Linearized model of the Yardstik aircraft

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1 Trim conditions

The trim condition used for getting the linearized model for the Yardstik from the nonlinear model is as follows:

1. INPUTS:
   δ_e = -0.2836
   δ_r = -0.0066
   δ_T = 0.2581

2. STATES:
   u = 9.00 m/s
   v = -0.03 m/s
   w = 0.16 m/s
   p = -0.00 deg/s
   q = 0.00 deg/s
   r = -0.00 deg/s
   ϕ = -0.16 deg
   θ = 0.99 deg
   ψ = 3.96 deg
   h = 50.00 m
   Ω = 2907 RPM

3. OUTPUTS:
   V_a = 9.00 m/s
   β = -0.22 deg
   α = 0.99 deg
   ϕ = -0.16 deg
   θ = 0.99 deg
   ψ = 3.96 deg

2 Longitudinal model

The longitudinal dynamics extracted from the decoupled linearized model is as follows:
State vector: x = $[u\ w\ q\ \theta\ h\ \Omega]^T$
Input vector: u = $[\delta_e\ \delta_T]^T$
Output vector: y = $[V_a\ \alpha\ q\ \theta\ h]^T$
The state matrix A and the control matrix B are given by:

$$A_{lon} = \begin{bmatrix} -0.8989 & 0.5473 & -0.2978 & -9.7809 & -0.0000 & 0.0274 \\ -1.9231 & -14.4997 & 5.1014 & -0.1687 & 0.0009 & 0 \\ 0.1198 & -6.9521 & -39.4935 & 0 & -0.0000 & 0 \\ 0 & 0 & 1.0000 & 0 & 0 & 0 \\ 0.0172 & -0.9998 & 0 & 9.0036 & 0 & 0 \\ 80.6358 & 1.3896 & 0 & 0 & 0.0354 & -6.0023 \end{bmatrix}$$

$$B_{lon} = \begin{bmatrix} 0.5 & 0 \\ -5.0 & 0 \\ -20.8 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1422.1 \end{bmatrix}$$

2.1 Short period and long period modes

From the linearized longitudinal model, we extract the u, w, q and θ states to compute the short period and long period modes.

2.1.1 Short period mode

The short period modes are given by:

1. Eigenvalue: -37.9840, Time constant = 0.0263 s
2. Eigenvalue: -15.9517, Time constant = 0.0627 s

2.1.2 Phugoid mode

The long period modes are given by:

1. Eigenvalue: -0.4780 + 0.1166i
2. Eigenvalue: -0.4780 - 0.1166i
   and this will give:
   (a) $\zeta = 0.9715$
   (b) $\omega_n = 0.4920$ rad/s

3 Lat/Directional model

The lat/directional dynamics extracted from the decoupled linearized model is as follows:
State vector: x = $[v\ \ p\ \ r\ \ \phi\ \ \psi]^T$
Input vector: u = δ_r
Output vector: y = $[\beta\ \ p\ \ r\ \ \phi\ \ \psi]^T$
The state matrix A and the control matrix B are given by:

$$A_{lat} = \begin{bmatrix} -1.1976 & 0.1551 & -9.0023 & 9.7809 & 0 \\ -37.9670 & -27.8060 & 7.4336 & 0 & 0 \\ 7.4503 & -4.0041 & -1.4873 & 0 & 0 \\ 0 & 1.0000 & 0.0172 & 0.0000 & 0 \\ 0 & 0 & 1.0001 & 0.0000 & 0 \end{bmatrix}$$

$$B_{lat} = \begin{bmatrix} 2.1372 \\ 12.6576 \\ -54.7350 \\ 0 \\ 0 \end{bmatrix}$$

3.1 Dutch roll, roll and spiral modes

3.1.1 Dutch roll mode

The dutch roll mode is given by eigenvalue at -0.8890 ± 11.3009i and this gives:

1. $\zeta = 0.0784$
2. $\omega_n = 11.3358$ rad/s

3.1.2 Roll mode

The roll mode is given by eigenvalue at -28.7265, which gives a time constant of 0.0348 s.

3.1.3 Spiral mode

The spiral mode is given by eigenvalue at 0.0135(unstable), which gives a time constant of 73.8613 s.

3.2 Data from other small aircraft for comparison

Data from [1] for small UAV is presented below for comparison:

Airplanes	Span (m)	Mean Geometric Chord (m)	Mass (kg)	Cruise Velocity (m/s)	Average Wing Sweep (deg)
Modified Zagi-400 (Flying Wing)	1.21	0.25	0.65	13	26
StablEyes (BYU Captsone 2004)	0.61	0.15	0.47	15	8
Procerus Prototype (Flying Wing)	0.60	0.23	0.57	16	43

Figure 1: Data for small UAV (Data source [1])

	Zagi 400	**StablEyes**	**Procerus**
$\omega_{n,\ sp}$ (rad/s)	12.4	14.9	16.6
ζ_{sp}	0.68	0.99	0.30
$\omega_{n,\ ph}$ (rad/s)	1.1	0.94	0.81
ζ_{ph}	0.05	0.16	0.05
$\omega_{n,\ d}$ (rad/s)	5.7	9.0	10.3
ζ_{d}	0.15	0.16	0.10
T_r (s)	0.10	0.06	0.09
T_s (s)	-0.15	-0.79	-0.11

Figure 2: Data for small UAV (Data source [1])

References

[1] Tyler M. Foster, ***DYNAMIC STABILITY AND HANDLING QUALITIES OF SMALL UNMANNED-AERIAL-VEHICLES***, Master of Science thesis, Brigham Young University, 2005.