Observations on the scaling of the streamwise velocity component in wall turbulence

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Synopsis

- Self-similarity – what should we be looking for?
- Log law: self-similar scaling - (y, u_τ)
- Local-equilibrium approximation as a self-similar energy balance in physical space.
- Spatial transport – interaction between inner and outer regions.
- Self-similarity of energy balance appears as inertial subrange in spectral space.
- Consistent physical- and spectral- space views.
- Self-similarity as a pre-requisite for universality.

What is self-similarity?

$$\frac{U}{u_\tau} = \frac{1}{\kappa}\ln\left(\frac{yu_\tau}{\nu}\right) + B, \quad \frac{U}{u_\tau} = \frac{1}{\kappa}\ln\left(\frac{y}{k}\right) + B^{**}$$

- *Simultaneous* overlap analysis for $\nu/u_\tau << y << R$, indicates motion independent of inner and outer lengthscales
- Therefore the constant in the log argument is merely a constant of integration and may be freely chosen.
- It is usually taken to be the dominant imposed lengthscale so that its influence on B or B^{**} is removed: as

$$y^+ \to \infty, \; B \to 5.6 \qquad \frac{y}{k} \to \infty, \; B^{**} \to 8.5$$

- Overlap analysis indicates κ is universal: but self-similarity is a pre-requisite.

The local-equilibrium approximation

- Application of self-similar $\left(y, u_\tau\right)$ scaling to the energy balance gives $P = \varepsilon$
- Therefore expect log-law and local-equilibrium regions to be coincident.
- Inertial subrange is self-similar spectral transfer, $T(k)$, as demonstrated by *simultaneous* overlap with inner and outer scaling.
- Wavelet decomposition (DNS data, *JFM 491*) shows $T(k)$ much more spatially intermittent than equivalent terms for either P or ε.
- Therefore $T(k)$ is unlikely to scale simply.
- Even then, energy balance at any point in space is an integration over all k – so $P = \varepsilon$ *will only ever be an approximation.*
- Usefulness of a “first-order” subrange (Bradshaw 1967)?

Self-similarity of the second moment

- Examine self-similarity using distinction between inner $(y^+ = yu_\tau/\nu >> 1)$ and outer $(R^+ = Ru_\tau/\nu)$ influences in wall region.
- Examine possibility of self-similarity in $\phi_{11}(k_1)$.
- If $\phi_{11}(k_1)$ not self-similar, then $\overline{u^2}^+$ is very unlikely to be either.
- Comparison of Townsend's 1956 ideas with those of 1976 – are outer-layer influences "inactive"?
- Use these ideas to highlight principal differences between scaling of $\phi_{11}(k_1)$ in pipes and boundary layers, and *even between different flows at the same* R^+.

“Strong” asymptotic condition: $R^+=\infty$

- As $R^+ \to \infty$, and $y/R \to 0$, “large eddies are weak” (Townsend 1956).
- “Neglecting this possibility of outside influence”:
$$\phi_{11}(k_1, y, u_\tau) = u_\tau^2 y \psi(k_1 y)$$
where ψ is a “universal” function.
- Therefore, provided ϕ_{11} is independent of y, collapse on inner variables <u>alone</u> is sufficient to demonstrate self-similarity.
- Then:
$$\phi_{11} \propto u_\tau^2 k_1^{-1}$$

"Strong" asymptotic condition

- Neglecting streamwise gradient of Reynolds stress:

$$\frac{\partial}{\partial y}(-\overline{uv}) = \overline{v\omega_z} - \overline{w\omega_y} + \nu \frac{\partial^2 U}{\partial y^2}$$
$$\approx \left[\overline{\mathbf{u}\times\boldsymbol{\omega}}\right]_x$$

- Write $\mathbf{u} = \mathbf{u_i} + \mathbf{u}_o$with $\boldsymbol{\omega}_o \approx 0$

$$\frac{\partial}{\partial y}(-\overline{uv}) \approx \left[\overline{\mathbf{u}_i \times \boldsymbol{\omega}_i}\right]_x + \cancel{\left[\overline{\mathbf{u}_o \times \boldsymbol{\omega}_i}\right]_x}$$

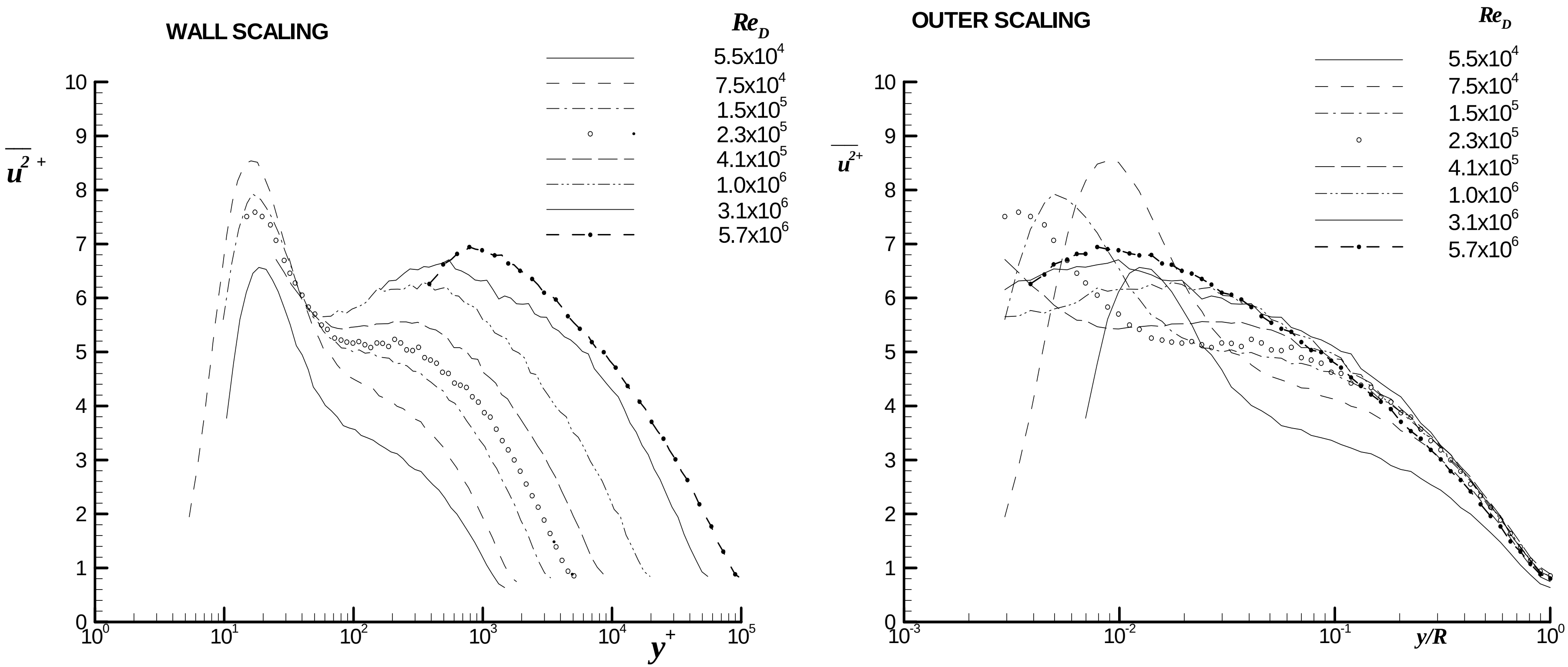
- Last term negligible (scale separation again) and removal of cross product linearizes the outer influence.
- "Inactive motion is a meandering or swirling made up from attached eddies of large size……" (Townsend 1961).

Conclusions from"strong" asymptotic condition

- Write $\mathbf{u} = \mathbf{u_i} + \mathbf{u}_o$: $\quad \overline{\mathbf{u}^2}^+ = \overline{\mathbf{u}_i^2}^+ + \overline{\mathbf{u}_0^2}^+ + \underbrace{\cancel{2\overline{\mathbf{u}_i \mathbf{u}_0}^+}}_{}^{0}$
- Blocking means that $v_0 \approx u_0 \dfrac{y}{x}$
- Therefore, $\overline{v^2}^+$ and $\overline{uv}^+$ are, to first order, $F\left(y^+\right)$ only.
- But: $\overline{u_i^2}^+ = F_i\left(y^+\right)$, $\overline{u_o^2}^+ = F_o\left(R^+\right)$ and outer influence appears as a linear superposition.
- Therefore, at the same R^+, internal and external flows are the same.

What's wrong with this picture?

Superpipe



What's wrong with this picture ?
Self-similar structure

- k_1^{-1} implies hierarchy of self-similar, non-interacting attached wall eddies that makes valid the assumption of linear superposition.

- Then:

$$\hat{R}_{vv} = \frac{\overline{v(y)v(y_1)}}{\overline{v^2}(y_1)} = \frac{y}{y_1}$$

- Even atmospheric surface layer show that this is not the case: the absence of direct viscous effects is an insufficient condition.

Hunt et al. (Adv. Turb 2, Springer 1989)

Linear superposition:

HUNT ET AL. 1989

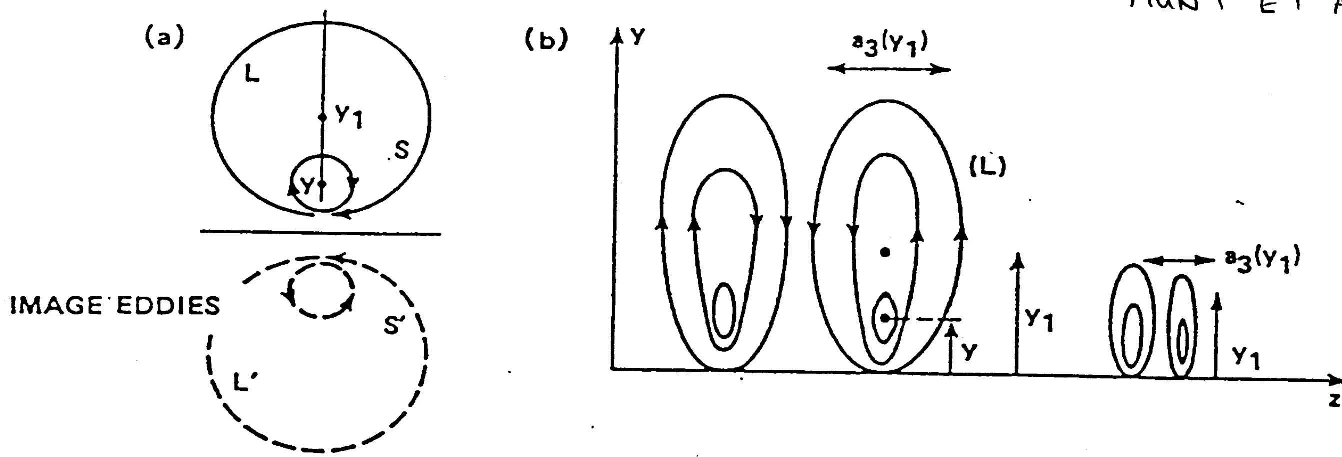


Fig.2 Schematic eddy shapes for analysing (a) shear-free boundary layers, and (b) shear boundary layers.

eddy below the boundary (which is present to satisfy the boundary condition (2.1c)). So at y, $v(y) \simeq v_S + (y/y_1)v_L$; and therefore (as the statistical theory confirms),

$$\overline{v(y)v(y_1)} \simeq (y/y_1)\overline{v^2}(y_1) \quad \text{or} \quad \hat{R}_{vv} = \frac{\overline{v(y)v(y_1)}}{\overline{v^2}(y_1)} \simeq y/y_1 \qquad (2.4)$$

Hunt et al. (Adv. Turb 2, Springer 1989)

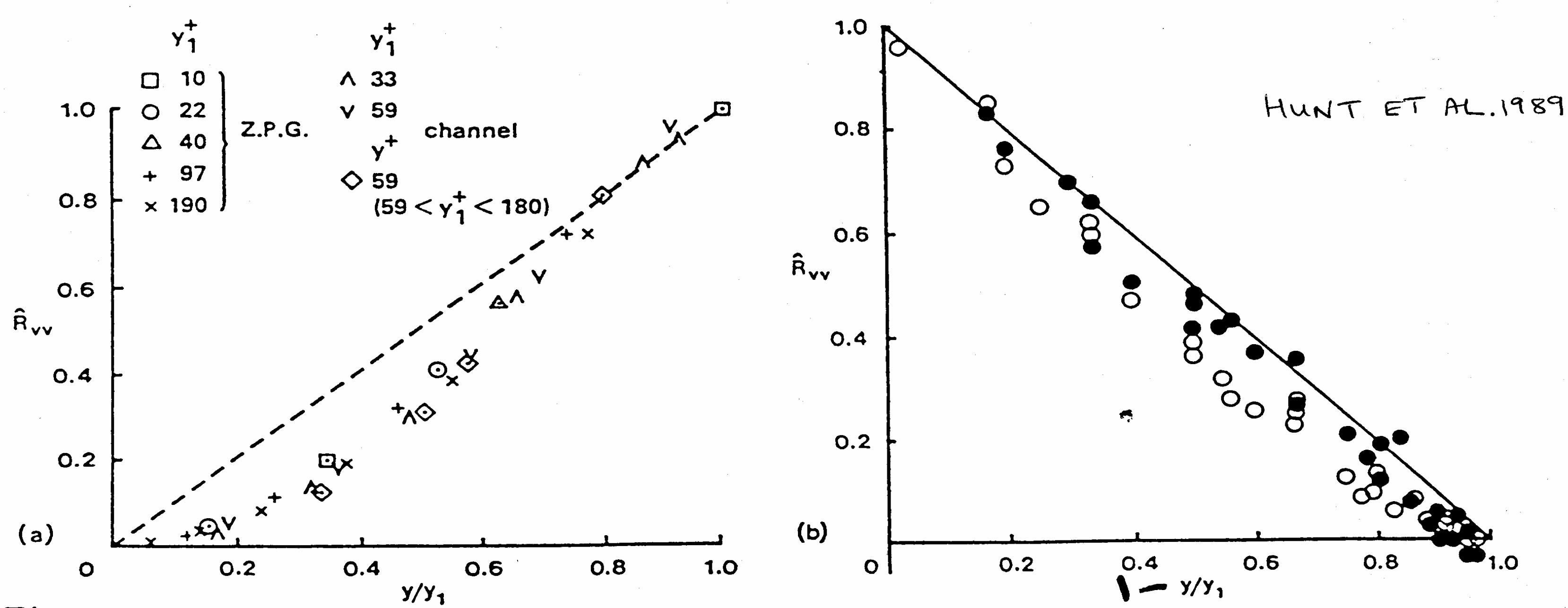


Fig.3

Cross correlation of v at heights y and y_1 normalised by $\overline{v^2}$ at y_1. (a) Computed from direct numerical simulations of the zero pressure gradient boundary layer (Spalart 1987), and the plane channel (Moin & Moser 1987). Also shown are the theoretical predictions of Hunt (1984) ($\hat{R}_{vv} \approx y/y_1$). (b) From atmospheric measurements at Boulder Atmospheric Observatory.

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What's wrong with this picture?

Wall-pressure fluctuations

- Integration of spectrum $\phi_{pp}(k_1) = A\dfrac{\tau_w^2}{k_1}$

  in the approximate range $(0.1R)^{-1} \le k_1 \le u_\tau/30\nu$

  gives: $\dfrac{\overline{p_w^2}}{\tau_w^2} = B \ln R^+ + C$

  where $B \approx 1.6$ and $C > 0$.

- Therefore, even consideration of the active motion alone shows that:

  1. wall-pressure fluctuations increase with R^+,

  2. large scales penetrate to the wall: the near-wall region is not "sheltered".

A "weak" asymptotic condition: $R^+ \to \infty$

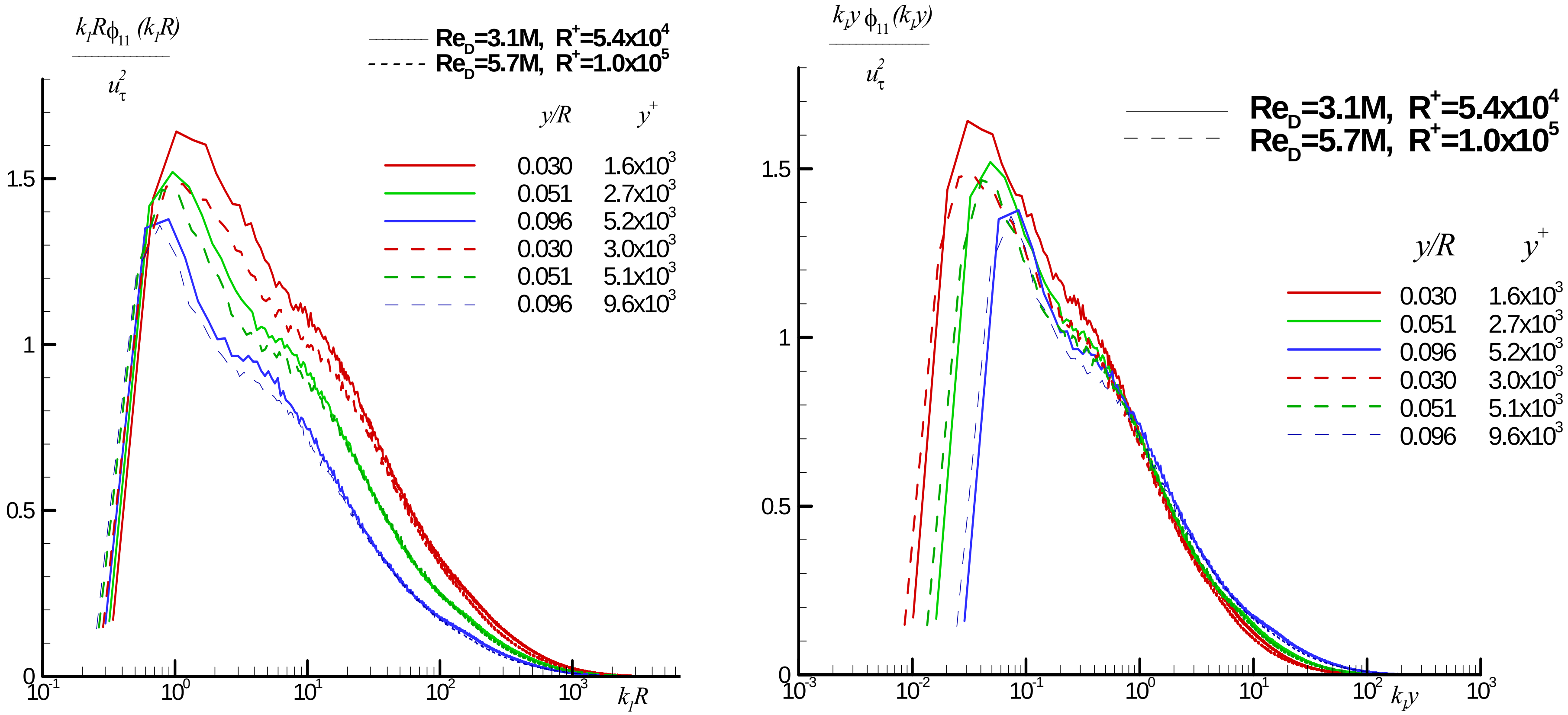
- "Superpipe" data show that outer influence:
  1. is not "inactive" and interacts with inner component
  2. increases with R^+
  3. increases with decreasing distance from the wall.
- Therefore, linear decomposition is not possible, i.e:

$$\overline{u^2}^+ = F(y^+; y/R) = G(y^+; R^+)$$

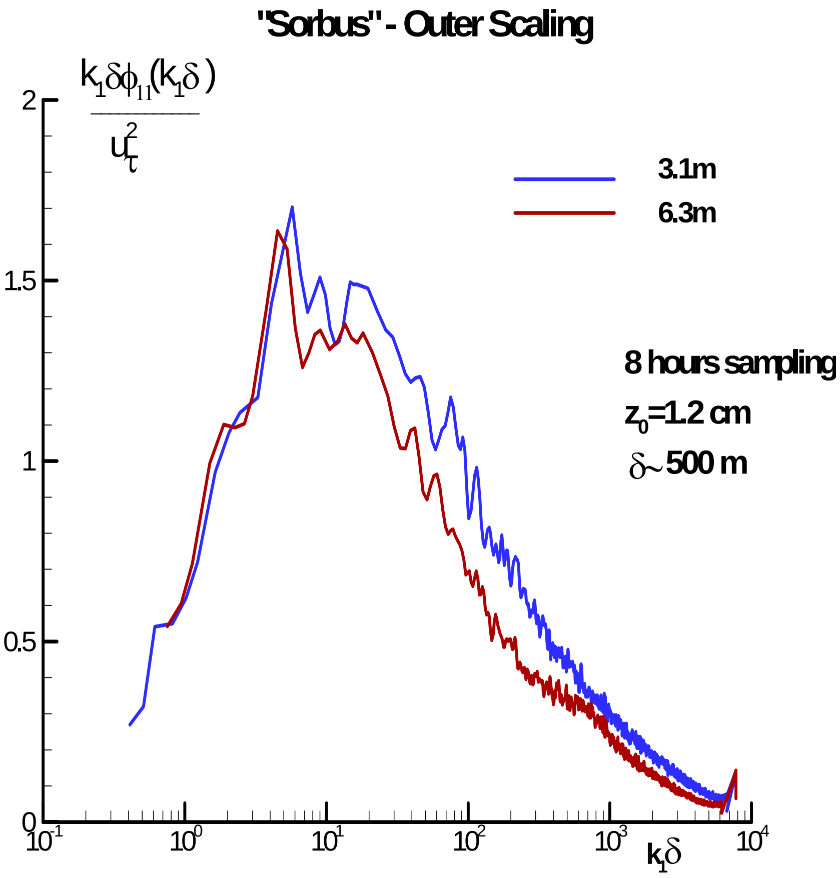
- Therefore to demonstrate complete similarity, we must have <u>simultaneous</u> collapse on inner and outer variables.
- "It now appears that simple similarity of the motion is not possible with attached eddies and, in particular, the stress-intensity ratio depends on position in the layer"

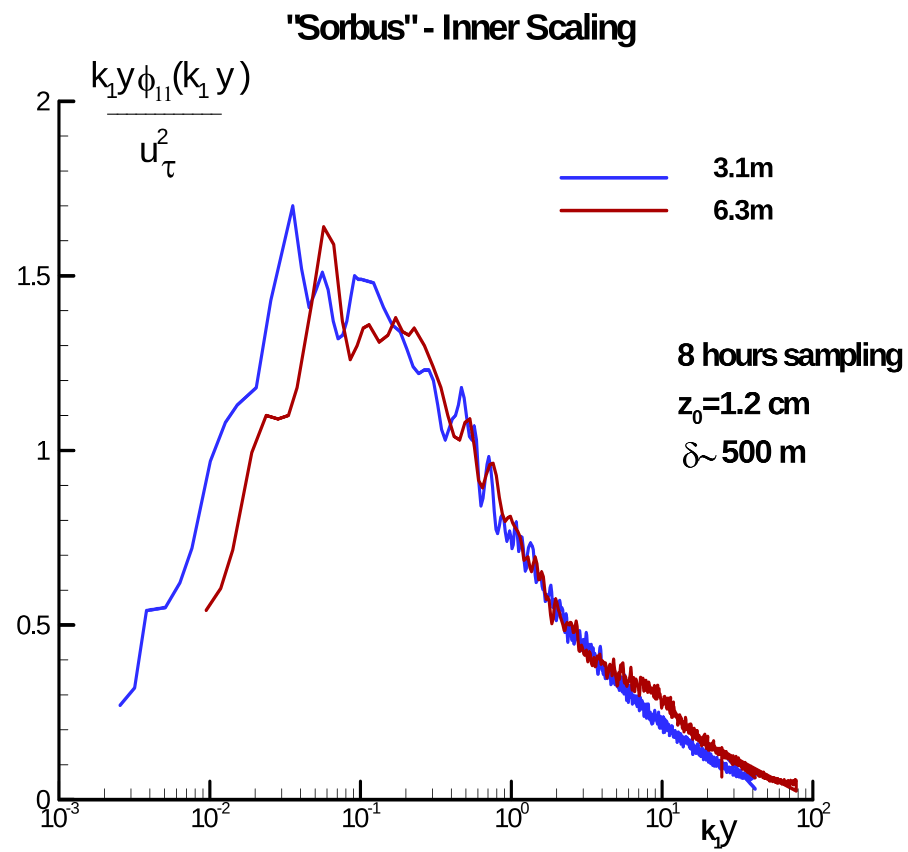
(Townsend 1976).

Superpipe spectra



Laban's Mills surface layer (Högström)





Observations

- Both the "strong" and "weak" asymptotic conditions lead to complete similarity.
- $\phi_{11}(k_1)$ spectra in both the "superpipe" and the atmospheric surface layer show only incomplete similarity. In both cases, R^+ is too low to show complete similarity.
- As the Reynolds number increases, the receding influence of direct viscous effects has to be distinguished from the increasing influence of outer-layer effects, because the inner/outer interaction is non-linear: $\overline{u^2}^+ = F(y^+; y/R)$

Nature of the inner-outer interaction

- Streamwise momentum: $\frac{\partial}{\partial y}(-\overline{uv}) = \overline{v\omega_z} - \overline{w\omega_y} + \nu\nabla^2 U$

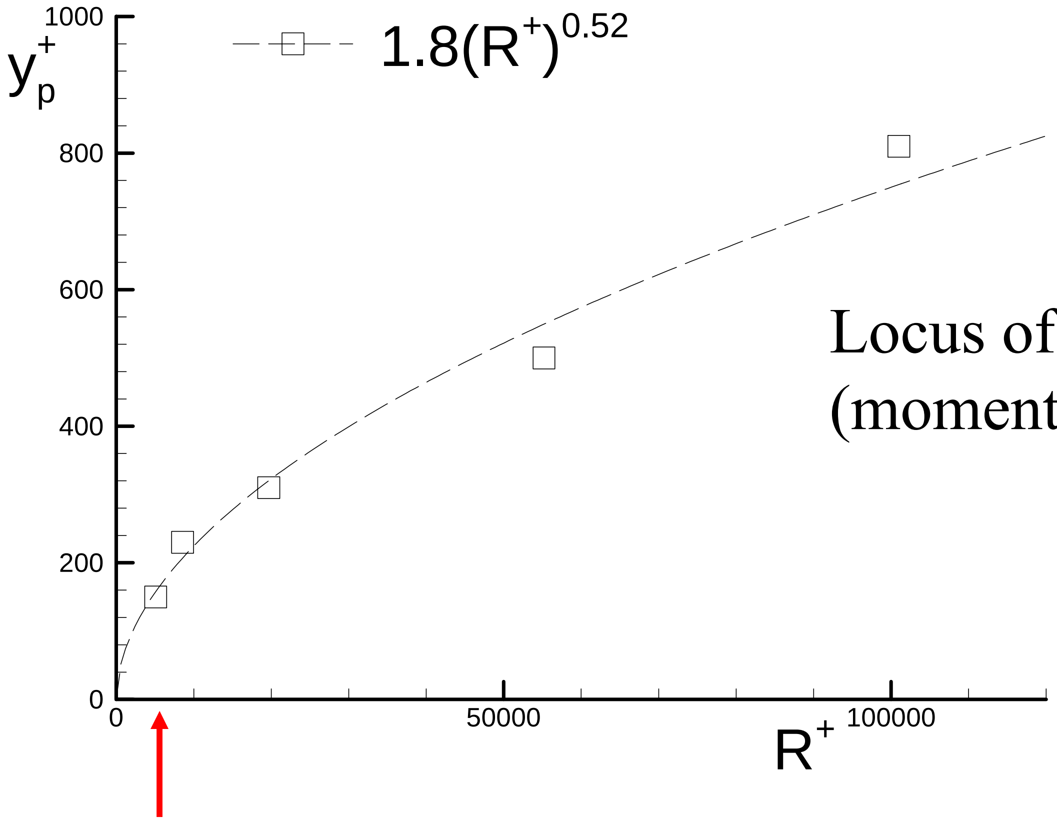
- The mesolayer defined by $\frac{\partial}{\partial y}(-\overline{uv}) = 0$
- Balance of viscous and inertial forces gives length scale

$$\delta_m^+ \propto \left(R^+\right)^{\frac{1}{2}}$$

- The energy balance in the mesolayer involves turbulent and viscous transport, as well as production and dissipation.
- Since $1 << R^{+\frac{1}{2}} << R^+$, a mesolayer exists at any R^+.
- The lower limit to the log law should be expected to increase approximately as $R^{\frac{1}{2}}$.

The mesolayer:

locus of outer peak in $\overline{u^2}^+$



Locus of $-\overline{uv}^+$: $1.5(R^+)^{0.5}$
(momentum equation and log law)

First appearance of log law

Nature of the inner-outer interaction

- The occurrence of a self-similar k_1^{-1} range cannot be expected in or below the mesolayer.
- The full decomposition

not small

$$\frac{\partial}{\partial y}(-\overline{uv}) \approx \left[\overline{\mathbf{u}_i \times \boldsymbol{\omega}_i}\right]_x + \left[\overline{\mathbf{u}_i \times \boldsymbol{\omega}_o}\right]_x + \left[\overline{\mathbf{u}_o \times \boldsymbol{\omega}_i}\right]_x + \left[\overline{\mathbf{u}_o \times \boldsymbol{\omega}_o}\right]_x$$

  shows how ideas concerning widely separated wavenumbers can be misleading – inner/outer interaction is a more important consideration if looking for self-similarity, $R^+ \to \infty$
- Does inner/outer interaction preclude self-similarity of inertial-range statistics?
- Is this most likely in the local-equilibrium region?

A first-order inertial subrange

- Bradshaw (1967) suggested that a sufficient condition for a "first-order" subrange is that $T(k) >>$ sources or sinks.
- This occurs in a wide range of flows for $R_\lambda > 100$
- No local isotropy: $R_{12}(k_1) > 0$, but decreasing rapidly as $k_1 \to \infty$
- Local-equilibrium region is a physical-space equivalent, where small spatial transport appears as a (small) source or sink at each k (*JFM 241*).
- Saddoughi and Veeravalli (1994) show two decades of -5/3: lower one, $R_{12}(k_1) > 0$: higher one $R_{12}(k_1) \approx 0$ for $R_\lambda > 1500$
- How does the requirement of self-similar $T(k)$ fit in?

A self-similar inertial subrange

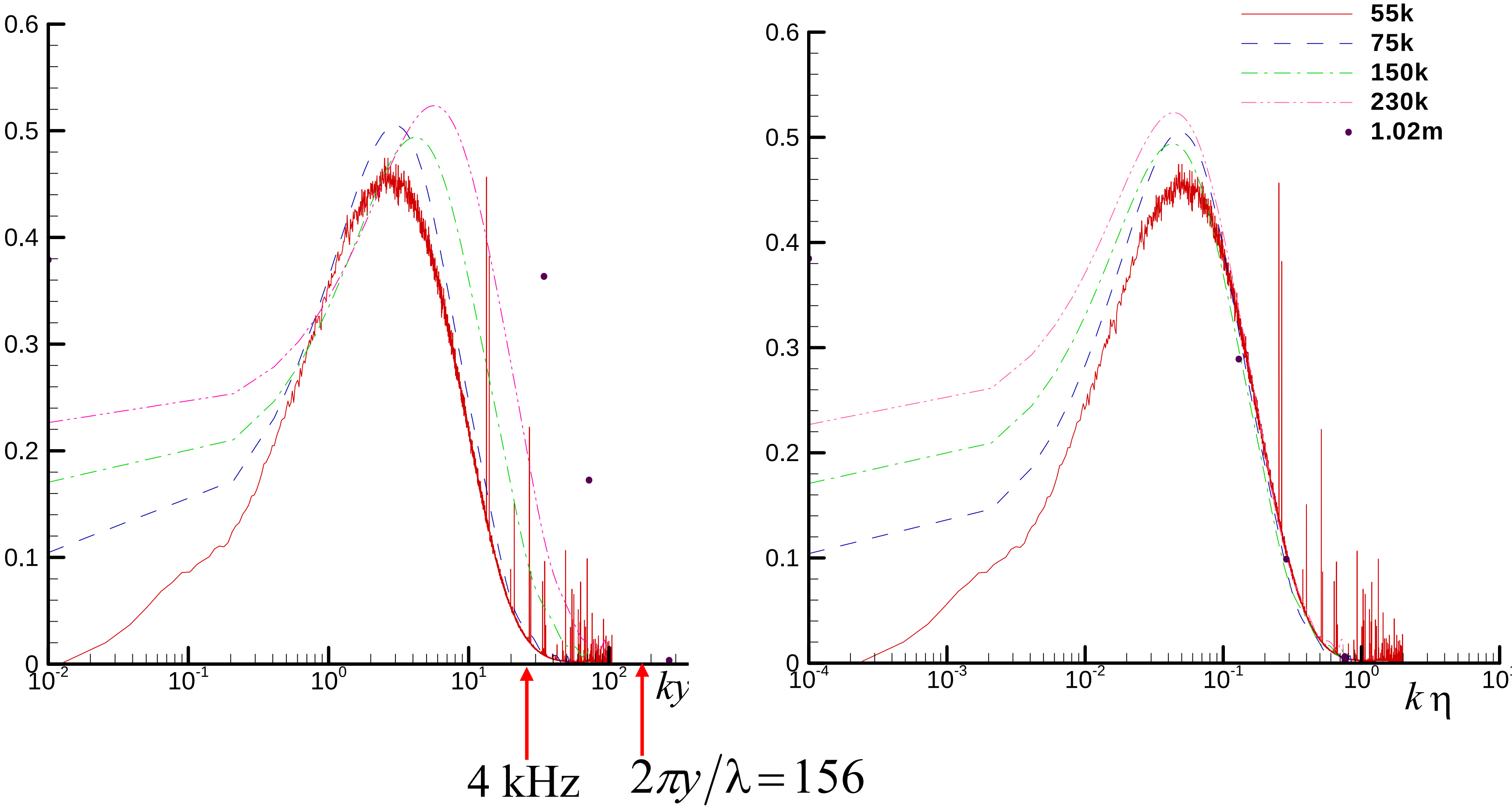
- *Simultaneous* collapse

$$\left(\frac{\kappa}{-\overline{uv}^{+}}\right)^{\frac{2}{3}} \frac{(k_1 y)^{\frac{5}{3}}}{u_\tau^2} \phi_{11}(k_1 y) = C = \frac{(k_1 \eta)^{\frac{5}{3}}}{\upsilon_\varepsilon^2} \phi_{11}(k_1 \eta)$$

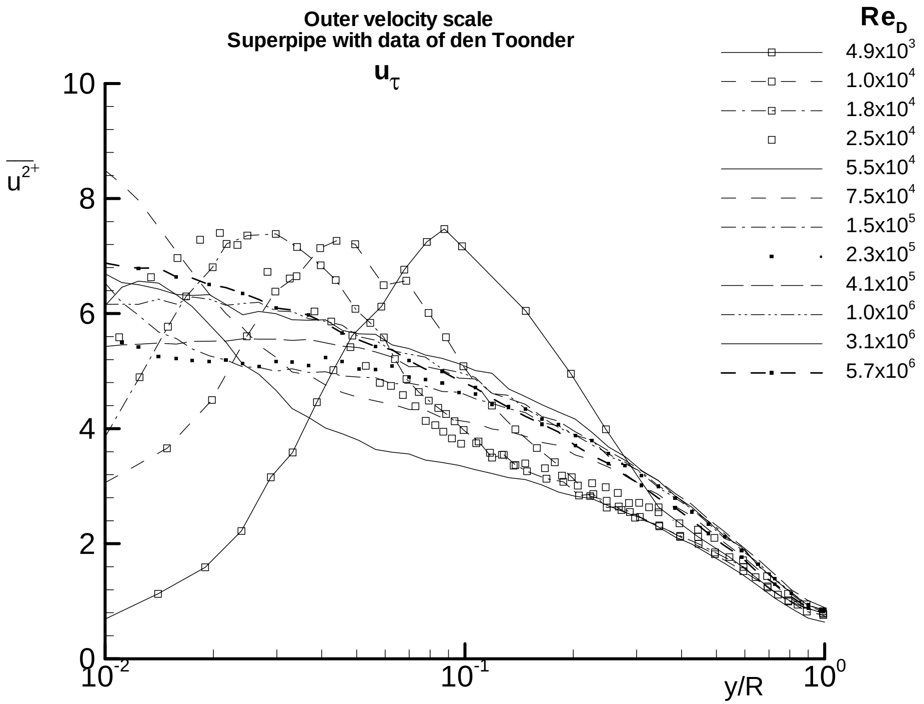
- ε and η can be estimated from local-equilibrium approximation and the log law: y/R=0.096
- No specific requirement for local isotropy:

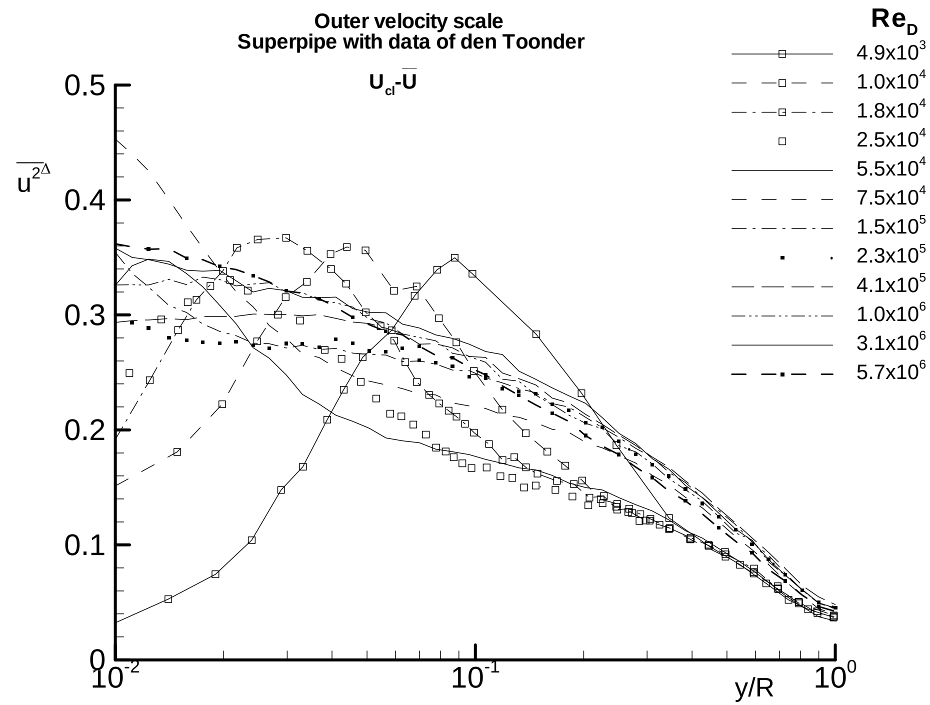
Re_D	R_λ
55k	105
75k	140
150k	210
230k	270
1.0m	575

Inertial subrange scaling: u_τ outer scale

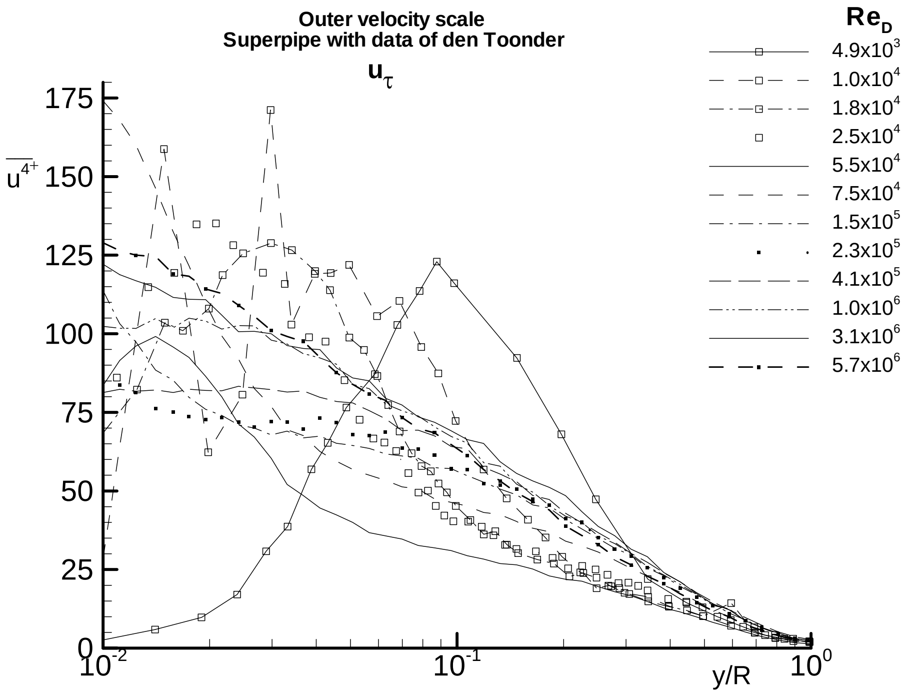


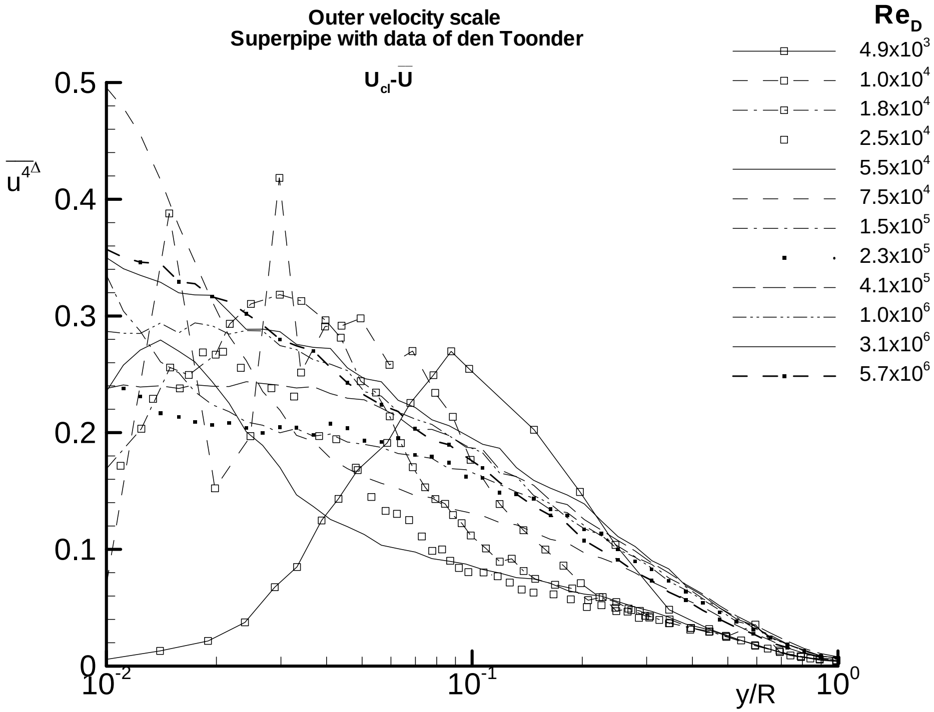
Outer velocity scale – second moment





Outer velocity scale – fourth moment



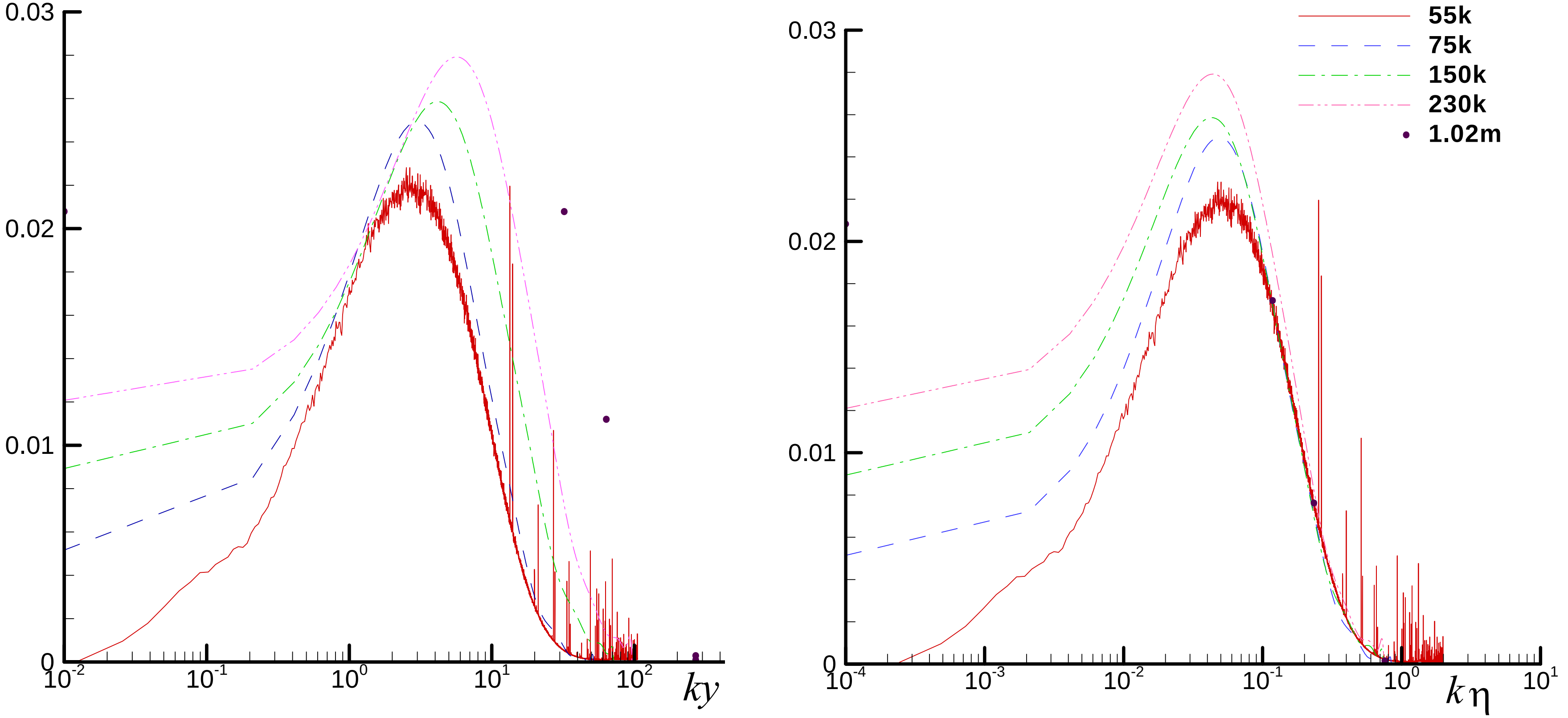


$\left(U_{cl} - \overline{U}\right)$ scaling

$$\left(\frac{\kappa}{-\overline{uv}^{+}}\right)^{\frac{2}{3}} \frac{\left(k_1 y\right)^{\frac{5}{3}}}{\left(U_{cl} - \overline{U}\right)^{2}} \phi_{11}\left(k_1 y\right) = \frac{C}{\xi^{2}} = \frac{\left(k_1 \eta\right)^{\frac{5}{3}}}{\xi^{2} \upsilon_{\varepsilon}^{2}} \phi_{11}\left(k_1 \eta\right)$$

$$4.56 \geq \xi = \frac{\left(U_{cl} - \overline{U}\right)}{u_{\tau}} \geq 4.28$$

Inertial subrange scaling: $\left(U_{cl} - \overline{U}\right)$ outer scale



Conclusions - I

- Statistics in boundary layers at short fetch and high velocity will not be the same as those at long fetch and low velocity.
- $\overline{u^2}^+$ and $\phi_{11}(k_1)$ in pipes and boundary layers at the same R^+ are not the same.
- "fully-developed pipe flow" does not appear to be a universal condition.
- But, self-similarity does lead to universal properties (log law, inertial subrange?), but R^+=constant does not.

Conclusions - II

- Inner/outer interaction dominates: “top-down” influence increases with increasing R^+, and decreasing y/R.
- Mesolayer $\sim \left(R^+\right)^{\frac{1}{2}}$ determines lower limit to log region.
- $\left(U_{cl} - \overline{U}\right)$ is a better velocity scale for $\mathrm{Re}_D \geq 75 \times 10^3$
- The pressure velocity scale $u_p^+ = \left(\frac{R^+}{2}\right)^{-\frac{1}{3}}$ is only a second-order correction:
  - at $R^+ = 5000$, $u_p^+ = 7\%$.

Conclusions III

- In local-equilibrium region, self-similar inertial subrange appears above $R_\lambda \approx 500$
- Departures from self-similarity: retain -5/3 scaling, but relax condition $\varepsilon =$ constant (Lumley 1964):

$$\chi = \frac{\varepsilon/k}{\partial T/\partial k} \sim \frac{\left[\varepsilon/k\right]^{\frac{1}{3}}}{\left[kE(k)\right]^{\frac{1}{2}}}$$

where $T(k) = E(k)^{\frac{3}{2}} k^{\frac{5}{2}}$.

- Then, if $T(k) = \varepsilon,\ \chi = 1$
- Need to look in outer region: larger spatial transport, but R_λ larger.