Determination of moment of inertia of Yardstik aircraft

23rd Jan 2007 by Paw Yew Chai

The moment of inertia of the Yardstik aircraft is determined using the method of a compound pendulum [1].

1. Location of center of gravity

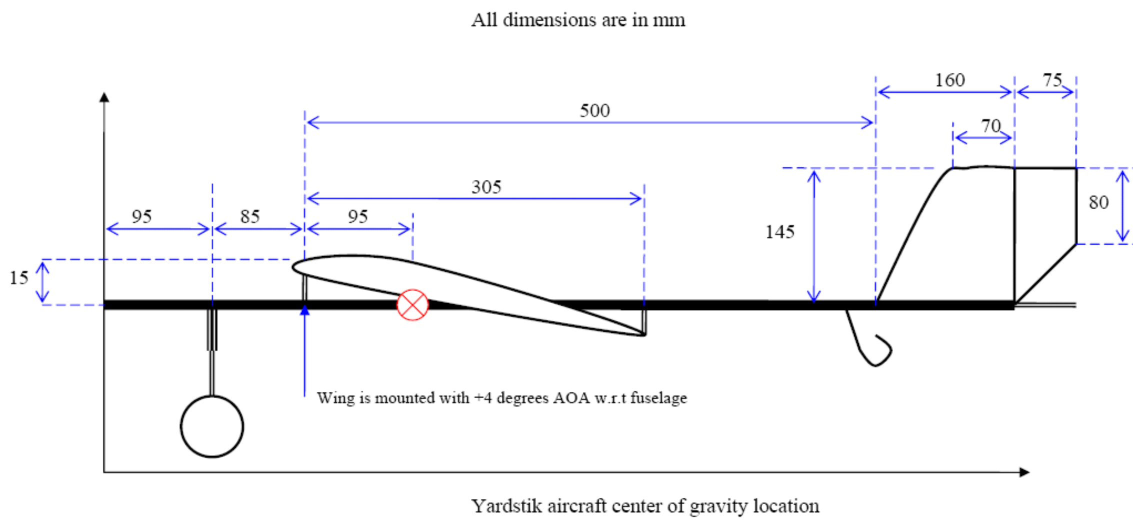

Figure 1 Center of gravity location of Yardstik aircraft

The total weight of the fully instrumented plane is 0.56kg.

2. Theory for the moment of inertia determination using the compound pendulum method

For compound pendulum as shown in figure 2, we can find the moment of inertia using the angular frequency of small amplitude oscillations of a compound pendulum given by:

$$\omega_n = \sqrt{\frac{MgL}{I_{xx} + ML^2}}$$

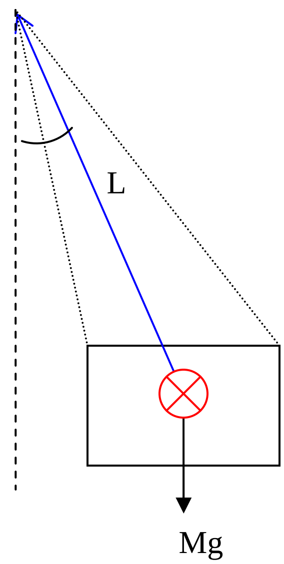

Figure 2 Compound pendulum

Therefore, the moment of inertia of the object is given by:

$$I_{xx} = \frac{MgL}{\omega_n^{\,2}} - ML^2$$

Since L, g and M are known, we have to determine the natural frequency ω_n.

For a second order equation of the pendulum, the relationship between the damped frequency (ω_d), natural frequency (ω_n) and the undamped frequency (ω) is given by:

$$\omega_n = \sqrt{\omega_d^{\,2} + \omega^2}$$

Hence, we have to find the damped frequency and undamped frequency.

Damped frequency

The damped frequency is given by the average time of a period of oscillation for the pendulum motion.

$$\omega_d = \frac{2\pi}{T_p}$$

Undamped frequency

The oscillation for the pendulum motion is given by the exponent equation $e^{-\omega t}$ which can be written as $e^{-t/\tau}$, where τ is the time constant which is the equal to $1/\omega$. For the oscillation motion to die out (in which the exponent equation becomes zero), we can approximate $t = 5\tau$. So, we will take the time for the oscillation to die down (denoted by T_f) and we can estimate the undamped natural frequency using $\omega = 5/T_f$.

3. Experiment

Figure 3. Pendulum swing to determine I_{zz}

Figure 4. Pendulum swing to determine I_{xx} and I_{yy}

Figure 3 and 4 shows the setup for the pendulum swing. In each of the swing, the time for 10 oscillation is taken and the time for the oscillation to die out is taken as well.

4. Result

Izz

Test	10 oscillation (s)	T-final (s)
1	22.18	115
2	22.22	116
3	22.1	114
Average	22.16666667	115

Period	2.216666667

Iyy

Test	10 oscillation (s)	T-final (s)
1	20	50
2	20.3	61
3	21	52
Average	20.43333333	54.333333

Period	2.043333333

Ixx

Test	10 oscillation (s)	T-final (s)
1	19.91	117
2	19.78	110
3	19.8	98
Average	19.83	108.33333

Period	1.983

Using the formulae from the theory, we have the result of the moment of inertia as:

$$I_{xx} = 0.034106513 \text{ kgm}^2$$
$$I_{yy} = 0.064500417 \text{ kgm}^2$$
$$I_{zz} = 0.045528878 \text{ kgm}^2$$

References:

[1] Nidal M.Jodeh, Paul A. Blue and Athon A Waldron, "**Development of Small Unmanned Aerial Vehicle Research Platform: Modeling and Simulating with Flight Test Validation**", AIAA, Aug 2006.