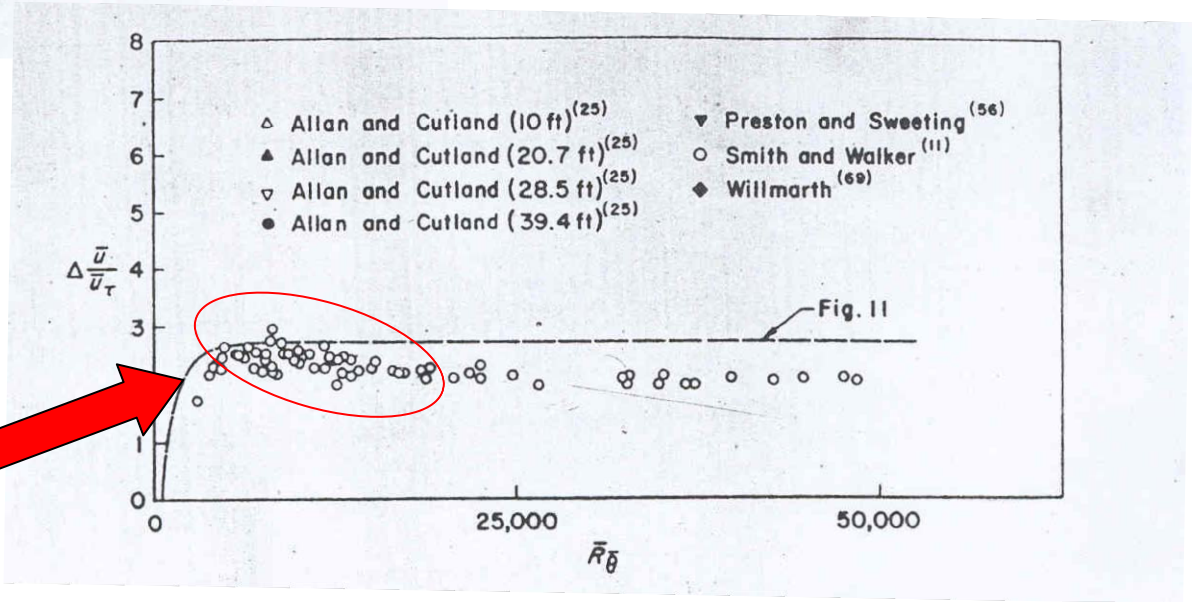


FIGURE 19. Reynolds number dependence of spatial scales for a turbulent jet.

Variation of wake factor with Re_θ





From Coles (1962)

Pipe equivalent: variation of ξ

- For boundary layers wake factor from

$$U^+ = \frac{1}{\kappa}\ln y^+ + B + \frac{\Pi}{\kappa}W_C\left(\frac{y}{\delta}\right)$$

$$2\frac{\Pi}{\kappa} = U_\infty^+ - \left(\frac{1}{\kappa}\ln\delta^+ + B\right)$$

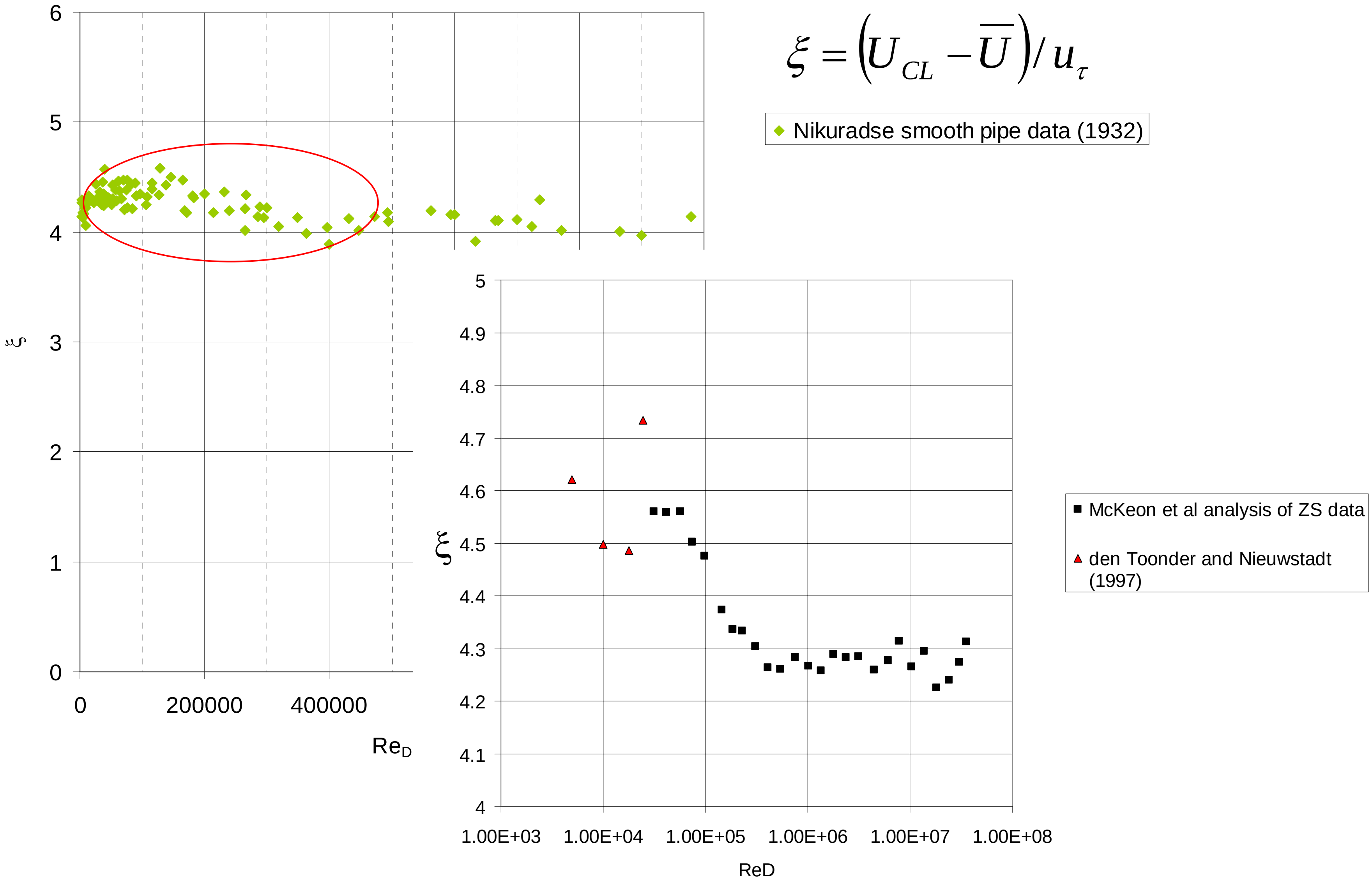
- In pipe $\xi = \left(U_{CL} - \overline{U}\right)/u_\tau$ related to wake factor

$$U_{CL}^+ - \overline{U^+} = U_{CL}^+ - \left(\frac{1}{\kappa}\ln R^+ + B - \frac{3}{2\kappa} + C_3 - C_4(\mathrm{Re}_D)\right)$$

- Note that ξ is ratio of ZS to traditional outer velocity scales

Same kind of Re variation in pipe flow

$$\xi = \left(U_{CL} - \overline{U}\right) / u_\tau$$



Identification with mixing transition

- Transition for $R_\lambda \sim 100 - 140$, or $Re_{\delta*} \sim 10^4$
- $Re_{R*} = 10^4$ when $Re_D \sim 75 \times 10^3$ ($R_\lambda = 110$ when $y^+ = 100$, approximately)
- This is Re_D where $\xi = \left(U_{CL} - \overline{U}\right)/u_\tau$ begins to decrease with Reynolds number
- R_λ varies across pipe – start of mixing transition when $R_\lambda = 100$
- Coincides with the appearance of a "first-order" subrange (Lumley `64, Bradshaw `67, Lawn `71)

Extension to inertial subrange?

- Mixing transition corresponds to decoupling of viscous and y scales - necessary for self-similarity
- Suggests examination of spectra, particularly close to dissipative range
- Inertial subrange: local region in wavenumber space where production=dissipation i.e. inertial transfer only
- "First order" inertial subrange (Bradshaw 1967): sources,sinks << inertial transfer

Scaling of the inertial subrange

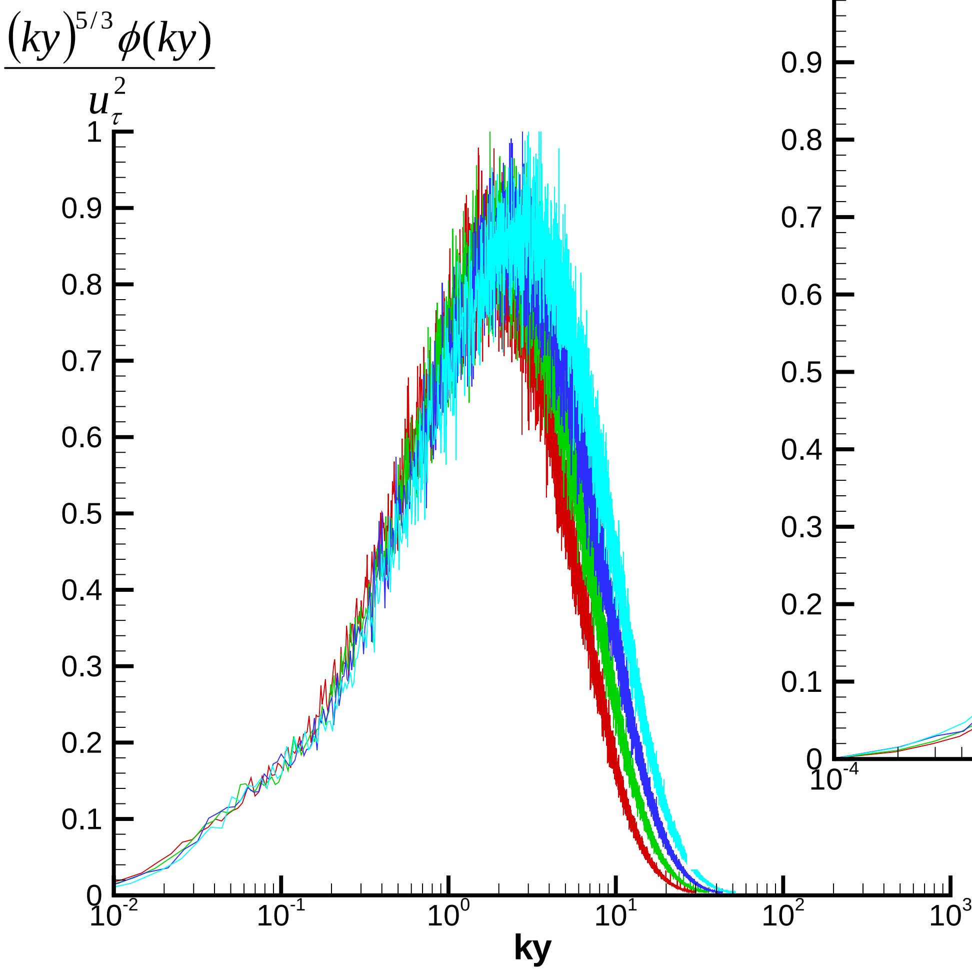
For $\quad {}^{\nu}\!/_{u_\tau} << y << R$

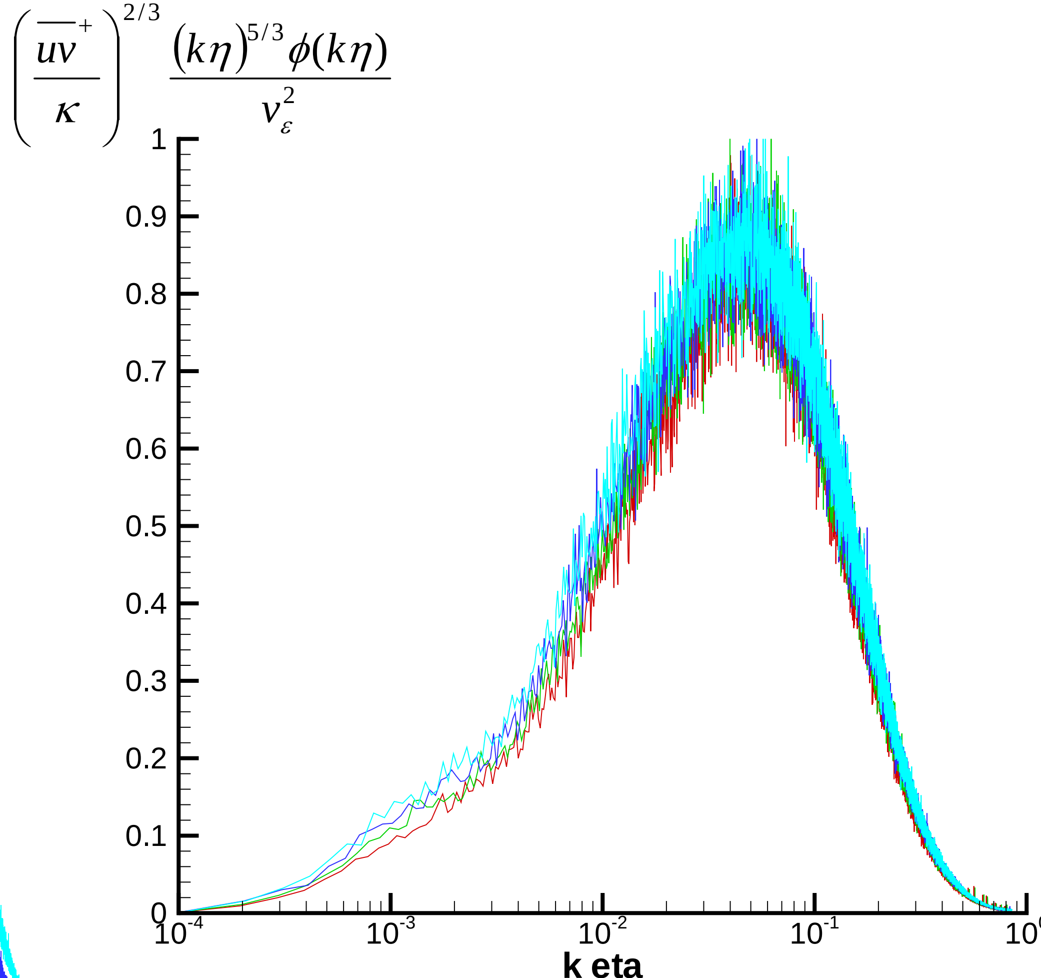
K41 overlap $\quad \phi_{11}(k_1) = A\varepsilon^{2/3}k_1^{-5/3} \qquad {}^{1}\!/_{y} << k_1 << {}^{1}\!/_{\eta}$

In overlap region, dissipation $\quad \varepsilon = \dfrac{-\overline{uv}^{+}u_\tau^3}{\kappa y}$

$$\boxed{\frac{(k_1 y)^{5/3}\phi_{11}(k_1 y)}{u_\tau^2} = C\left(\frac{-\overline{uv}^{+}}{\kappa}\right)^{2/3} = \left(\frac{-\overline{uv}^{+}}{\kappa}\right)^{2/3}\frac{(k_1\eta)^{5/3}\phi_{11}(k_1\eta)}{v_\varepsilon^2}}$$

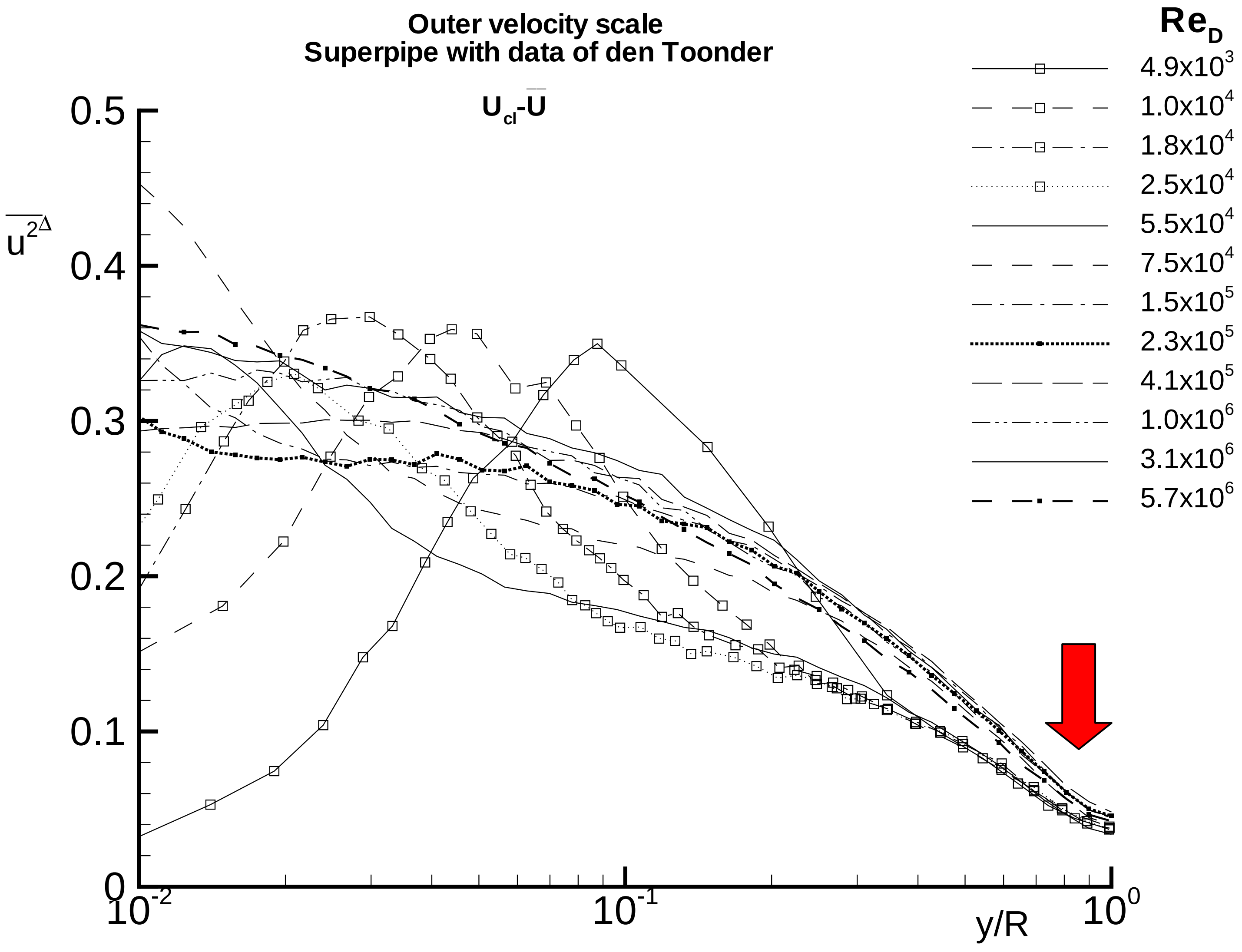
Inertial subrange – $Re_D = 75 \times 10^3$



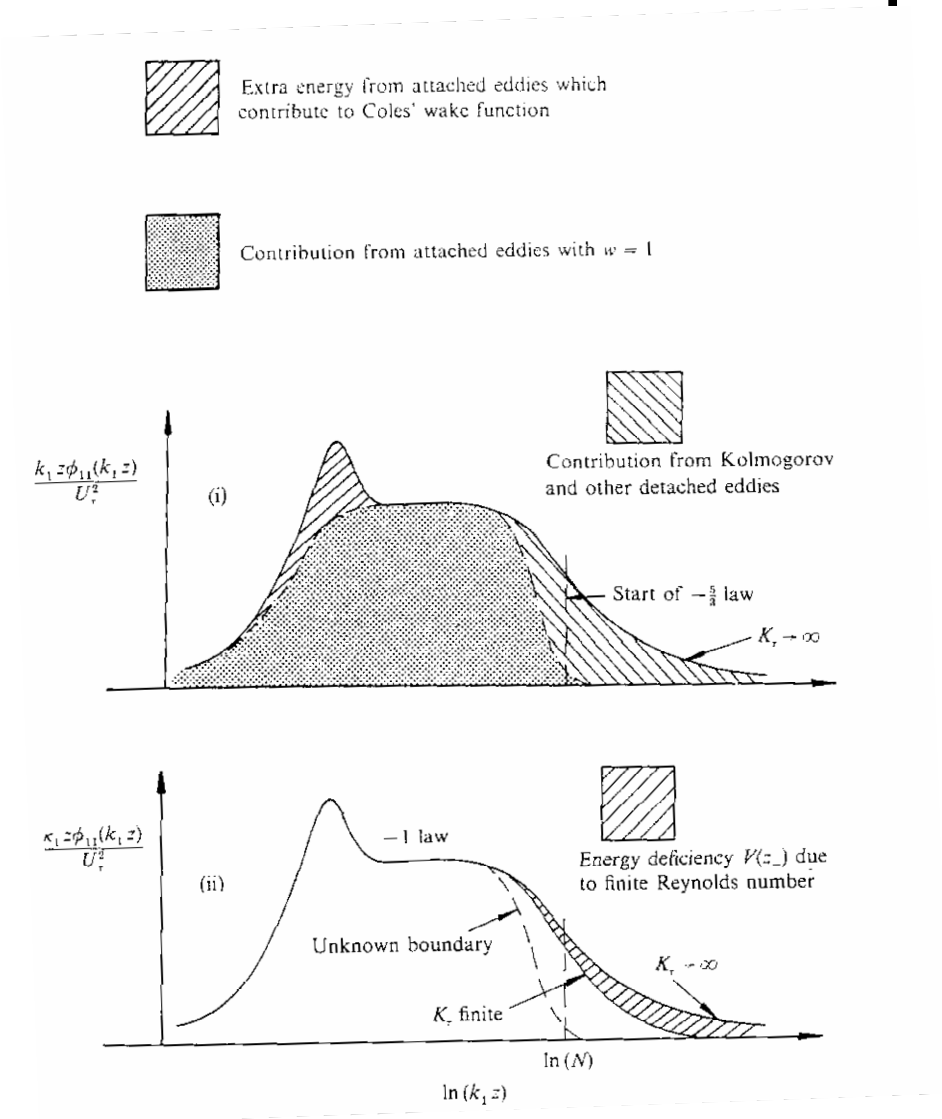


$y^+ = 100 - 200$ i.e. traditionally accepted log law region

Scaling of streamwise fluctuations $\overline{u^2}$

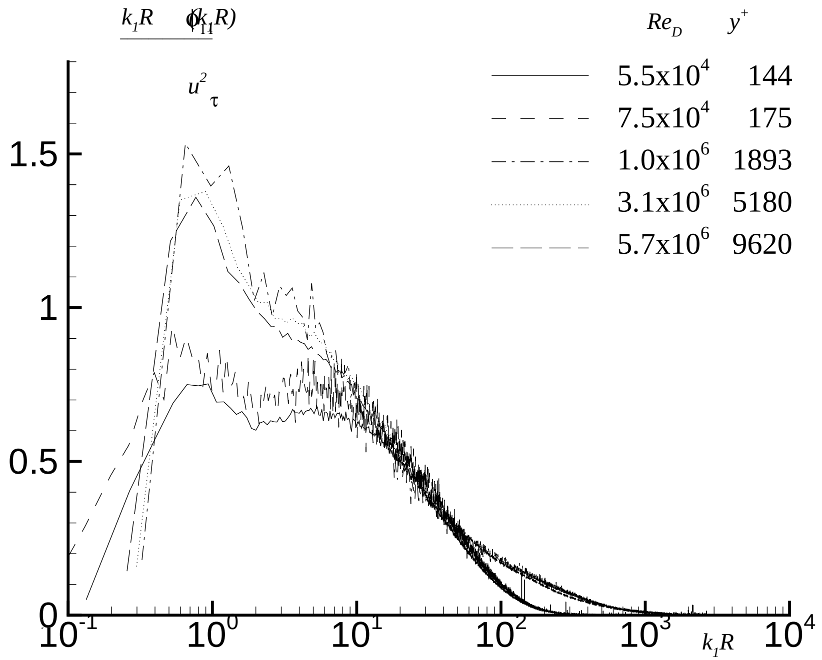


Similarity of the streamwise fluctuation spectrum I

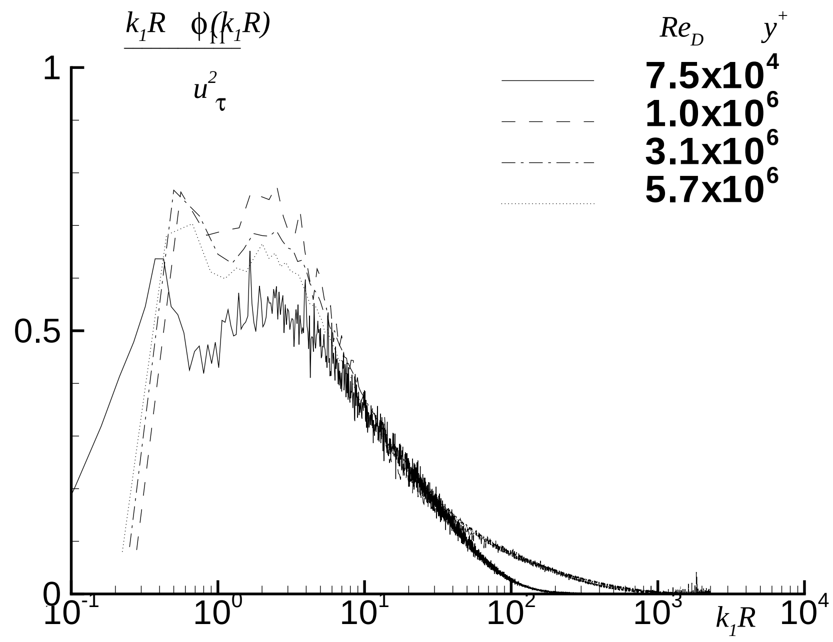


Perry and Li, J. Fluid Mech. (1990). Fig. 1b

Similarity of the streamwise fluctuation spectrum II

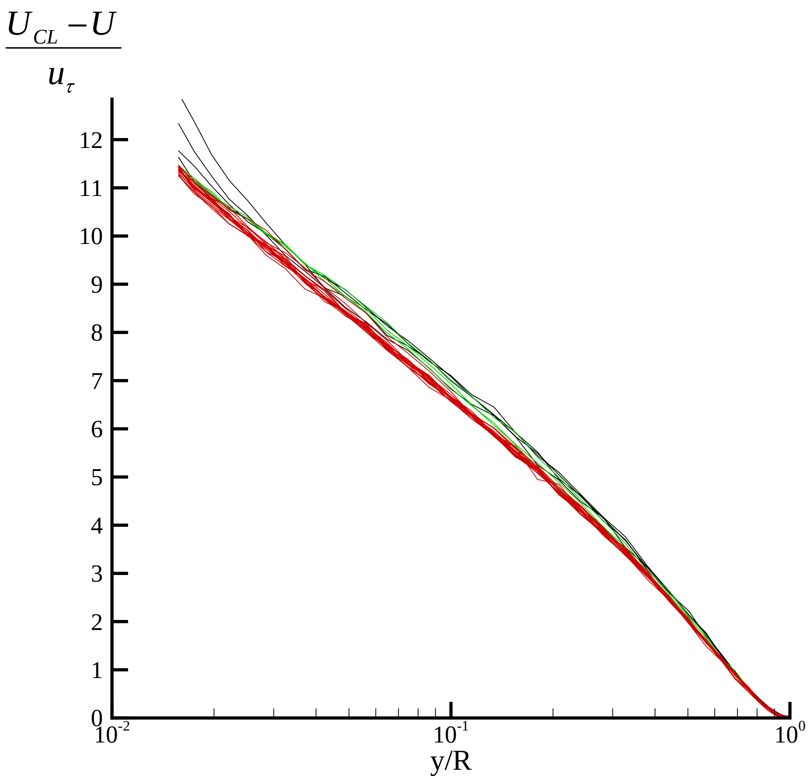


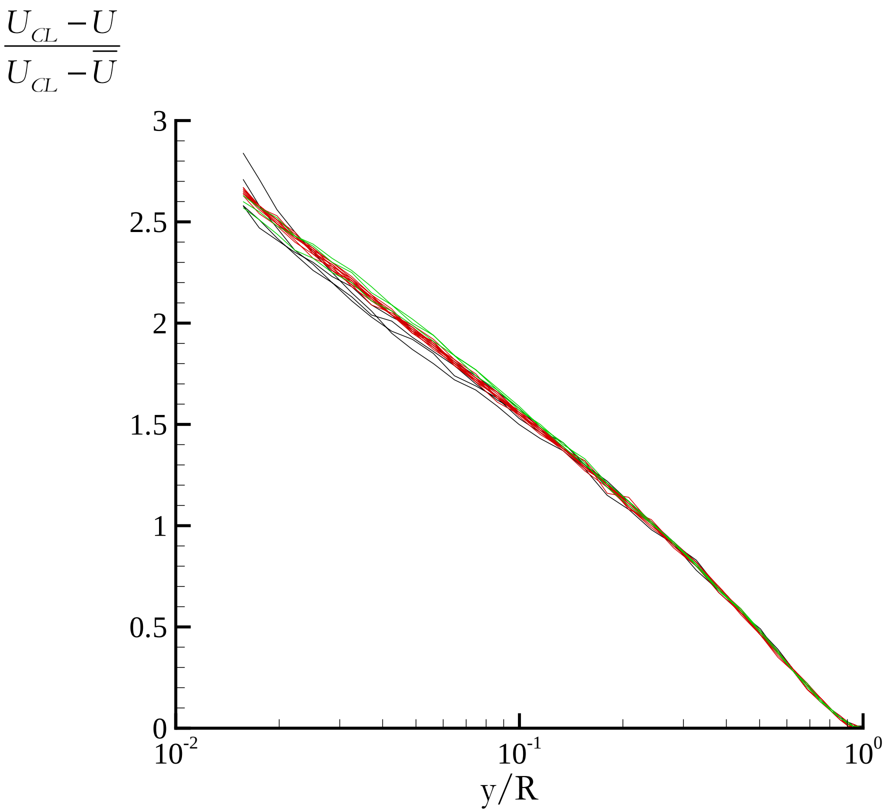
y/R = 0.10

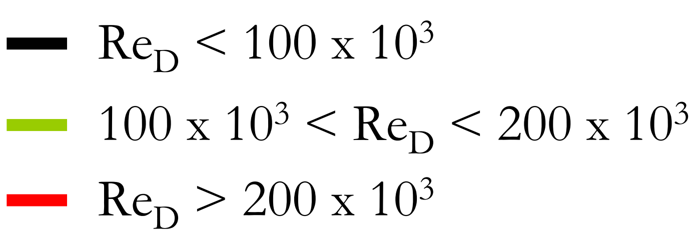


y/R = 0.38

Outer velocity scale for the pipe data





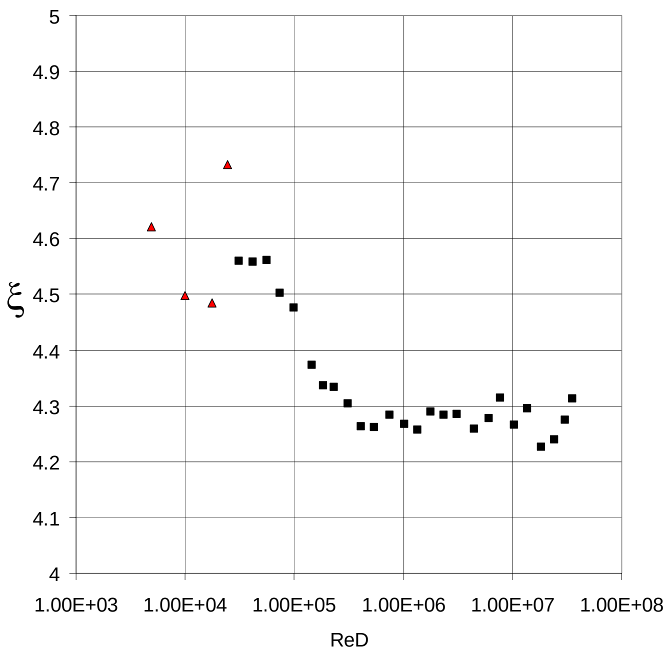


ZS outer scale gives better collapse in core region for all Reynolds numbers

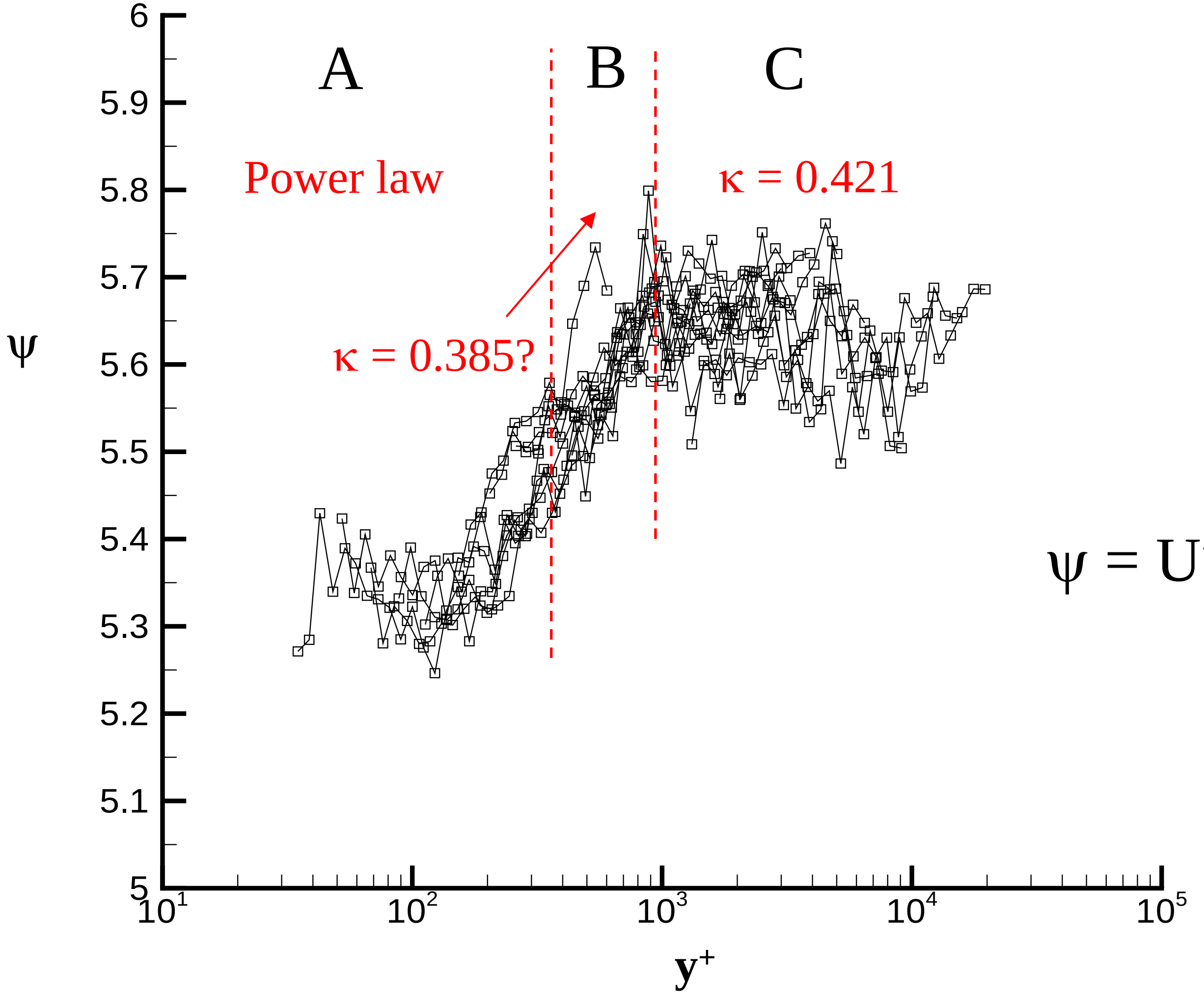
Self-similarity of mean velocity profile requires ξ = const.

- Addition of inner and outer log laws shows that U_{CL}^{+} scales logarithmically in R^{+}
- Integration of log law from wall to centerline shows that $\overline{U}^{+}$ also scales logarithmically in R^{+}

- Thus for log law to hold, the difference between them, ξ, must be a constant
- True for $Re_D > 300 \times 10^3$



Inner mean velocity scaling



$$\psi = U^+ - 1/\kappa \ln y^+$$

Relationship with “mesolayer”

- e.g. Long & Chen (1981), Sreenivasan (1997), Wosnik, Castillo & George (2000), Klewicki *et al*
- Region where separation of scales is too small for inertially-dominated turbulence OR region where streamwise momentum equation reduces to balance of pressure and viscous forces $\left(d\overline{uv} \big/ dy \approx 0 \right)$
- Observed below mixing transition
- Included in generalized log law formulation (Buschmann and Gad-el-Hak); second order and higher matching terms are tiny for $y^+ > 1000$

Summary

- Evidence for start of mixing transition in pipe flow at $Re_D = 75 \times 10^3$. (Not previously demonstrated)
- Correspondence of mixing transition with emergence of the “first-order” inertial subrange, end of mesolayer
- Importance of constant ξ for similarity of mean velocity profile (Reynolds similarity)
- Difference between Re_D for mixing transition and complete similarity